# Unit 4

# Ratios and percentages



# **Unit Competencies**

- Use the ratios to safely express and resolve environmental situations.
- Solve problems in everyday life with interest, using the calculation of quantities corresponding to different percentages.

# Sequence and Scope



Quantity per unit

Unit Plan

Lesson	Class	Title
	1	Comparison between quantities: number of times
	2	Calculation of quantity to compare
	3	Calculating the base quantity
1	4	Ratio and ratio value
Ratios	5	Ratio between heterogeneous quantities
	6	Antecedent and consequent
	7	Consequent calculation
	8	Practice what you learned

	1	Percent or percentage
	2	Relationship between ratios and percentages
	3	Percentages greater than 100%
	4	Calculating the antecedent using percentages less than 100%
	5	Calculating the antecedent using percentages greater than 100%
2	6	Calculating prices with VAT
Percentages	7	Calculating prices and discounts
	8	Calculating the consequent using percentages
	9	Calculating percentage and the consequent
	10	Calculating the consequent using percentages less than 100%
	11	Practice what you learned
	12	Practice what you learned
	1	Unit 4 assessment





# Lesson 1

## **Ratios (8 classes)**

This lesson introduces the ratio concept using the number of times studied in fourth and fifth grade. In the first class, a review is to refresh how to calculate the number of times and visualize that it can be a natural or decimal number (greater or less than 1); while in the following two classes, it is remembered how to calculate the quantity to be compared and the base quantity, respectively.

Up to class 1.4, the concept of ratio and value ratio is formally defined. Previously stated as, the number of times is a comparison between quantities, through their quotient between); the second is directly related to the number of times when the quantities being compared have the same unit (cm, km, hours, days, dollars, etc.). In addition, it is necessary to express the value of a ratio as a fraction when the quotient turns out to be an infinite decimal number.

In the following classes, we work on situations where the quantities to be compared are in different types of units, interpreting the ratio value as quantity per unit. The terms antecedent and consequent are also introduced. Students must identify them in a ratio and determine the unknown quantity in the problems addressed in class, since they will continue to be used in both Lesson 2 and Unit 5.

Equivalent ratios will not be part of this lesson as this topic is directly related to proportions, content to be worked on in the next unit. However, to calculate the ratio value, students can simplify the calculations (if writing the value of a ratio as a fraction).

# Lesson 2

## Percentages (12 classes)

The lesson begins calculating the percentage as the ratio value multiplied by 100 and its respective interpretation: m % means m of 100. The relationship between the ratio value and the percentage associated with it; is also done by using the double number line to obtain either one. This resource also works on percentages greater than 100%, whose meaning arises from situations where the ratio value exceeds 1 (studied in lesson 1)

Throughout the lesson, situations are solved where the unknown quantity corresponds either to the antecedent or the consequent of the ratio associated with a percentage; the percentage may be less than or greater than 100% and be given explicitly or not. It is important to note that the percentage approach in this lesson does not involve the concepts of proportion, as is traditionally worked (using the so-called "rule of three"), but is directly related to a ratio and its value.



# 1.1 Comparison between quantities: number of times

#### Analyze

Look at the bars and the number line.



a. How many times is the length of the pink ribbon relative to the size of the green ribbon?

b. How many times is the length of the orange ribbon relative to the length of the green ribbon?

c. How many times is the length of the blue ribbon compared to the size of the green Ribbon?

**Solution** a. **PS:** 15 ÷ 5 15 ÷ 5 = 3

The length of the pink ribbon is three times the length of the green ribbon. **A:** Three times.

In the diagram, the number of times the pink ribbon is relative to the green ribbon is represented by a. So, a is equal to three.



#### b. **PS:** 12 ÷ 5



The length of the orange ribbon is 2.4 times the length of the green ribbon.

#### A: 2.4 times.

In the diagram, the number of times the orange ribbon is relative to the green ribbon is represented by b. So, b is equal to 2.4.



Unit 4

c. **PS:** 4 ÷ 5

4 ÷ 5 = 0.8

The length of the blue ribbon is 0.8 times the length of the green ribbon.

A: 0.8 times.

In the chart, c is equal to 0.8.



#### Understanding

A number of times is also a comparison between quantities through the quotient between them; it can be a natural number, a decimal number, or a fraction.

The number of times one quantity is calculated over another:

#### Number of times = Quantity to compare ÷ Base quantity



1.1 Calculates the number of times that one quantity is relative to another.

**Objective:** Remember the concepts: number of times, quantity to compare, base quantity, and to calculate the number of times from the other two.

**Key Points:** The concepts "number of times," "quantity to compare," and "base quantity" have been worked on since fourth grade; and set the standard to introduce the concept of "ratio." This class recalls the procedure to calculate the number of times; if the quantity to compare and the base quantity are known.

In ①, the number of times is verified to show it will not always result in a natural number, but in a decimal number (i.e., cases b. and c.), And therefore in a fraction. In ②, the situation addressed in 1. is similar to the one used in Analyze section, since lengths are compared; In contrast, in problems 2. and 3., the quantities to be compared are given in other units (years in case 2. and number of goals in 3.), but they are always homogeneous.

**Methodological advice:** In Analyze, it is essential to use the tape graph to visualize the relationship between the quantity to be compared and the base quantity and to intuitively determine if the number of times will be a number greater or less than 1. Emphasis should also be placed on the expression "in respect to "used in all problems to identify the quantity to compare and the base quantity (not necessarily the first will be greater than the second).

**Problem Solving:** 2. **PS:** 42 ÷ 10 1. **PS:** 8 ÷ 2 3. **PS:** 9 ÷ 12  $8 \div 2 = 4$  $42 \div 10 = 4.2$  $9 \div 12 = 0.75$ A: 4 times. A: 4.2 times. A: 0.75 times. Date: **Class:** 1.1 15 m c. **PS:** 4 ÷ 5 (A)Pink 12 m  $4 \div 5 = 0.8$ Orange **A:** 0.8 times (c = 0.8). Blue Green (R)a (times) **c** 1 b 1. **PS:** 8 ÷ 2 2. **PS:** 42 ÷ 10 a. How many times is the length of the pink ribbon 8 ÷ 2 = 4  $42 \div 10 = 4.2$ compared to the length of the green ribbon? A: 4.2 times. **A:** 4 times. b. And the orange ribbon with respect to the green ribbon? c. And the blue ribbon compared to the green one ribbon? 3. **PS:** 9 ÷ 12  $9 \div 12 = 0.75$ S a. **PS:** 15 ÷ 5 b. **PS**: 12 ÷ 5 A: 0.75 times.  $12 \div 5 = 2.4$  $15 \div 5 = 3$ **A:** 2.4 times (**b** = 2.4). **A:** 3 times (a = 3). Homework: Page 70

Materials: Poster with the graph of Analyze or colored ribbons to represent it on the blackboard.



## 1.2 Calculation of the quantity to compare

#### Analyze

Charles and Mary went for a run together. Charles completed 4 km, while Mary covered 2.5 times what Charles did. How many kilometers did Mary run?





#### Solution



Do the multiplication to find the number of kilometers that Mary traveled:

$$4 \times 2.5 = 10$$

Therefore, Mary traveled 10 km. A:10 km

**PS:** 4 × 2.5

In the figure, the number of kilometers traveled by Mary is represented by a. Thus, a = 10:



I can also verify that, by dividing the quantity to be compared (10 km) by the base quantity (4 km), the number of times (2.5) is obtained.

#### Understanding

When the base quantity and the number of times are known, then the quantity to compare is calculated as follows: Quantity to compare = Base quantity × Number of times

#### Solve

68

- 1. Joseph weighs 45 kg, while Martha weighs 0.8 times what Joseph weighs. How much does Martha weigh?
- Remember that the base quantity may be greater than the quantity to be compared.
- 2. A red tank has a total capacity of 300 liters, while a yellow tank has 1.75 times the capacity of the red tank. What is the capacity of the yellow tank?



3. Carmen and Beatrice competed in the long jump. Carmen jumped 2m, and Beatrice jumped 0.75 times what Carmen jumped. How many meters did Beatrice jump?

1.2 Find the amount to compare, multiplying the base quantity by the number of times.

**Objective:** Remember how the quantity to compare is calculated when the base quantity and the number of times are known.

**Key Points:** This class remembers the formula (studied in fifth grade) to calculate the amount to compare, from the base amount and the number of times. In **1**, the graph helps visualize the relationship between the quantities and determine the unknown quantity. In **2**, the emphasis should be on checking the solution and verifying if the answer obtained is correct. In the problems proposed in **3**, use the Understand section's information to calculate the quantity to be compared. Instruct the students they can also check if their answers are correct as per the solution of the initial problem.

**Methodological advice:** For problems in **3**, students can make a tape graph if this helps them visualize the relationship between the quantities and which of them is unknown. It is important that next to the graphic resource, the solution is also found using the algorithm.

Materials: Poster with the graph of the Analyze or colored ribbons to represent it on the board.

#### **Problem Solving:**

1. <b>PS:</b> 45 × 0.8	2. <b>PS:</b> 300 × 1.75	3. <b>PS:</b> 2 × 0.75
45 × 0.8 = 36	300 × 1.75 = 525	2 × 0.75 = 1.5
<b>A:</b> 36 kg	<b>A:</b> 525 liters.	<b>A:</b> 1.5 m





# 1.3 Calculating the base quantity

#### Analyze

On one day, Carmen toured 1.5 times what Anthony did. If Carmen traveled 9 km, how many kilometers did Anthony travel?





#### Solution

FOO

I divide to find the number of kilometers Anthony traveled:

9÷1.5=6

Then, Anthony traveled 6 km.

PS: 9 ÷ 1.5

**A:**6 km

As per the graph, the number of kilometers traveled by Anthony is represented by b. Thus, b = 6:



## 2

I can also see that by dividing the quantity to be compared (9 km) by the base amount (6 km), you get the number of times (1.5).

#### Understanding

When the quantity to be compared and the number of times is known, then the base quantity calculated: Base quantity = Quantity to compare ÷ Number of times

#### Solve

1. In a swimming class, Martha swam three times what Ana swam. If Martha swam 1.5 km, how many kilometers did Ana swim?

2. In a classroom, the number of boys is 1.4 times the number of girls. If there are 21 boys, how many girls are there in the room?

3. In a rectangle, the length is 3.5 times the width. If the length is 42 cm, how much is the width?

4. At a parents' meeting, the number of men was 0.4 times the number of women. If 32 men attended, how many women attended?



1.3 Find the base quantity, dividing the amount to compare by the number of times.

**Objective:** Remember how the base quantity is calculated when the quantity to be compared and the number of times is known.

**Key Points:** This class remembers the formula (studied in fifth grade) to calculate the base amount from the amount to be compared and the number of times. As in the previous classes, in 1, the relationship between the quantities is visualized with the help of the graph to determine the unknown quantity. Again, students can determine if the solution to the initial problem is correct by dividing the quantity to be compared by the base quantity and check if the number of times is obtained, as mentioned in 2. For the problems in 3, use the information from Understand section to calculate the base quantity.

**Methodological advice:** For problems in **③**, students can make a tape graph to visualize the relationship between the quantities and which is unknown. It is important that next to the graphic resource, the solution is also found using the algorithm.

Materials: Poster with the graph of the Analyze or colored ribbons to represent it on the board.



Jnit 4



## 1.4 Ratio and Ratio value

#### Analyze



- 1. Joseph saved \$ 8 and Julia \$ 3. She writes the ratio of the amount saved by Joseph and the amount saved by herself and calculates the ratio value. What is the interpretation of this result, using the number of times?
  - 2. A tank has a total capacity of 2 liters, and a pot, a full capacity of 7 liters. Write the ratio between the tank's capacity and the pot's capacity, then find the ratio value. What is the interpretation of this result, using the number of times?



1.4 Find the ratio of two quantities with the result of a fraction.

**Objective:** Define ratio and ratio value, associating it with the number of times, and write the ratio value using fractions.

**Key Points:** In this class, the concept of reason is introduced. The initial problem in **1** resembles the one worked in class 1.1; on this occasion, an infinite decimal number is obtained when calculating the quotient. Therefore, it is convenient to write it as a fraction, as shown in **2**. With the commentary of the parakeet in **3**, the number of times is related to the value of the ratio. If the quantities being compared are in the same unit, the ratio value is the number of times. This information will help solve the problems in **4**, since students must not only write the ratio using the notation *a* : *b* and calculate the value of the ratio as a fraction, but also make sense of these concepts. The values of the ratios worked in this class turn out to be irreducible fractions.

Materials: Poster with the graph of the Analyze or colored ribbons to represent it on the board.

#### **Problem Solving:**

1.Ratio → 8:3

Ratio value  $\longrightarrow 8 \div 3 = \frac{8}{3}$  (It is written this way since  $8 \div 3 = 2.66666...$ )

en this Ratio value  $\longrightarrow 2 \div 7 = \frac{2}{7}$  (it is written this way since  $2 \div 7 = 0.28571428...$ )

2. Ratio -> 2 : 7

This means that the money saved by Joseph is  $\frac{8}{3}$  times the money saved by Julia.

This means that the capacity of the tank is  $\frac{2}{7}$  times the capacity of the pot.







1.5 Find the ratio and the ratio value between two heterogeneous quantities.

**Objective:** Interpret the ratio and the ratio value between two heterogeneous quantities as quantity per unit.

**Key Points:** In previous classes, the quantity units used have been equal (meters, kilometers, years, kilograms, etc.), and the ratio has been interpreted as "number of times." In this class, the ratio is related to the concept "quantity per unit" studied in fifth grade (unit 6); thus, the ratio value is interpreted as the number of elements in each unit of measure.

In 1, speed (distance traveled ÷ time) is used to calculate ratios with heterogeneous quantities and determine the fastest, as seen in fifth-grade unit 6. The example presented in 2 Relates the ratio value with the number of meters traveled in one second and will help students draw similar conclusions about the problems in 3. Furthermore, although in problem 2. Both quantities are in the same units (one could speak in general of the number of students), the interpretation using the quantity per unit and not the number of times; will be used later on with ratios and percentages.

#### **Problem Solving:**

- 1. a. Ratio  $\longrightarrow$  298 : 4 Ratio value  $\longrightarrow$  298 ÷ 4 = 74.5 The ratio value can also be written as a fraction.
  - b. A: The car travels 74.5 km in 1 hour.
- 2. a. Ratio  $\longrightarrow$  20 : 10 Ratio value  $\longrightarrow$  20 ÷ 10 = 2 The ratio value can also be written as a fraction.

A: There are 2 girls for every boy.







#### Analyze

In an old lemonade recipe, the number of lemons and cups of water are in a ratio of 3 : 2. If 6 cups of water are used, how many lemons should be used?



#### Solution



The ratio values is  $\frac{3}{2}$  (or 1.5). Then, for each cup of water you need  $\frac{3}{2}$  lemons. And, for 6 cups of water,  $6 \times \frac{3}{2}$  lemons will be used.

$$\int_{0}^{3} \times \frac{3}{\chi} = 3 \times 3 = 9$$

The 3:2 ratio indicates that two cups of water are used for every three lemons.



- · For six lemons, four cups of water are used (both portions double).
- · For nine lemons, six cups of water are used (both portions triple).

A: 9 lemons.

## Understanding

In a ratio *a* : *b*, quantity *a* is called the antecedent, and quantity b is called the consequent. In addition, it is true that:

antecedent = consequent × ratio value

A: 9 lemons.

Note that calculating the antecedent is similar to calculating the quantity to compare: Quantity to =

Base Number of × compare quantity times

Replace base quantity for consequent and number of times for ratio value.

#### Solve

- 1. Twenty tickets are placed in a bag for a raffle. The number of winning tickets and the total number of tickets in the bag has a ratio of 1:4. How many winning papers are there?
- 2. Anthony practices basketball. One day, he made 15 throws. If the ratio of the shots made to the total number of shots was 4 : 5, how many shots did he make?



3. A restaurant estimated the ratio for the number of people served in one night to the profit they made was 1:10. If the restaurant's profit was \$300 that night, how many people did they serve?

174

1.6 Calculate the antecedent from the ratio value and the consequent.

**Objective:** Identify the antecedent and consequent in a ratio, and find the antecedent using the consequent and the ratio value.

**Key Points:** In this class, the terms antecedent and consequent are introduced. They will continue to be used in the remainder of this unit and unit 5 on proportionality. Unlike the problems previously completed, in **1**, the reason and the consequent are given; The student must remember the interpretation of the ration value as quantity per unit and solve it similar to Joseph in **2**.

In (3), the armadillo's comment shows the relationship between the calculation of the antecedent and the quantity to compare seen in class 1.2; This information is useful to complete to solve ccccc and 2 directly. In (4), since the quantities compared in both cases are in the same units. In 3. The quantities have different units (number of people and profit in dollars).

#### **Problem Solving:**

1. The number of awarded tickets is the antecedent of the ratio 1:4 Consequent  $\rightarrow 20$ Ratio value  $\rightarrow \frac{1}{4}$ 

- Antecedent =  $20 \times \frac{1}{4} = 5$
- A: 5 Awarded tickets

 The number of successful shots is the antecedent of the ratio 4: 5 Consequent → 15

Ratio value  $\rightarrow \frac{4}{5}$ Antecedent  $= \frac{3}{15} \times \frac{4}{5} = 12$ 

A: 12 successful shots

3. The number of people served is the antecedent of the ratio of 1: 10

Consequent 
$$\rightarrow$$
 300  
Ratio value  $\rightarrow \frac{1}{10}$   
Antecedent  $= 3\frac{30}{400} \times$ 

edent = 
$$300 \times \frac{1}{10}$$
  
= 30 1

A: 30 people.

Date:Class: 1.6(A) In a recipe, the number of lemons and cups of water  
are found in a ratio of 3: 2. If 6 cups of water are  
needed, how many lemons should be used?(B) 1. The number of awarded tickets is the  
antecedent of the ratio 1: 4  
Consequent 
$$\rightarrow 20$$
  
Ratio value  $\rightarrow \frac{1}{4}$ (S) The ratio value is  $\frac{3}{2}$ . For every cup of water  $\frac{3}{2}$  of  
lemons are needed. For 6 cups of water will be used  
 $6 \times \frac{3}{2}$  lemons,  
 $\frac{3}{5} \times \frac{3}{\frac{7}{1}} = 3 \times 3 = 9$   
A: 9 lemons(B) 1. The number of awarded tickets is the  
antecedent of the ratio 1: 4  
Consequent  $\rightarrow 20$   
Ratio value  $\rightarrow \frac{1}{4}$   
Antecedent  $= \frac{5}{20} \times \frac{1}{\frac{7}{1}} = 5$   
A: 5 awarded tickets(S) The ratio value is  $\frac{3}{2}$ . For every cup of water  $\frac{3}{2}$  of  
lemons are needed. For 6 cups of water will be used  
 $6 \times \frac{3}{2}$  lemons,  
 $\frac{3}{5} \times \frac{3}{\frac{7}{1}} = 3 \times 3 = 9$   
A: 9 lemons(Class: 1.6  
(R) 1. The number of awarded tickets  
 $2.0 \times \frac{1}{4} = 5$   
(R)  $2. \times 12$  successful shots  
 $3. A: 30$  people  
Homework: Page 75



## **1.7 Consecuent calculation**

#### Analyze

ก

The length and width of a rectangle are at a ratio of 7 : 4. If the length measures 14 cm, how much does the width measure?



#### Solution



The ratio value is  $\frac{7}{4}$  (or 1.75), so the length is  $\frac{7}{4}$  times the width. Then, Mario divides the length by  $\frac{7}{4}$  and the result will be the width:

$$14 \div \frac{7}{4} = 1\frac{2}{4} \times \frac{4}{7} = 2 \times 4 = 8$$

The ratio 7 : 4 indicates that, for every 7 cm of the length, there is 4 cm of width. Then:



• For 14 cm in length, there is 8 cm in width (quantities doubled).



A: 8 cm



1.7 Calculate the consequent from the ratio value and the antecedent

**Objective:** Identify the antecedent and consequent in a ratio, and find the consequent using the antecedent and the ratio value.

**Key Points:** Unlike the previous class, in the case presented in **1**, the ratio and the antecedent are given; the student must remember the interpretation of the ratio value as quantity per unit and solve it similar to Mario's in **2**.

In ③, the armadillo's comment shows the relationship between the calculation of the consequent and the base quantity seen in class 1.3. In item 1. From ④, the formula must be used directly, while in 2. It is necessary to identify the antecedent and calculate the ratio value.

#### **Problem Solving:**

1. a. Consequent = $1 \div \frac{1}{2} = 2$	b. Consequent = $6 \div \frac{3}{4} = \cancel{6} \times \frac{4}{\cancel{3}} = 8$
<b>A:</b> 2	<b>A:</b> 8
c. Consequent = $10 \div 2 = 5$	d. Consequent = $12 \div \frac{4}{3} = \frac{3}{12} \times \frac{3}{4} = 9$
<b>A:</b> 5	<b>A:</b> 9

2. The amount of milliliters of red paint is the consequent of the ratio 4 : 5

Antecedent  $\rightarrow 12$ Ratio value  $\rightarrow \frac{4}{5}$ 

Antecedent = 
$$12 \div \frac{4}{5} = 12 \times \frac{5}{4} = 15$$

**A:** 15 ml





## 1.8 Practice what you learned

1. Write the length ratio between the green and red ribbons. Then calculate the value of the ratio:



Solve the following problems:

- In the Salvadoran diet, two tortillas provide 31 g of carbohydrates, 1 g of fat, 3 g of protein, and 150 calories.
  - a. Write the ratios, and calculate the ratio value between the number of carbohydrates, the number of tortillas, the amount of fat, and the number of tortillas
  - b. How do you interpret the above results?
- 3. Anthony saved \$ 15, and then he spent \$ 5. What is the ratio and ratio value between the money spent and money saved? How do you interpret this result?
- 4. The ratio of the length to the width of a rectangle is 3 : 2. If the width is 10 cm, how long is the length?
- 5. On a bus, the ratio between the number of seats occupied and unoccupied is 6 : 5; If 24 seats are occupied, how many seats are vacant?
- 6. The ratio between the number of calories a person burns and the time (in minutes) they spend running is 10 : 1. If a person burned 150 calories, how many minutes did they spend running?
- 7. A soccer team determined that the ratio between the total games in a championship and the games they won was 5 : 3. If they won 6 games, how many games were played during the tournament?

1.8 Solve ratio problems.

#### **Problem Solving:**

- - →  $3 \div 2 = 1.5 \left( \text{or} \frac{3}{2} \right)$ Ratio value
  - c. Ratio 12:4

tortillas

Ratio value

→ 12 ÷ 4 = 3 Ratio value

→ 31:2

- b. Ratio → 2 : 5 Ratio value
  - $\rightarrow$  2 ÷ 5 = 0.4 (or  $\frac{2}{5}$ )
- 2. a. The ratio between the number of carbohydrates and b. On the number of carbohydrates and tortillas: 1 tortilla provides 15.5 g of carbohydrates.

Regarding the amount of fat and the number of tortillas: 1 tortilla provides 0.5 g of fat.

The ratio between the amount of fat and the number of tortillas  $\rightarrow 1:2$  $1 \div 2 = 0.5 \left( \text{or} \frac{1}{2} \right)$ Ratio value

 $\rightarrow$  31 ÷ 2 = 15.5 (or  $\frac{31}{2}$ )

3. Ratio → 5 : 15

 $\rightarrow 5 \div 15 = \frac{1}{15} = \frac{1}{3}$ Ratio value

The ratio value indicates that the money Anthony spent is one-third of what he saved.

5. Antecedent -> 24  $\rightarrow \frac{6}{5}$ Ratio value Consequent =  $24 \div \frac{6}{5} = 24 \times \frac{5}{8} = 4 \times 5 = 20$ 

A: 20 seats.

7. Consequent --- 6  $\rightarrow \frac{5}{3}$ Ratio value

Antecedent  $= \oint_{\alpha}^{2} \times \frac{5}{\beta} = 2 \times 5 = 10$ 

A: 10 games.

## Notes:

- 4. Consequent → 10  $\rightarrow \frac{3}{2}$ Ratio value Antecedent =  $1\frac{5}{20} \times \frac{3}{2} = 5 \times 3 = 15$ A: 15 cm
- 6. Antecedent → 150 → 10 Ratio value Consequent  $= 150 \div 10 = 15$

A: 15 minutes.



## 2.1 Percent or percentage

#### Analyze \_\_\_\_

The following table contains the notes of the number of goals and the number of attempts that John made in his last two football training sessions:

0

Training	Goals	Attempts
First	5	10
Second	9	12

In which training would you say John was most successful?



The ratios between the number of goals and the number of attempts for the first and second training sessions are 5:10 and 9:12, respectively. I calculate the ratio value:

2

#### First training session $5 \div 10 = 0.5$

Second training session  $9 \div 12 = 0.75$ 



During the first training session, John succeeded in half of the attempts. In the second practice, he succeeded 0.75 times the number of attemps.

A: In the second training session.

#### **Understanding**

The **percent** or **percentage** is obtained by multiplying the ratio value by 100, i.e.: **Percentage = Ratio value × 100** 

After the last digit of the number indicating percentage, the symbol "%" is written. For example, if the ratio value between the number of goals and the number of attempts (in the first training) is multiplied by 100, you get:

porcentage = 
$$0.5 \times 100 = 50$$

It is written "50%" and reads: "fifty percent ."This number indicates that 50 out of every 100 attempts are successful.

#### Solve

4

3

1. The following table contains Michael's results in the last two basketball games.

Game	Baskets	Throws
First	12	16
Second	9	15

a. Find the value of the ratio between the number of baskets and total throws attempts.

b) What percentage of baskets did you get in each game?, how is this result interpreted?

2. Joseph wrote down the results he got by playing cup and ball on Monday, Tuesday , and Wednesday:

Day	Successful	Attempts
Monday	8	20
Tuesday	10	25
Wednesday	8	16

a. Between Monday and Wednesday, which day did you get the best results? Explain using percentages.

b. Between Monday and Tuesday, which day did you get the best results? Explain using percentages.

2.1 Calculate the percentage that a quantity represents, finding the ratio value and multiplying by 100.

Objective: Introduce the concept of percent or percentage and calculate it using the ratio and the ratio value between two quantities.

**Key Points:** The case presented in **1** is similar to the work done in class 1.5. Two sets of quantities are given to compare and determine; which training session John was more successful. To solve the problem, students must identify that it is necessary to calculate the ratios between the number of goals and the number of attempts in each case (similar to how Antonio does in (2)). In (3), emphasis should be placed on the interpretation of a percentage (50% indicates 50 out of 100) since similar analyzes must be carried out in the problems presented in **4**.

**Methodological advice:** To solve the initial problem in **1**, instruct the students to express the ratio value as a decimal number; This can facilitate the value comparisons in both cases and determine which is the greater. In this lesson, the ratio interpretation as the number of times or quantity per unit will be used interchangeably; both definitions are analogous and should not represent ant difficulty for the students.

#### **Problem Solving:**

- 1. a. In the first game, the ratio is 12: 16, and its value is  $12 \div 16 = 0.75$ In the second game, the ratio is 9:15, and its value is 9 ÷ 15 = 0.6
  - b. First game:  $0.75 \times 100 = 75$ ; the percentage of baskets is 75%, that is, he makes 75 of 100 shots. Second game:  $0.6 \times 100 = 60$ ; the percentage of baskets is 60%, that is, he makes 60 of 100 shots.
- 2. a. On Monday, the ratio for the number of goals versus the number of attempts is 8:20, its value is 0.4, and its percentage is 40%. On Wednesday, the ratio is 8:16, its value is 0.5, and its percentage is 50%. So on Wednesday, he got better results because the success rate is higher.
  - b. A: He got the same results on both days, as Tuesday's is also 40%.

Date: A Numb John in	er of goals and n n two trainings:	umber of	attempts n	Class: nade by	2.1 R
	Training First Second	Goals 5 9	Attempts 10 12		<ol> <li>a. In the first game, the ratio its value is 12 ÷ 16 = 0.75 In the second game, the rat its value is 9 ÷ 15 = 0.6</li> </ol>
In which training was John most successful?			b. First game: 0.75 × 100 percentage of baskets is 3 scores 75 of 100 throws.		
Ratio → Ratio valu	5 : 10 ie 5 ÷ 10 = 0.5	Ratio Ratio	→ 9 : 12 value → 9 ÷ 12 =	0.75	Second game: 0.6 × 10 percentage of baskets is 6 scores 60 of 100 throws.
A: In the	second training.				Homework: Page 78

- is 12 : 16 and io is 9 : 15 and
  - = 75; the '5 %, that is,

0 = 60: the 50 %, that is,





2.2 Find the percentage corresponding to a certain ratio and vice versa.

**Objective:** Calculate the percentage corresponding to the ratio value and vice versa.

**Key Points:** Operations in **1** will serve to remember the procedure for multiplying a decimal by 100 (it will be used from this class when calculating the percentage associated with a ratio). To solve the initial problem in **2**, students must use the algorithm seen in class 2.1 to calculate the percentage. In **3**, the relationship between the value of a ratio and the percentage associated with it is consolidated, and how to calculate one from the other. Finally, in **4**, for the problems in 1. And 2. must apply the algorithms directly in each case. Still, in 3., the procedure is similar to the initial problem solution: the ratio value and then the percentage.

**Methodological advice:** Since the goal of ① is to remember what happens when a decimal is multiplied by 100 (the decimal point is moved two spaces to the right), students can be directly told how to solve each exercise. In the problems posed in both ② and ③, we must remember the importance of the order of quantities when writing the ratio and calculating its value; for example, if it is asked to calculate "the ratio value between the area of the court (252 m<sup>2</sup>) and the total area of the school (1,200 m<sup>2</sup>)" then the corresponding division is 252 ÷ 1,200. If you are not careful with the above, you will get a percentage greater than 100% and confuse the student.

Problem Solving: 1. a. 0.01 × 100 = 1 A: 1 %	b. 0.07 × 100 = 7 A: 7 %	c.	0.75 × 100 = 75 <b>A:</b> 75 %	d. 1 × 100 = 100 A: 100 %
2. a. 5 ÷ 100 = 0.05 A: 0.05	b. 9 ÷ 100 = 0.09 A: 0.09	c.	12 ÷ 100 = 0.12 <b>A:</b> 0.12	d. 54 ÷ 100 = 0.54 A: 0.54
3. a. Ratio value → A:0.21	- 252 ÷ 1,200 = 0.21	b.	Percentage: 0.21 × 100 A: 21 %	= 21
Date:	C	lass: 2.2	2	
Re Perform: a. $0.01 \times 100 = 1$ A In the classroom the children. What is the class? S Ratio $\rightarrow 7:20$	b. 0.2 × 100 = 20 ere are 20 students, and 7 a e percentage of children in t	nre he	<ul> <li>R 1. Find the percentag</li> <li>a. 0.01 × 100 = 1</li> <li>A: 1 %</li> <li>c. 0.75 × 100 = 75</li> <li>A: 75 %</li> </ul>	ge b. 0.07 × 100 = 7 A: 7 % d. 1 × 100 = 100 A: 100 %
Ratio value -	7 ÷ 20 = 0.35		2. Find the ratio valu	ie
Then, percenta <b>A:</b> 35% of students in	ge = 0.35 × 100 = 35 the classroom are children		a. 5 ÷ 100 = 0.05 A: 0.05 c. 12 ÷ 100 = 0.12	b. 9 ÷ 100 = 0.09 A: 0.09 d. 54 ÷ 100 = 0.54
			Homework: Page 79	<b>~</b> · 0.54

Unit 4





2.3 Calculate percentages greater than 100% in exercises and problems.

**Objective:** Relate ratios values greater than 1 with percentages greater than 100%.

**Key Points:** In lesson 1, the unit presented some situations where the antecedent was greater than the consequent. Therefore, the ratio value was greater than 1; these situations are related to percentages greater than 100%. In the initial problem in ①, students must identify that the antecedent of the ratio is the number of people served on Saturday. The restaurant's capacity is the consequent (the double number line graph is used to visualize the relationship between the quantities and estimate that the percentage will be greater than 100%). Calculate the percentage in the same way it has been done. In ②, the double number line graph is used to relate the ratio value with the percentage and consolidate their calculation: the ratio value is multiplied by 100 to obtain the percentage. The percentage is divided by 100 to get the ratio value. In exercise 1. ③ Must use the multiplication or division by 100 to complete the spaces. In 2. And 3. The percentage must be calculated applying the algorithm.

**Methodological advice:** If students have difficulty identifying the antecedent and consequent in the initial problem in **1**, the statement-question should be analyzed; As the percentage of people who attended will be calculated **concerning** the capacity of the restaurant, then the 90 people attended is the antecedent, and the capacity of 60 people that the restaurant has is the consequent.

#### **Problem Solving:**

2. Ratio  $\rightarrow$  2.5 : 2 Ratio value  $\rightarrow$  2.5 : 2 = 1.25 Percentage = 1.25 × 100 = 125

**A:** 125 %

3. Ratio  $\rightarrow$  6 : 4 Ratio value  $\rightarrow$  6 ÷ 4 = 1.5 Percentage = 1.5 × 100 = 150

**A:** 150 %





-	

78

1. Calculate:

a. 20 % of 80 liters.

b. 90 % of 120 liters.

- 2. Out of a classroom of 30 students, 80% of students passed the subject of Mathematics. How many students passed?
- 3. In a parking lot, there are 80 vehicles of which, 5% are green. How many green cars are in the parking lot?



2.4 Calculate the antecedent of a ratio when the percentage is less than 100%.

**Objective:** Find the antecedent of a ratio from the consequent and the percentage associated with the ratio value.

**Key Points:** In **1** the algorithm to calculate the antecedent from the consequent and the ratio value (antecedent = consequent × ratio value, class 1.6) and how to calculate the ratio value corresponding to a percentage (ratio value = percentage ÷ 100, class 2.2). In **2**, unlike the previous classes, the percentage that one quantity represents concerning another is provided; To solve this type of problem, it is not necessary to resort to the well-known "rule of three"; but rather relate the percentages with the calculation of ratios and identify what quantities are given (if the antecedent or the consequent). Students must solve similarly to Julia (see **3**) since it is the most general and feasible process for all cases. In **4**, students must apply the steps outlined in Understand.

**Methodological advice:** As in Lesson 1, the students might have difficulty identifying whether the quantities given in the statements correspond to the antecedent or the consequent. The following can be indicated: "when you have the percentage that represents a quantity *a* concerning another quantity *b*, then the quantity *a* is the antecedent and *b* the consequent."

#### **Problem Solving:**

- 1. a. Ratio value  $= 20 \div 100 = 0.2$ Antecedent  $= 80 \times 0.2 = 16$ A: 16 liters
- 2. Ratio value  $= 80 \div 100 = 0.8$ Antecedent  $= 30 \times 0.8 = 24$ A: 24 students

- b. Ratio value  $= 90 \div 100 = 0.9$ Antecedent  $= 120 \times 0.9 = 108$ A: 108 liters 3. Ratio value  $= 5 \div 100 = 0.05$
- 3. Ratio value  $= 5 \div 100 = 0.05$ Antecedent  $= 80 \times 0.05 = 4$ A: 4 vehicles.

Date: Class	: 2.4
Re 1. Antecedent = consequent × ratio 2. a. 35 % → 35 ÷ 100 = 0.35 b. 100 % → 100 ÷ 100 = 1	$ \begin{array}{ c c c c } \hline R & 1. a. Ratio value & = 20 \div 100 = 0.2 \\ & Antecedent & = 80 \times 0.2 = 16 \\ \hline A: 16 \text{ liters.} \end{array} $
A How much is 35% of 200 ml? Represents the amount corresponding to 35% as $a$ .	b. Ratio value = 90 ÷ 100 = 0.9 Antecedent = 120 × 0.9 = 108 A: 108 liters.
S Ratio value = $35 \div 100 = 0.35$ This value corresponds to the ratio <i>a</i> : 200; then: $a = 200 \times 0.35 = 70$	2. <b>A:</b> 24 students 3. <b>A:</b> 4 vehicles
<b>A:</b> 70 ml	Homework: Page 81



# 2.5 Calculating the antecedent using percentages greater than 100%

#### Analyze

Martha's parents must pay \$250 a monthly payment for their home mortgage. Also, they have to pay a 4% fixed interest. How much should you pay each month?

#### Solution .....

A 100% of payment is \$250; "4% interest rate per month" indicates that 4% of \$250 is added. So, I have to calculate each monthly payment, including the interest.

1) The total percentage is: 100% + 4% = 104%

I use the previous lesson information:

2 Calculate the ratio value (percentage ÷ 100): 104 ÷ 100 = 1.04

③ Calculate 104 % of 250 (consequent × ratio value): 250 × 1.04 = 260

Martha's parents must pay \$260 each month, which corresponds to the monthly mortgage payment, plus 4% fixed interest.

A: \$260 monthly

#### Understanding

In situations involving increases to the percentage, and you want to find the ratio antecedent, do the **2** following.

1) Find the total percentage: 100% + percentage increase.

(2) Calculate the ratio value: percentage ÷ 100.

③ Calculate the antecedent: antecedent = consequent × ratio value.

#### Solve

1. A pineapple juice typically containing 800 ml is on offer, with 20% more than usual. How many milliliters of juice does it have when on offer?



- 2. A small printing press wants to purchase a batch of paper that costs \$720. As you want to import it from another country, you must pay a 5% tax on import duties over the original price. How many dollars must the printing press pay for the batch of paper, including taxes?
- 3. In a restaurant, 9% of the consumption is paid as a tip. If someone consumes \$30, how much will they have to pay, including the tip?





2.5 Calculate the antecedent of a ratio when its percentage is greater than 100%.

**Objective:** Find the antecedent of a ratio when there is an increase in the percentage concerning the consequent of the ratio.

**Key Points:** One more step needs to be completed before applying what was learned previously. Do it as Beatrice did it in **1**. Since each monthly payment represents 104% of the monthly amount to be paid (\$ 250). The ratio value is calculated using that percentage and following the procedure of the previous class. In **2**, the process to calculate the quantity (antecedent) corresponding to a percentage greater than 100% is consolidated (steps (2) and (3) coincide with those studied in class 2.4). For **3**, use the description shown in the Understand section.

**Methodological advice:** This class should emphasize the expressions "percentage, percent, additional percentage..., etc." since they indicate that the percentage increases, as per step (2) of the Understand section. It should be noted; as in the previous class, the given quantity corresponds to the consequent and the unknown to the antecedent.

#### **Problem Solving:**

- 1. (1) Total percentage = 100 % + 20 % = 120 %
  - (2) Ratio value =  $120 \div 100 = 1.2$
  - (3) Antecedent =  $800 \times 1.2 = 960$

**A:**960 ml

- 3. (1) Total percentage = 100 % + 9 % = 109 %
  - (2) Ratio value =  $109 \div 100 = 1.09$
  - (3) Antecedent =  $30 \times 1.09 = 32.7$
  - **A:**\$32.70

2. 1 Total percentage = 100 % + 5 % = 105 %
2 Ratio value = 105 ÷ 100 = 1.05
3 Antecedent = 720 × 1.05 = 756

**A:**\$756







c. A television for \$449.



2.6 Calculates the price of a product considering the Value Added Tax (VAT).

**Objective:** Find the price of an item by including the Value Added Tax (VAT).

**Key Points:** The content of this class is a particular situation developed in class 2.5, where the percentage of increase is fixed for all problems (13%). In **1**, Anthony's solution applies previous knowledge, making it relevant for the students. In **2**, the two possible procedures are taken up to solve the initial problem (Anthony's and Carmen's). To solve the situations raised in **3**, students can use either of the two ways described in **2**; the important thing is to ensure they understand the process and apply it correctly.

**Methodological advice:** In the initial problem situation (see ①), tell the students that VAT is the main source of funds in a country, with which expenses are covered in public schools, national hospitals, public lighting, etc. In addition, students must be clear that this tax is always 13% of the original price; that is, the original price necessarily corresponds to 100%. The above indicates that in the first form presented in ②, step ① can be omitted, knowing the new price results from multiplying the original price by 1.13

#### **Problem Solving:**

	Second option:
= 113 ÷ 100 = 1.13	<ol> <li>13 % of the original price: 525 × 0.13 = 68.25</li> </ol>
= 525 × 1.13 = 593.25	(2) New price = 525 + 68.25 = 593.25
	<b>A:</b> \$593.25
	c. First option:
= 1.13	(1)Ratio value = 1.13
= 30 × 1.13 = 33.9	(2)New price = 449 × 1.13 = 507.37
	<b>A:</b> \$507.37
	= 113 ÷ 100 = 1.13 = 525 × 1.13 = 593.25 = 1.13 = 30 × 1.13 = 33.9

Date: Class	: 2.6
A dining set costs \$160 without VAT. How much will the dining set cost including VAT (13%)?	(2) The amount corresponding to VAT (\$20.80) is added to the original price:
	160 + 20.8 = 180.8
	<b>A:</b> \$180.80
<ul> <li>(5) First option:         <ul> <li>(1) Total percentage = 100 % + 13 % = 113 %</li> <li>(2) Ratio value = 113 ÷ 100 = 1.13</li> <li>(3) Antecedent = 160 × 1.13 = 180.8</li> <li>A: \$180.80</li> </ul> </li> </ul>	Ratio value       = 113 ÷ 100 = 1.13         (2) New Price       = 525 × 1.13 = 593.25         A: \$593.25
Second option: (1) Amount of money corresponding to 13 %: Ratio value = 13 ÷ 100 = 0.13 antecedent = 160 × 0.13 = 20.8	Second option: (1) 13 % of the original price: 525 × 0.13 = 68.25 (2) New price = 525 + 68.25 = 593.25 A: \$593.25
	Homework: Page 83





2.7 Calculate the price of an item offering a discount percentage.

**Objective:** Find the price of an item (antecedent) when a discount is applied.

**Key Points:** In the two previous classes, it has been necessary to increase the discount based on a certain percentage, resulting in an amount greater than 100%; in this class, 100% is decreased due to a discount, thus resulting in an amount less than 100%. In **1**, students must visualize for the given case; they must subtract the discount percentage from 100% (the turtle provides a clue for this); Furthermore, it is expected that the majority will solve as Mario did in **2** since what was seen in previous classes is used. For problems in **3**, students can use either of the two forms described in the Understand section but ensuring they understand the process and apply it correctly.

#### **Problem Solving:**

a. First option

- (1) Percentage : 100 % 30 % = 70 %
- (2) Ratio value =  $70 \div 100 = 0.7$
- (3) Discounted price =  $20 \times 0.7 = 14$

**A:**\$14

- b. First option:
  - (1) Percentage: 100 % 20 % = 80 %
  - (2) Ratio value = 80 ÷ 100 = 0.8
  - (3) Discounted price =  $20 \times 0.8 = 12$

**A:**\$12

c. **A:** \$4.75

Second option:

- (1) Ratio value corresponding to 30 %: 0.3
- (2) Amount corresponding to 30%:  $20 \times 0.3 = 6$
- (3) Discounted price = 20 6 = 14

**A:**\$14

Second option:

- (1) Ratio value corresponding to 20 %: 0.2
- (2) Amount corresponding to 20%:  $15 \times 0.2 = 3$
- (3) Discounted price = 15 3 = 12
- **A:**\$12







2.8 Calculates the consequent when the antecedent corresponding to a percentage greater than 100% is known

**Objective:** Find the consequent of a ratio from the antecedent and the percentage, when it's greater than 100%.

**Key Points:** In this and the following two classes, problems will be solved where the data provided correspond to the antecedent and the percentage, and the unknown quantity is the consequent. In **1** it is recalled how to calculate the consequent from the antecedent and the ratio value, the algorithm studied in lesson 1 (consequent = antecedent ÷ ratio value). Will use this information to solve the initial problem in **2**; the armadillo's comment provides a clue to visualize who the unknown quantity is (students are expected to solve in a similar way to what Carlos did in **3**). In **4**, must apply the steps described in Understand section.

**Methodological advice:** Again, the students should be told that when having a percentage representing a quantity a in respect to another quantity b, then the quantity a is the antecedent and b the consequent, so that they can identify that now the unknown quantity is the consequent ratio. On 1st of (4), you should remember that VAT corresponds to an increase of 13% on the original price of an item.

#### **Problem Solving:**

1. The \$678 corresponds to 113%, since the VAT is 13% of the original price.

(1) Ratio value = 113 ÷ 100 = 1.13

- 2 Consequent = 678 ÷ 1.13 = 600
- **A:**\$600

2. (1) Ratio value = 120 ÷ 100 = 1.2 (2) Consequent = 60 ÷ 1.20 = 50

**A:** 50 kg

Date: Class: 2.8	
Re Julia read 200 pages of a book; this is 5 times what Joseph read. How many pages did Joseph read?	R 1. The \$678 corresponds to 113%, since the VAT is 13% of the original price.
Consequent=200 ÷ 5=40 <b>A:</b> 40 pages.	(1) Ratio value = $113 \div 100 = 1.13$ (2) Consequent = $678 \div 1.13 = 600$
A one-month-old giraffe measures 260 cm; this is 130% of his height right at birth. What was the height ( <i>b</i> cm) of the giraffe after birth?	A:\$600
S The ratio value of 260:b is calculated using the percentage: Ratio value = $130 \div 100 = 1.3$ As b is the consequent :	<ul> <li>2. 1 Ratio value = 120 ÷ 100 = 1.2</li> <li>2 Consequent = 60 ÷ 1.20 = 50</li> <li>A:50 kg</li> </ul>
<i>b</i> = 260 ÷ 1.3 = 200	
<b>A:</b> 200 cm	Homework: Page 85



## 2.9 Calculating percentage and consequent

#### Analyze ....

1 At Ana's school this year, there are 390 students. If this number is 25% more students than the previous year, how many students were there last year? It represents the number of students last year as *b*.



#### Solution \_\_\_\_\_

"25% more students than last year" indicates that last year's number of students (b students) represents 100%. This year there are 100% + 25% = 125% students compared to last year.

**2** This year's 390 students correspond to 125%, and the ratio value 390 : b is equal to:

$$125 \div 100 = 1.25$$

I apply previous lesson knowledge, Consequent = Antecedent ÷ Ratio value:

A: 312 students



#### Understanding

In problems where the percentage increases, the amount corresponding to that increase is known as (antecedent); the original (consequent) amount is still unknown. Perform the following:

(1) Find the total percentage corresponding to the increase: 100% + percentage increase.

- (2) Calculate the ratio value: total percentage ÷ 100
- (3) Calculate the original (consequent) quantity: **Consequent = Antecedent** ÷ **Ratio value**

#### Solve

1. Joseph's height is 156 cm, 20% more than his sister Julia. What is Julia's height in centimeters?

- 2. After receiving a 10% increase from his previous salary, John's salary is \$440. What was the previous wage?
- 3. A puppy weighs 168 g a week after birth. This amount is 60% more than the puppy's birth weight. How many grams did it weigh at birth?

2.9 Calculate the consequent of a ratio when the antecedent is known and the percentage that the antecedent has increased in respect to the consequent.

**Objective:** Find the consequent of a ratio of the increase, in percentage terms, of the antecedent n respect to the known consequent is known.

**Key Points:** Unlike the previous class, in the initial problem in **1**, the percentage is not explicitly presented. Still, the number of students in Ana's school this year (antecedent) is compared with that of last year (later) using the expression "25% more"; In the parakeet's comment, the relationship between the quantities and the percentage of increase is graphically observed. In this way, before applying what was seen in the previous class, the students must complete the addition 100% + 25% as Julia does in **2**. For problems in **3**, you should use what is described in Understand.

#### **Problem Solving:**

- 1. (1) Total percentage: 100 % + 20 % = 120 %
  - (2) Ratio value: 120 ÷ 100 = 1.2
  - (3) Consequent = 156 ÷ 1.2 = 130

#### **A:** 130 cm

- 3. (1) Total percentage: 100 % + 60 % = 160 %
  - (2) Ratio value : 160 ÷ 100 = 1.6
  - (3) Consequent= 168 ÷ 1.6 = 105

#### **A:** 105 g

Notes:

- 2. ① Total percentage: 100 % ÷ 10 % = 110 %
  - (2) Ratio value: 110 ÷ 100 = 1.1
  - (3) Consequent = 440 ÷ 1.1 = 400

**A:** \$400





# 2.10 Calculating the consequent using percentages less than 100 %

#### Analyze

The owner of a piece of land decides to sell it on lots for a more significant profit. So far, it has sold a lot of 80 m<sup>2</sup>, which represents 40% of the total land. What is the total area of the land? Represent the entire area as  $b \text{ m}^2$ .







The ratio value 80 : b equal a:  $40 \div 100 = 0.4$ 

To calculate the quantity b, I Use:

Consequent = Antecedente ÷ Ratio value

 $b = 80 \div 0.4 = 200$ 

#### A: 200 m<sup>2</sup>



Remember, the antecedent may be greater than the consequent.

The total area ( $b \text{ m}^2$ ) represents 100 %. Like 100 % = 40 % + 40 % + 20 %, then I can find b by adding the corresponding areas to 40 % and 20 %.



- 40 % --> 80 m<sup>2</sup>
- 20 % -> 40 m<sup>2</sup> (Is half of 40%)

A:200 m<sup>2</sup>

## Understanding

Even if the percentage is less than 100%, the consequent is always calculated with the formula: **Consequent = Antecedent ÷ Ratio value** 

## Solve

2 1. A farmer plants 55 ha of corn representing 20% of his land. How many hectares is the land?



2. A worker saves \$56, which is 10% of her monthly salary. How much is her monthly salary?

2.10 Calculate the consequent of a ratio when the antecedent corresponding to a percentage less than 100% is known.

**Objective:** Find the consequent of a ratio if increases, in terms of percentages, of the antecedent concerning the consequent.

**Key Points:** In the initial problem of **1**, the quantity provided (80 m<sup>2</sup>) is the antecedent; Although the percentage corresponding to this is less than 100%, students should be reminded the antecedent may be greater than the consequent in a ratio. Therefore the percentage will be less than 100% (the turtle icon presents the relationship between the areas). Students are expected to solve similarly to Joseph. In **2** previous classes, applied knowledge. In **3**, use the description shown in the Understand section. It is reaffirmed that the process to calculate the consequent is the same.

**Methodological advice:** Analyze the initial problem in detail and generate questions for students to identify who the antecedent and consequent are and the unknown quantity. Regardless of its measurements, the way to calculate the consequent remains.

Suppose students have doubts about whether the procedure they have carried out is correct. In that case, they can verify their answer by multiplying it by the ratio value equivalent to the percentage and comparing if that result is equal to the one given in the problem statement.

#### **Problem Solving:**

1. (1) Ratio value =  $20 \div 100 = 0.2$ 

- (2) Consequent = 55 ÷ 0.2 = 275
  - **A:**275 ha

2. 1 Ratio value = 10 ÷ 100 = 0.1
2 Consequent = 56 ÷ 0.1 = 560
A: \$560





## 2.11 Practice what you learned

- 1. In the Math exam, Martha got eight correct questions out of a total of ten. What is the percentage of correct answers?
- In a cinema room, 42 seats out of the 120 available are occupied. What is the percentage of occupied seats?

3. Fill in the missing ratio and percentage values:



- 4. A spa served 250 people on August 5th and 300 people on August 6th.
  - a. Calculates the ratio value between the number of people who attended on August 6th and those who attended on August 5th.
  - b. What percentage of people who attended on the 6th compared to those who attended on the 5th?
- 5. At John's nursery, there are 420 plants of which, 25% are roses. How many roses are in the nursery?
- 6. While waiting for photos to download to his computer; John notices so far, 30% of 50 megabytes have been downloaded. How many megabytes have been downloaded so far?

#### **\*Self-challenge**

1. Calculate the percentage represented by the shaded area with lines of the rectangle in relation to the area of the blue rectangle.



2. Calculate the percentage represented by the shaded area with lines of the rectangle in relation to the area of the blue square.



Unit 4

2.11 Solve percentage problems

**Problem Solving:** 

1. Ratio  $\rightarrow 8:10$ Ratio value  $\rightarrow 8 \div 10 = 0.8$ Then, percentage =  $0.8 \times 100 = 80$ 

**A: 80 %** 

- 4. a. Ratio → 300 : 250 Ratio value → 300 ÷ 250 = 1.2 A: 1.2
- 5. Ratio value = 25 ÷ 100 = 0.25 Antecedent = 420 × 0.25 = 105 A: 105 roses.

## **Self-challenge**

1. Area of rectangle shaded with lines:  $8 \times 2 = 16;$  16 cm<sup>2</sup>

Blue rectangle area:  $10 \times 2 = 20; 20 \text{ cm}^2$ 

The ratio between the areas of the rectangle shaded with lines, and the one colored blue is 16:20, its value is:

16 ÷ 20 = 0.8

Then,

percentage =  $0.8 \times 100 = 80$ 

A: 80 %

## Notes:



- b. The ratio value is multiplied by 100: percentage = 1.2 × 100 = 120
   A: 120 %
- 6. Ratio value =  $30 \div 100 = 0.3$ Antecedent =  $50 \times 0.3 = 15$ A: 15 megabytes.

2. Area of rectangle shaded with lines :  $8 \times 5 = 40;$  40 cm<sup>2</sup>

Blue square area:

$$4 \times 4 = 16; 16 \text{ cm}^2$$

The ratio between the areas of the rectangle shaded with lines, and the one colored blue is 40: 16, its value is:  $40 \div 16 = 2.5$ 

Then,

percentage =  $2.5 \times 100 = 250$ 

A: 250 %



## 2.12 Practice what you learned

 A brown bear (living in the mountains of Cantabria, Spain) within a few months of birth reaches 150% of its initial weight. It is known that the birth weight of such bears is approximately 350 grams. How many grams is 150% of his weight equivalent?



2. A shirt that costs \$40 is on sale with 15% off. How many dollars does the shirt cost when you apply the discount.?

- 3. At the end of the year, John managed to save \$70, and this represents 140% of what was planned. How many dollars had he planned to save?
- 4. Anna sold a TV for \$240; this amount is 20% more than the price for which she purchased the TV. How many dollars did Anna pay when she purchased the TV?



5. When a grizzly bear (subspecies of the North American brown bear) hibernates, its heart rate drops to 10 beats per minute, 20% of its standard value. What is the normal heart rate of the grizzly bear?

#### ✓ Self-challenge



- Anthony is building a wall for which he needs eight bags of cement. If each bag costs \$5 without tax, how much will you have to pay for the eight bags after adding 13% tax?
- 2. A train has covered 65% of its route. If you still have 70 km left to travel, how many kilometers is the total route?



2.12 Solve percentage problems

## Problem Solving:

 Ratio value = 150 ÷ 100 = 1.5 Antecedent = 350 × 1.5 = 525
 A: 525 g 2. (1) Percentage: 100 % - 15 % = 85 %
(2) Ratio value = 85 ÷ 100 = 0.85
(3) Discounted price = 40 × 0.85 = 34

**A:** \$34

- 4. (1) Percentage total: 100 % ÷ 20 % = 120 %
  - (2) Ratio value 120 ÷ 100 = 1.2

(3) Consequent = 240 ÷ 1.2 = 200

A: \$200

5. (1) Ratio value = 20 ÷ 100 = 0.2 (2) Consequent = 10 ÷ 0.2 = 50

3. (1) Ratio value = 140 ÷ 100 = 1.4

(2) Consequent =  $70 \div 1.4 = 50$ 

A: 50 beats per minute

## **\*** Self-challenge

#### 1. Option 1

A:\$50

The price, including VAT, is calculated for one bag of cement.

Ratio value: 1.13 Price including VAT 5 × 1.13 = 5.65

To calculate the total to pay for the 8 bags, multiply the above result by 8:

5.65 × 8 = 45.2

#### **A:** \$45.20

#### Option 2

The total payable for eight bags of cement is calculated without VAT as follows:

5 × 8 = 40; \$40

The price for the eight bags, including VAT, is calculated

Ratio value: 1.13 Price including VAT 40 x 1.13 = 45.2 A: \$45.20

2. If a train has covered 65% of its route, it still has 100% - 65% = 35% of its route to travel. Then, 70 km represents 35% of the total journey (that is the antecedent).).

Ratio value: 35 ÷ 100 = 0.35 Consequent: 70 ÷ 0.35 = 200

#### A: 200 km

#### Notes: