

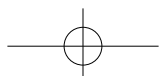
Practice Book for Mathematics

Answer Book

Grade
5



Japan International
Cooperation Agency



1 - 1 Whole Numbers and Decimal Numbers

Structure of Decimal Numbers (1)

Instruction The official distance for a marathon is 42.195 km. The structure of this number is as shown below.

Tens Place	Ones Place	Tenths Place	Hundredths Place	Thousandths Place
4	2	1	9	5

We can use the numerals 0 through 9 and a decimal point to express whole numbers and decimal numbers of any size.

$$42.195 = 10 \times 4 + 1 \times 2 + 0.1 \times 1 + 0.01 \times 9 + 0.001 \times 5$$

Example Look at the structure of the number 23.567, write the correct number in the .

1 23.456

$$= 20 + 3 + 0.4 + 0.05 + 0.006$$

2 23.456 = 20 + 3 + 0.4 + 0.05 + 0.006

$$= 10 \times 2 + 1 \times 3 + 0.1 \times 4 + 0.01 \times 5 + 0.001 \times 6$$

Look at the structure of the following numbers and write the correct number in the .

1 3.579

$$= 3 + 0.5 + 0.07 + 0.009$$

2 10.975

$$= 10 + 0.9 + 0.07 + 0.005$$

3 2.863 = 2 + 0.8 + 0.06 + 0.003 = 1 × 2 + 0.1 × 8 + 0.01 × 6 + 0.001 × 3

4 30.17 = 30 + 0 + 0.1 + 0.07 = 10 × 3 + 1 × 0 + 0.1 × 1 + 0.01 × 7

1

1 - 2 Whole Numbers and Decimal Numbers

Structure of Decimal Numbers (2)

Example Solve the math sentence. Write the answer in the .

$$10 \times 4 + 1 \times 2 + 0.1 \times 1 + 0.01 \times 9 + 0.001 \times 5 = 42.195$$

It might be easier to understand if it is written this way.

Tens Place	Ones Place	Tenths Place	Hundredths Place	Thousandths Place
4	2	1	9	5

1 Write each number in the following math sentences.

1 $10 \times 2 + 1 \times 6 + 0.1 \times 5 + 0.01 \times 3 + 0.001 \times 4$

26,534

2 $10 \times 3 + 1 \times 1 + 0.1 \times 2 + 0.01 \times 7 + 0.001 \times 8$

31,278

3 $100 \times 1 + 10 \times 2 + 1 \times 4 + 0.1 \times 3 + 0.01 \times 5 + 0.001 \times 6$

124,356

4 $100 \times 2 + 10 \times 5 + 1 \times 3 + 0.01 \times 4 + 0.001 \times 8$

253,048

5 $1 \times 4 + 0.1 \times 7 + 0.001 \times 5$

4,705

2 Write the math sentence that equals the number given. Focus on each individual number. Otherwise, there could be many answers.

1 5.64 = 5 + 0.6 + 0.04 =

$1 \times 5 + 0.1 \times 6 + 0.01 \times 4$

2 24.789 = 20 + 4 + 0.7 + 0.08 + 0.009 =

$10 \times 2 + 1 \times 4 + 0.1 \times 7 + 0.01 \times 8 + 0.001 \times 9$

3 61.035 = 60 + 1 + 0.03 + 0.005 =

$10 \times 6 + 1 \times 1 + 0.01 \times 3 + 0.001 \times 5$

4 987.021 = 900 + 80 + 7 + 0.02 + 0.001 =

$100 \times 9 + 10 \times 8 + 1 \times 7 + 0.01 \times 2 + 0.001 \times 1$

5 402.104 = 400 + 2 + 0.1 + 0.004 =

$100 \times 4 + 1 \times 2 + 0.1 \times 1 + 0.001 \times 4$

2

1 - 3 Whole Numbers and Decimal Numbers

Structure of Decimal Numbers (3)

Example Make the largest and smallest possible numbers by placing the numbers in the .

<input type="text"/>	<input type="text"/>	.	<input type="text"/>	<input type="text"/>	<input type="text"/>
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Largest number 96.542 Smallest number 24.569

When creating the largest possible number, place the highest value digit on the left. In decreasing value order, place the lower digit on the right. When creating the lowest possible number, place the lowest value digit on the left. In increasing value order, place the highest digit on the right.

1 Make the following numerals by placing the numbers in the . Use each number only once and fill in all the .

<input type="text"/>	<input type="text"/>	.	<input type="text"/>	<input type="text"/>
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1 Largest number 75.31

2 Smallest number 13.57

3 The number that is closest to 30. 31.57

2 Make the following numerals by placing the numbers in the . Use each number only once and fill in all the .

<input type="text"/>	<input type="text"/>	.	<input type="text"/>	<input type="text"/>
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1 Largest number 98.654

2 Smallest number 20.456

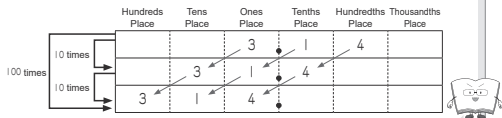
3 The number that is closest to 40. 40.256

4 The number that is closest to 50. 49.865

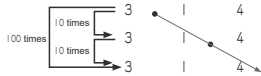
3

1 - 4 Whole Numbers and Decimal Numbers
Structure of Decimal Numbers (4)

Instruction What happens when 3.14 is multiplied by 10? When it is multiplied by 100?



The same can be expressed below.



When a decimal number is multiplied by 10, the digits move to the left one place. When a decimal number is multiplied by 100, the digits move to the left two places.

Example Multiply the decimal number.

10 times 3.14 = 31.4 100 times 3.14 = 314

1 Multiply the decimal number.

- ① 2.37 times 10 = 23.7 times 100 = 237
- ② 0.468 times 10 = 4.68 times 100 = 46.8
- ③ 0.095 times 10 = 0.95 times 100 = 9.5

2 How many times was 1.34 multiplied to get the following numbers?

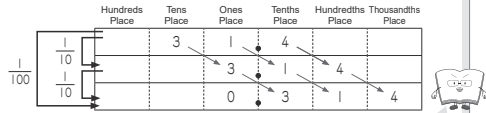
- ① 13.4 = 10 times ② 134 = 100 times

3 How many times was 0.76 multiplied to get the following numbers?

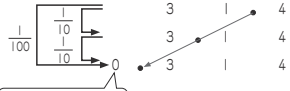
- ① 76 = 100 times ② 7.6 = 10 times

1 - 5 Whole Numbers and Decimal Numbers
Structure of Decimal Numbers (5)

Instruction What happens when 31.4 is multiplied by $\frac{1}{10}$? By $\frac{1}{100}$?



The same can be expressed below.



When a decimal number is multiplied by $\frac{1}{10}$, the digits move to the right by one place. When a decimal number is multiplied by $\frac{1}{100}$, the digits move to the right by two places.

Example Write the decimal number.

$\frac{1}{10}$ times 31.4 = 3.14 $\frac{1}{100}$ times 31.4 = 0.314

1 Multiply the decimal numbers.

- ① 71.5 times $\frac{1}{10}$ = 7.15 times $\frac{1}{100}$ = 0.715
- ② 2.83 times $\frac{1}{10}$ = 0.283 times $\frac{1}{100}$ = 0.0283
- ③ 10.06 times $\frac{1}{10}$ = 1.006 times $\frac{1}{100}$ = 0.006

2 What fraction was 12.4 multiplied by to get the following numbers?

- ① 1.24 = $\frac{1}{10}$ ② 0.124 = $\frac{1}{100}$

3 What fraction was 8.09 multiplied by to get the following numbers?

- ① 0.0809 = $\frac{1}{100}$ ② 0.809 = $\frac{1}{10}$

1 - 6 Whole Numbers and Decimal Numbers
Review

1 Look at the structure of the following numbers. Write the correct numbers in the .

- ① 56.789 = 50 + 6 + 0.7 + 0.08 + 0.009 = 50 × + 6 × + 0.7 × + 0.08 × + 0.009 ×
- ② 9.876 = 9 + 0.8 + 0.07 + 0.006 = 9 × + 0.8 × + 0.07 × + 0.006 ×

Focus on each individual number. Otherwise, there could be many answers.

2 Write the correct numbers in the .

- ① 357.9 = 300 + 50 + 7 + 0.9 = 100 × + 10 × + 1 × + 0.1 ×
- ② 12.34 = 10 + 2 + 0.3 + 0.04 = 10 × + 1 × + 0.1 × + 0.01 ×
- ③ 4.157 = 4 + 0.1 + 0.05 + 0.007 = 1 × + 0.1 × + 0.01 × + 0.001 ×
- ④ 7.067 = 7 + 0.06 + 0.007 = 1 × + 0.01 × + 0.001 ×
- ⑤ 0.503 = 0.5 + 0.003 = 0.1 × + 0.001 ×

3 Write math sentences for the following decimal numbers.

- ① 6.92 = 6 + 0.9 + 0.02 =
- ② 2.835 = 2 + 0.8 + 0.03 + 0.005 =
- ③ 60.107 = 6 + 0.1 + 0.007 =

4 Multiply the decimal numbers by the following numbers.

- ① 5.08 times 10 = 50.8 times 100 = 508
- ② 0.102 times 10 = 1.02 times 100 = 10.2
- ③ 20.03 times 10 = 200.3 times 100 = 2003

5 How many times was 37.4 multiplied by to get the following numbers?

- ① 374 = 10 times ② 374 = 100 times

6 Multiply the decimal number by the fraction.

- ① 29.7 times $\frac{1}{10}$ = 2.97 times $\frac{1}{100}$ = 0.297
- ② 6.03 times $\frac{1}{10}$ = 0.603 times $\frac{1}{100}$ = 0.0603
- ③ 40.05 times $\frac{1}{10}$ = 4.005 times $\frac{1}{100}$ = 0.4005

7 What fraction was 30.8 multiplied by to get the following answers?

- ① 3.08 = $\frac{1}{10}$ ② 0.308 = $\frac{1}{100}$

8 Make the following numerals by placing the numbers in the . Use each Number only once and fill in all the .



- ① Largest number = 97.643
- ② Smallest number = 10.346
- ③ The number that is closest to 20. = 19.764
- ④ The number that is closest to 70. = 70.134

2 - 1 Volume


How to Express the Amount of Space


Instruction How to compare the amount of space

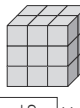
Which cuboid is larger?

Compare them using 1 cm cubic blocks. It is the similar way to calculate the "Area" as we learned in the previous grades.

Example Find the number of blocks with 1 cm sides in the following figures below.


1  6 blocks


2  4 blocks


3  18 blocks


There are 6 blocks with 3 layers. 2 blocks in length and 3 blocks in width.

Find the number of blocks with 1 cm sides in the following figures below.

1  12 blocks

2  12 blocks

3  27 blocks

4  9 blocks

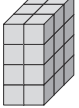
2 - 2 Volume

Volume of Cuboids and Cubes (1)

Instruction Volume

- The size of a solid is called its "volume".
- The volume of a cube with sides of 1 cm is called 1 cubic centimeter, and it is written as 1 cm³.
- "Cubic centimeter" is a unit of volume.


Example 1 If you stack 4 layers of the following blocks, what is the volume? Fill in the blanks.

 $3 \times 2 \times 4 = 24 \text{ cm}^3$

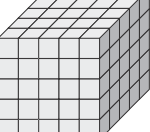
Number for length: 3, Number for width: 2, Number for height: 4

1 Find the volume.

1 When you stack 3 layers of the following blocks.

 $4 \times 3 \times 3 = 36 \text{ cm}^3$

2 Find the volume when you stack 5 layers of the following blocks.

 $5 \times 5 \times 5 = 125 \text{ cm}^3$


Instruction How to Calculate Volume

(Volume of Cuboid) = (Length) × (Width) × (Height)
 (Volume of Cube) = (Length of Side) × (Length of Side) × (Length of Side)

2 - 3 Volume

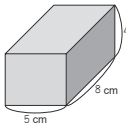
Volume of Cuboids and Cubes (2)


Example 1 Find the volume of the following figure

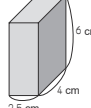
 $5 \times 4 \times 3 = 60$

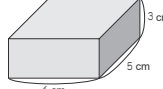
Math sentence: $5 \times 4 \times 3 = 60$
 Answer: 60 cm^3

1 Find the volume of the following figures.

1  $5 \times 8 \times 4 = 160$
 Math sentence: $5 \times 8 \times 4 = 160$
 Answer: 160 cm^3

2  $4 \times 4 \times 4 = 64$
 Math sentence: $4 \times 4 \times 4 = 64$
 Answer: 64 cm^3

3  $2.5 \times 4 \times 6 = 60$
 Math sentence: $2.5 \times 4 \times 6 = 60$
 Answer: 60 cm^3

4  $6 \times 5 \times 3 = 90$
 Math sentence: $6 \times 5 \times 3 = 90$
 Answer: 90 cm^3

Example 2 The volume of the following cuboid is 150 cm³, the length and the width of the cuboid are 6 cm and 5 cm.

1 Letting the height be □ cm, make a math sentence to find the volume.

(Length) × (Width) × (Height) = (Volume)
 Math sentence: $6 \times 5 \times \square = 150$
 $30 \times \square = 150$

2 Find the height.

Since $30 \times \square = 150$, dividing 150 by 30 to find the value of □.

Math sentence: $30 \times \square = 150$
 $\square = 150 \div 30$
 $= 5$
 Answer: 5 cm

2 The volume of the following cuboid is 36 cm³, the length and the width of the cuboid are 3 cm and 2 cm.

1 Letting the height be □ cm, make a math sentence to find the volume.

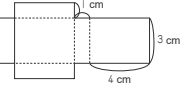
Math sentence: $3 \times 2 \times \square = 36$
 $6 \times \square = 36$

2 Find the height

Since $6 \times \square = 36$, dividing 36 by 6 to find the value of □.

Math sentence: $6 \times \square = 36$
 $\square = 36 \div 6$
 $= 6$
 Answer: 6 cm

Example 3 Find the volume of the cuboid that can be assembled with the following net.

 $4 \times 3 \times 1 = 12$

Math sentence: $4 \times 3 \times 1 = 12$
 Answer: 12 cm^3

3 Find the volume of the cuboid that can be assembled with the following net.

1 What is the dimension of the cuboid?

Length: 6 cm Width: 5 cm
 Height: 2 cm

2 Find the volume of the cuboid

Math sentence: $6 \times 5 \times 2 = 60$
 Answer: 60 cm^3

2 - 4 Volume Ideas for Finding Volume

Example Find the volume of the figure below.

How can we think about the volume of this kind of solid figure?
How about dividing the figure? Let's think.

Idea 1

1. Divide a given figure into two cuboids and calculate.

Cuboid A: $5 \times 2 \times 8 = 80$
Cuboid B: $5 \times 6 \times 4 = 120$

2. Sum up the two volumes and find the total volume.
 $80 + 120 = 200$ 200 cm^3

Idea 2

1. The given figure is obtained by subtracting the part of the dotted line from the large cuboid.

Cuboid A: $5 \times 8 \times 8 = 320$
Cuboid B: $5 \times 6 \times 4 = 120$

2. Calculate the volumes separately.

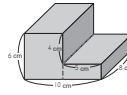
3. Subtract the dotted line from the larger one.
 $320 - 120 = 200$ 200 cm^3

Alternatively, given the height of the solid figure is 8 cm, I wonder we could cut and paste as follows and find the volume.

Cut and paste

Math sentence: $(2 + 8) \times 5 \times 4 = 200$
Answer: 200 cm^3

1 Find the volume of the following figure by using the following ways.



1 By dividing the figure into two cuboids as follows. Calculate the volumes separately.

Cuboid A: $8 \times 5 \times 6 = 240$
Cuboid B: $8 \times 5 \times 2 = 80$

Math sentence: $8 \times 5 \times 6 = 240$ $240 + 80 = 320$
 $8 \times 5 \times 2 = 80$ Answer: 320 cm^3

2 By subtracting a part from the whole.

1. Calculate the volumes separately.

Cuboid C: $8 \times 10 \times 6 = 480$
Cuboid D: $8 \times 5 \times 4 = 160$

2. Subtract cuboid D from cuboid C.
Math sentence: $480 - 160 = 320$
Answer: 320 cm^3

3 By cutting the figure and paste to make a cuboid.

Cut and paste

Math sentence: $8 \times 5 \times (6 + 2) = 320$
Answer: 320 cm^3

2 - 5 Volume Various Units of Volume and Capacity (1)

Example 1 Answer the following questions.

1 Find the volume of the cuboid on the right.

Math sentence: $3 \times 2 \times 2 = 12$
Answer: 12 m^3

2 How many cm^3 is 1 m^3 ?

$1 \text{ m} = 100 \text{ cm}$

$100 \times 100 \times 100 = 1000000 \text{ cm}^3$

Length Width Height

- The volume of a cube with sides of 1 m is called 1 cubic meter, and it is written as 1 m^3 .
- "Cubic meter" is a unit of volume.
- $1 \text{ m}^3 = 1000000 \text{ cm}^3$

1 Find the volume of the following figures.

1 $5 \times 5 \times 5 = 125$ Answer: 125 m^3

2 $3 \times 0.5 \times 0.5 = 0.75$ Answer: 0.75 m^3

2 How many cm^3 is 24 m^3 ?

Since $1 \text{ m}^3 = 1000000 \text{ cm}^3$,

Math sentence: $24 \times 1000000 = 24000000$
Answer: 24000000 cm^3

Example 2 There is a container with the shape of a cuboid that is made of 1 cm thick wood as shown on the right.

1 Fill in the blanks with numbers.

Since the container is made of 1 cm thick wood, the size of the inside container is as follows:

Length: $7 - 2 = 5$
Width: $7 - 2 = 5$
Height: $5 - 1 = 4$

In length, the edge of the container has 2, so subtract 2 cm.

2 How many cm^3 is the volume of water that fills this container?

Math sentence: $5 \times 5 \times 4 = 100$ Answer: 100 cm^3

- The inside length, width, and height of the container are called the inside measures. The inside height is also called the depth.
- The size of a container is measured by the volume of water that it can hold. This volume is the capacity of the container.

3 Fill in the blank with a word or numbers.

When water is put into a container such as a cup or mass, the volume of water to be put is called **capacity**.

4 Answer the following questions.

1 What is the dimension of the inside measures?

Length: 6 cm Width: 6 cm
Depth: 5 cm

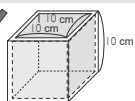
2 How many cm^3 is the capacity of the container?

Math sentence: $6 \times 6 \times 5 = 180$
Answer: 180 cm^3

2 - 6 Volume
Various Units of Volume and Capacity (2)

Example 1 There is a 1 L container whose inside length, width, and height has 10 cm. Find the capacity of the container.

Math sentence $10 \times 10 \times 10 = 1000$ Answer 1000 cm³



- 1 L = 1000 cm³
- 1 mL = 1 cm³

1 Find the capacity of the following containers. How much mL or L of water can hold?

1 Math sentence $12 \times 8 \times 6 = 576$
Answer 576 mL

2 Math sentence $40 \times 30 \times 25 = 30000$
Answer 30 L

2 There is a container with the shape of a cuboid that is made of 2 cm thick wood as shown below. Answer the following questions.

1 What is the dimension of the inside measures?
Length: 16 cm Width: 6 cm
Depth: 4 cm

2 How many cm³ is the capacity of the container?
Math sentence $16 \times 6 \times 4 = 384$
Answer 384 cm³

3 How much mL or L of water can hold?

384 mL

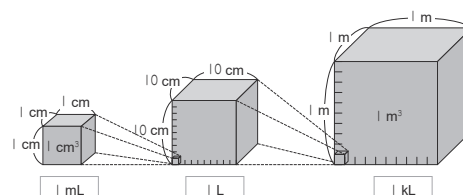
Example 2 How many L is 1 m³?

Since 1 m = 100 cm,

$1 \text{ m}^3 = 100 \times 100 \times 100 = 1000000 \text{ cm}^3$

Since 1 L = 1000 cm³,

$1000000 \div 1000 = 1000$ 1000 L 1000 L = 1 kL



2 Fill in the blanks with numbers.

Length of side	1 cm	10 cm	1 m
The area of the square	1 cm ²	100 cm ²	1 m ²
The volume of the cube	1 cm ³	1000 cm ³	1 m ³
The capacity of the cube	1 mL	1 L	1 kL

When the length of a side is 10 times, the area makes (10 × 10) times, and the volume makes (10 × 10 × 10) times.

2 - 7 Volume
Finding the Approximate Volume and Capacity

Example 1 Find the approximate volume of the rock on the picture. The rock with length is 52 cm, width is 18 cm, and height is 21 cm. Let length be 50 cm, width and height be 20 cm and calculate.



Math sentence $50 \times 20 \times 20 = 20000$ Answer 20000 cm³

1 Find the approximate volume of the following sizes of figures.

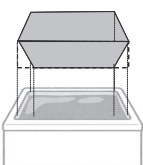
A block with length is 9 cm, width is 11 cm, and height is 5 cm. Let length and width be 10 cm and calculate.

Math sentence $10 \times 10 \times 5 = 500$ Answer 500 cm³

Example 2 Find the approximate capacity of the bathtub to find out how much water it can store.

1 What shape does the container look like?

Cuboid



2 How many L of water can this container hold?

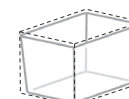
Math sentence $100 \times 80 \times 50 = 400000$
Since, 1 L = 1000 cm³
 $400000 \text{ cm}^3 = 400 \text{ L}$
Answer 400 L

You can calculate the volume by approximating the objects into the figures you have learned so far.

2 Approximate the volume of the container to know how much it can store.

1 What shape does the container look like?

Cuboid



2 Consider the container has the following dimensions. How many cm³ is it?

Math sentence $60 \times 30 \times 30 = 54000$
Answer 54000 cm³

3 Approximate the volume of the container to know how much water it can store.

1 What shape does the container look like?

Cube



2 Consider the container has the following dimensions. Approximate how many L it can store?

Math sentence $30 \times 30 \times 30 = 27000$
Since, 1 L = 1000 cm³ $27000 \text{ cm}^3 = 27 \text{ L}$
Answer 27 L

4 Approximate the volume of the following objects.

1 A loaf of bread

Math sentence $20 \times 12 \times 12 = 2880$
Answer 2880 cm³

2 A milk carton

Math sentence $7 \times 7 \times 20 = 980$
Answer 980 cm³

2 - 8

Volume

Review

1 Find the number of blocks with 1 cm sides in the following figures below.

1 12 blocks

2 24 blocks

2 Find the volume of the following figures.

1 2 cm

2 3 cm

Math sentence $7 \times 5 \times 2 = 70$

Answer 70 cm^3

Math sentence $3 \times 3 \times 3 = 27$

Answer 27 cm^3

3 3 m

4 0.7 m

Math sentence $8 \times 2 \times 3 = 48$

Answer 48 m^3

Math sentence $0.7 \times 0.7 \times 10 = 4.9$

Answer 4.9 m^3

3 Find the volume of the cuboid that can be assembled with the following net.

1 What is the dimension of the cuboid?

Length: 7 cm Width: 5 cm

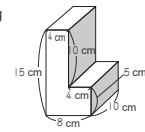
Height: 2 cm

2 Find the volume of the cuboid

Math sentence $7 \times 5 \times 2 = 70$

Answer 70 cm^3

4 Find the volume of the following figure by dividing the figure into two cuboids.



1. Divide a given figure into two cuboids.

2. Calculate the volumes separately.

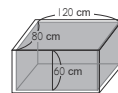
Cuboid A $10 \times 4 \times 10 = 400$

Cuboid B $10 \times 8 \times 5 = 400$

Math sentence $400 + 400 = 800$

Answer 800 cm^3

5 Answer the following questions.



1 What is the dimension of the inside measures?

Length: 120 cm Width: 80 cm

Depth: 60 cm

2 How many cm^3 is the capacity of the container?

Math sentence $120 \times 80 \times 60 = 576000$

Answer 576000 cm^3

3 How many L of water can hold this container?

576 L

$1000 \text{ cm}^3 = 1 \text{ L}$

4 If you use a 20 L container, how many times you should fetch water to fill the container?

Math sentence $576 \div 20 = 28 \text{ R } 16$

Answer 29 times

3 - 1

Proportion

Proportion (1)

Example 1 Investigate the relationship of following (a) and (b).

(a) Length and width of a rectangle with a perimeter of 30 cm.

1 cm 2 cm 3 cm 4 cm

14 cm 13 cm 12 cm 11 cm

Perimeter is the length around shape.

1 Summarize the relationship between the number of bricks piled up and total height. Fill in the table.

There are two lengths and two width, so... $10 \times 2 + 5 \times 2 = 30 \text{ (cm)}$

Width (cm)	1	2	3	4	5	...
Length (cm)	14	13	12	11	10	...

2 If a number increase by 1, how does the other number change? Fill in the blank of the table above and write the answer.

Answer Decrease by 1

(b) Build a wall by piling bricks. The height of bricks is 6 cm.



1 Summarize the relationship between the number of bricks piled up and total height. Fill in the table.

This is different from (a).

Number of bricks	1	2	3	4	5	...
Total height (cm)	6	12	18	24	30	...

2 If the number increase by 1, how does the other number change? Fill in the blank of the table and write the answer.

Answer Increase by 6

There is a relationship between the two quantities that changes as one increases and the other increases, or as one increases and the other decreases.

Investigate the relationship of following (a) and (b).

(a) The relationship between your age and the age of your sister, who is four years younger than you, when you celebrates your birthday.

1 Summarize the relationship between your age and the age of your sister who is four years younger than you.

Your age	4	5	6	7	8	9	10	11	12	13	...
The age of your sister	0	1	2	3	4	5	6	7	8	9	...

2 If the number increase by 1, how does the other number change? Fill in the blank of the table.

Answer Increase by 1

(b) Relationship between the width and area of a rectangle with a length of 4 cm.

1 cm 2 cm 3 cm 4 cm

4 cm 4 cm 4 cm 4 cm

1 Summarize the relationship between the length and width. Fill in the table.

Width (cm)	1	2	3	4	5	6	7	...
Area (cm^2)	4	8	12	16	20	24	28	...

2 If the number increase by 1, how does the other number change? Fill in the blank of the table and write the answer.

Answer Increase by 4

3 - 2 Proportion (2)

Example There is a ribbon that cost 80 zeds per meter. There's also a multiplication relationship!

1 Summarize the relationship between the length of ribbon and the price of ribbon. Fill in the table and blank.
(*zed (s) is the fictional currency unit.)

Length of ribbon (cm)	1	2	3	4	5	6	...
Price of ribbon (zeds)	80	160	240	320	400	480	...

2 If you buy a tape that is four times as long, how many times will the price of the tape be? Choose the symbol for your answer.
(a) 2 times (b) 3 times (c) 4 times. Answer (c)

The relationship between doubling or tripling the length of a tape and correspondingly doubling or tripling the price is called proportion. If the length of tape is \bigcirc and the price is \square , \bigcirc proportional to \square .

3 Write a math sentence by using the words and \bigcirc for the length of ribbon and \square for the price of ribbon.
Math sentence by words
Price for 1 m of ribbon \times Length = Price Answer $80 \times \bigcirc = \square$

4 What is the price of ribbon of 9 cm length of ribbon?
Math sentence
 $80 \times 9 = 720$
Answer 720 zeds

5 How long will be when the price of the ribbon is 1600 zeds?
Math sentence
 $1 \times 20 = 20$
Answer 20 cm

Find the surrounding length of a square that increases its length side by 1 cm. Let's think about the two quantities that change together.

1 Summarize the relationship between the number of pencils and the price.

Length of one side (cm)	1	2	3	4	5	6	7	8
Perimeter (cm)	4	8	12	16	20	24	28	32

2 If length of one side is 5 times long how many times will the perimeter be? Choose the symbol.
(a) 2 times (b) 4 times (c) 5 times. Answer (c)

3 Write a math sentence by using words and \bigcirc for the length of one side and \square for perimeter.
Math sentence by words
4 sides \times Length of one side = Perimeter Answer $4 \times \bigcirc = \square$

4 If one side is 11 cm long, how long is the perimeter?
Math sentence
 $4 \times 11 = 44$
Answer 44 cm

5 If the perimeter is 160 cm, how long is one side?
Math sentence
 $1 \times 40 = 40$
Answer 40 cm

3 - 3 Proportion Review

1 On the following (a)~(c), in which case is \bigcirc proportional to \square ?

(a) Length \square cm and width \bigcirc cm when a rectangles' perimeter is 26 cm.

Width \bigcirc (cm)	1	2	3	4	5	6	...
Length \square (cm)	12	11	10	9	8	7	...

(b) \bigcirc number of balls and total cost \square zeds when the cost of one ball is 300 zeds.

Number of balls \bigcirc	1	2	3	4	5	...
Total cost \square (zeds)	300	600	900	1200	1500	...

(c) \bigcirc number of candies and total cost \square zeds when the cost of one candy is 8 zeds.

Number of candies \bigcirc	1	2	3	4	5	...
Total cost \square (zeds)	8	16	24	32	40	...

Answer (b), (c)

2 Water is poured into a water tank so that the depth of water increases 2 cm in 1 minute.

1 Summarize the relationship between time to pour water and the depth of water in following the table.

Time to pour water \bigcirc (min)	1	2	3	4	5	6	...
Depth of water \square (cm)	2	4	6	8	10	12	...

2 If \bigcirc increase by 1, how much does \square increase?
Answer Increase by 2 cm

3 Write the math sentence for the depth of water by word and using \bigcirc min as the time to pour water and \square as the depth of water.
Math sentence by words
Depth of water per minutes \times time = Depth of water
Answer $2 \times \bigcirc = \square$

4 If the 8 minutes pass, what is the depth of the water?
Math sentence
 $2 \times 8 = 16$
Answer 16 cm

5 If the depth of water is 28 cm, how many minutes have passed?
Math sentence
 $28 \div 2 = 14$
Answer 14 minutes

3 On the following (a)~(c), in which case is \bigcirc proportional to \square ?

(a) \bigcirc number of notebooks and total cost \square zeds when the cost of one notebook is 120 zeds.

(b) Your brother is age \bigcirc years and his 2 year old younger sitter is age \square years when your brother's birthday reached.

(c) \bigcirc number of practice days and \square total minutes of practice when the practice is everyday for 30 minutes.

(d) Length of one side of a square is \bigcirc cm and area \square m².
Answer (a), (c)

4 Rectangles with length 3 and width 4 cm are piled up.

1 Summarize the relationship between the length and the width in following the table.

Length \bigcirc (cm)	3	6	9	12	15	18	...
Area \square (cm ²)	12	24	36	48	60	72	...

2 Write the math sentence by using words length \bigcirc cm and area \square cm².
Math sentence by words
 $12 \times \text{Length} = \text{Area}$
Answer $4 \times \bigcirc = \square$

4 - 1 Multiplication of Decimal Numbers
Multiplying with Decimal Numbers (1)

Example A 1 m long ribbon costs 60 zeds*. How much does a 2.3 m ribbon cost? (*zed(s) is the fictional currency unit.)

The math sentence is "60 × 2.3," but how can we calculate it?

According to the above diagram, we can make a math sentence of 60×2.3 . This multiplication problem can be solved in the following way.

$$60 \times 2.3 = 138$$

$$60 \times 23 = 1380$$

Reference: The multiplication algorithm can be shown as follows:

$$\begin{array}{r} 60 \\ \times 2.3 \\ \hline 180 \\ 1200 \\ \hline 1380 \end{array}$$

The multiplication algorithm with decimal numbers will be explained in detail from the section 4.4.

Math sentence
 $60 \times 2.3 = 60 \times 23 \div 10 = 1380 \div 10 = 138$ **Answer** 138 zeds

1 A 1 m long hose costs 80 zeds. How much does a 3.5 m hose cost? (*zed(s) is the fictional currency unit.)

Math sentence
 80×3.5
 $= 80 \times 35 \div 10$
 $= 2800 \div 10 = 280$

Answer 280 zeds

2 A 1 m long stick weighs 180 g. How much does a 1.6 m stick weigh?

Math sentence
 180×1.6
 $= 180 \times 16 \div 10$
 $= 2880 \div 10 = 288$

Answer 288 g

Complete the number line diagram and table.

4 - 2 Multiplication of Decimal Numbers
Multiplying with Decimal Numbers (2)

Example A 1 m long pipe weighs 1.8 kg. How much does a 4.2 m pipe weigh?

The math sentence is "1.8 × 4.2," but how can we calculate it?

According to the above diagram, we can make a math sentence of 1.8×4.2 . This multiplication problem can be solved in the following way.

$$1.8 \times 4.2 = 7.56$$

$$18 \times 42 = 756$$

Reference: The multiplication algorithm can be shown as follows:

$$\begin{array}{r} 1.8 \\ \times 4.2 \\ \hline 36 \\ 720 \\ \hline 7.56 \end{array}$$

The multiplication algorithm with decimal numbers will be explained in detail from the section 4.4.

Math sentence
 $1.8 \times 4.2 = 18 \times 42 \div 100 = 756 \div 100 = 7.56$ **Answer** 7.56 kg

1 A 1 m long iron pipe weighs 2.1 kg. How much does a 3.5 m pipe weigh?

Math sentence
 2.1×3.5
 $= 21 \times 35 \div 100$
 $= 735 \div 100 = 7.35$

Answer 7.35 kg

2 1 dL of paint was used to paint 1.5 m² of wall. How many m² can be painted with 2.7 dL of paint?

Math sentence
 1.5×2.7
 $= 15 \times 27 \div 100$
 $= 405 \div 100 = 4.05$

Answer 4.05 m²

Complete the number line diagram and table.

4 - 3 Multiplication of Decimal Numbers
Multiplication of Decimal Numbers (1)

Example Find the product of each of the following based on $176 \times 54 = 9504$.

1 $17.6 \times 54 = 950.4$

2 $1.76 \times 5.4 = 9.504$

1 Find the product of each of the following based on $254 \times 39 = 9906$.

1 $25.4 \times 39 = 990.6$

2 $254 \times 3.9 = 990.6$

3 $25.4 \times 3.9 = 99.06$

4 $2.54 \times 3.9 = 9.906$

5 $2.54 \times 39 = 99.06$

Pay attention to the location of the decimal point.

2 Find the product of each of the following based on $312 \times 56 = 17472$.

1 $31.2 \times 56 = 1747.2$

2 $312 \times 5.6 = 1747.2$

3 $3.12 \times 5.6 = 17.472$

4 $31.2 \times 5.6 = 174.72$

5 $3.12 \times 56 = 174.72$

6 $0.312 \times 56 = 17.472$

3 Find the product of each of the following based on $47 \times 851 = 39997$.

1 $4.7 \times 851 = 3999.7$

2 $47 \times 85.1 = 3999.7$

3 $4.7 \times 85.1 = 399.97$

4 $47 \times 8.51 = 399.97$

5 $4.7 \times 8.51 = 39.997$

6 $0.47 \times 851 = 399.97$

4 - 4 Multiplication of Decimal Numbers
Multiplication of Decimal Numbers (1)

Example Calculate 1.8×4.2 by using the multiplication algorithm.

Even when the multiplier is a decimal number, we can calculate just like we did with whole numbers!

Disregard the decimal points for now. Multiply as if they were whole numbers.

To determine the location of the decimal point of the product, add the number of places that are to the right of the decimal points of the multiplicand (1 place) and the multiplier (1 place). Then move the decimal point of the product from right to left the same number of places (2 places).

Calculate the following multiplication problems by using the multiplication algorithm.

1 2.1×3.8

2 5.4×1.9

3 1.3×2.7

4 0.8×4.4

5 4.1×3.7

6 2.7×3.4

7 0.7×6.8

8 3.5×2.3

9 2.5×3.3

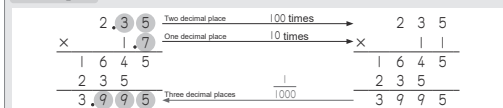
10 4.4×5.2

11 0.5×7.3

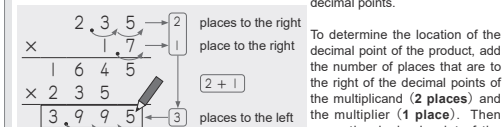
12 7.6×1.2

4 - 5 Multiplication of Decimal Numbers
Multiplication of Decimal Numbers (2)

Example Calculate 2.35×1.7 by using the multiplication algorithm.



Multiply as if there were no decimal points.



To determine the location of the decimal point of the product, add the number of places that are to the right of the decimal points of the multiplicand (2 places) and the multiplier (1 place). Then move the decimal point of the product from right to left the same number of places (3 places).

Calculate the following multiplication problems by using the multiplication algorithm.

1 1.69×2.5 2 2.04×9.2 3 0.53×4.3 4 0.17×6.2

5 2.36×4.6 6 3.61×2.5 7 4.75×1.5 8 9.99×9.9

9 5.03×2.8 10 4.18×7.5 11 6.54×3.13 12 8.25×4.44

We can cross out "0" at the end of the decimal numbers.

Regarding Problems 11 and 12, the decimal point moves 4 places to the left!

5	10.856	6	9.025	7	7.125	8	98.901
9	14.084	10	31.350	11	20.4702	12	36.6300

4 - 6 Multiplication of Decimal Numbers
Multiplier and the Size of the Product

Example A 1 m long iron bar weighs 20 kg. How much does a 1.8 m long iron bar weigh? How much does a 0.8 m long iron bar weigh?



Weight of the 1.8 m iron bar

Math sentence: $20 \times 1.8 = 36$
Answer: 36 kg

Weight of the 0.8 m iron bar

Math sentence: $20 \times 0.8 = 16$
Answer: 16 kg

When we multiply by a number less than 1, the product will be less than the multiplicand.

1 There is a 1 m wire that weighs 400 g. How many g does 0.7 m of wire weigh? In addition, how many does 1.7 m of wire weigh?

Weight of 1.7 m wire

Math sentence: $400 \times 1.7 = 680$
Answer: 680 g

Weight of 0.7 m wire

Math sentence: $400 \times 0.7 = 280$
Answer: 280 g

2 Which of the following will have a product that is less than 15?

(a) 15×0.9 (b) 15×1.4 (c) 15×2.08 (d) 15×0.76 Answer: (a), (d)

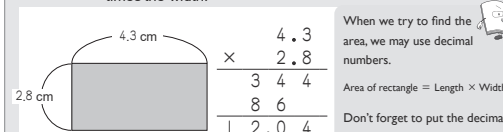
3 Calculate the following multiplication problems by using the algorithm.

1 4.18×0.3 2 1.94×0.6 3 0.56×0.49 4 0.37×0.85

Are all the products smaller than the multiplicand?

4 - 7 Multiplication of Decimal Numbers
Calculating Area

Example To find the area of the following shape, multiply the length times the width.



Math sentence: $4.3 \times 2.8 = 12.04$
Answer: 12.04 cm²

When we try to find the area, we may use decimal numbers.

Area of rectangle = Length \times Width

Don't forget to put the decimal point in the correct place in your answer!

To find the area of the following shapes, multiply the length times the width.

Area of rectangle = Length \times Width

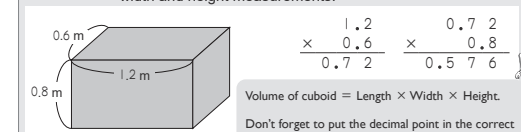
1 $7.2 \times 5.6 = 40.32$ Math sentence: $7.2 \times 5.6 = 40.32$ Answer: 40.32 cm²

2 $4.8 \times 4.3 = 23.04$ Math sentence: $4.8 \times 4.3 = 23.04$ Answer: 23.04 cm²

3 $12.4 \times 3.5 = 43.40$ Math sentence: $12.4 \times 3.5 = 43.40$ Answer: 43.4 cm²

4 - 8 Multiplication of Decimal Numbers
Calculating Volume

Example To find the volume of the following shape, multiply its length, width and height measurements.



Math sentence: $1.2 \times 0.6 \times 0.8 = 0.72 \times 0.8 = 0.576$
Answer: 0.576 m³

Volume of cuboid = Length \times Width \times Height.

Don't forget to put the decimal point in the correct place in your answer!

To find the volume of the following shapes, multiply its length, width and height measurements.

Volume of cuboid = Length \times Width \times Height

1 $1.8 \times 0.9 \times 1.3 = 2.106$ Math sentence: $1.8 \times 0.9 \times 1.3 = 1.62 \times 1.3 = 2.106$ Answer: 2.106 m³

2 $0.7 \times 0.7 \times 0.7 = 0.343$ Math sentence: $0.7 \times 0.7 \times 0.7 = 0.49 \times 0.7 = 0.343$ Answer: 0.343 m³

3 $3.6 \times 0.8 \times 0.4 = 1.152$ Math sentence: $3.6 \times 0.8 \times 0.4 = 2.88 \times 0.4 = 1.152$ Answer: 1.152 m³

4 - 9 Multiplication of Decimal Numbers
Usage of Properties of Operations

Example The properties of operations for whole numbers also applies to decimal numbers.

The properties of operations are

- ① $\blacksquare \times \bullet = \bullet \times \blacksquare$
- ② $(\blacksquare \times \bullet) \times \blacktriangle = \blacksquare \times (\bullet \times \blacktriangle)$
- ③ $(\blacksquare + \bullet) \times \blacktriangle = \blacksquare \times \blacktriangle + \bullet \times \blacktriangle$
- ④ $(\blacksquare - \bullet) \times \blacktriangle = \blacksquare \times \blacktriangle - \bullet \times \blacktriangle$

Example Rewrite the following math sentences using the properties of operations. Then solve the problem.

- ① $4.8 \times 4 \times 2.5 = 4.8 \times (4 \times 2.5) = 4.8 \times 10 = 48$
- ② $2.4 \times 1.8 + 2.6 \times 1.8 = (2.4 + 2.6) \times 1.8 = 5 \times 1.8 = 9$
- ③ $5.7 \times 1.8 - 3.7 \times 1.8 = (5.7 - 3.7) \times 1.8 = 2.0 \times 1.8 = 3.6$

Rewrite the following math sentences using the properties of operations. Then solve the problem.

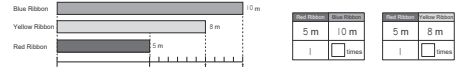
Think about to which example problem the following problems are similar.

- ① $9.2 \times 4 \times 2.5 = 9.2 \times (4 \times 2.5) = 9.2 \times 10 = 92$
- ② $2.5 \times 4 \times 8.4 = (2.5 \times 4) \times 8.4 = 10 \times 8.4 = 84$
- ③ $4 \times 6.8 \times 2.5 = (4 \times 2.5) \times 6.8 = 10 \times 6.8 = 68$
- ④ $0.7 \times 9.8 + 0.3 \times 9.8 = (0.7 + 0.3) \times 9.8 = 1 \times 9.8 = 9.8$
- ⑤ $2.7 \times 0.21 + 0.3 \times 0.21 = (2.7 + 0.3) \times 0.21 = 3 \times 0.21 = 0.63$
- ⑥ $1.3 \times 4.1 + 1.3 \times 5.9 = 1.3 \times (4.1 + 5.9) = 1.3 \times 10.0 = 13$
- ⑦ $3.5 \times 4.3 - 2.5 \times 4.3 = (3.5 - 2.5) \times 4.3 = 1.0 \times 4.3 = 4.3$
- ⑧ $0.3 \times 5.9 - 0.3 \times 2.9 = 0.3 \times (5.9 - 2.9) = 0.3 \times 3.0 = 0.9$
- ⑨ $25.5 \times 8 = (25 + 0.5) \times 8 = (25 \times 8) + (0.5 \times 8) = 200 + 4 = 204$

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4 - 10 Multiplication of Decimal Numbers
Times as Much with Decimal Numbers (1)

Example I have three ribbons. The red ribbon is 5 m long, the yellow ribbon is 8 m long and the blue ribbon is 10 m long.



- ① How many times longer is the white ribbon than the red ribbon?
Math sentence $10 \div 5 = 2$ Answer 2 times
- ② How many times longer is the yellow ribbon than red ribbon?
Math sentence $8 \div 5 = 1.6$ Answer 1.6 times

① I have three pieces of tape. The red tape is 40 m long, the blue tape is 60 m long, the green tape is 80 m long.



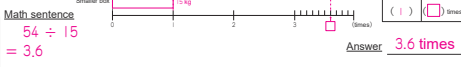
- ① How many times longer is the green tape than the red tape?
Math sentence $80 \div 40 = 2$ Answer 2 times
- ② How many times longer is the blue tape than the red tape?
Math sentence $60 \div 40 = 1.5$ Answer 1.5 times

② I have 28 cards, my older brother has 84 cards, and my older sister has 70 cards.



- ① How many times more cards does my older brother have than me?
Math sentence $84 \div 28 = 3$ Answer 3 times
- ② How many times more cards does my older sister have than me?
Math sentence $70 \div 28 = 2.5$ Answer 2.5 times

③ I have two boxes. The weight of bigger box is 54 kg and the weight of smaller box is 15 kg. How many times heavier is the bigger box than the smaller box?



Complete the number line diagram and table.

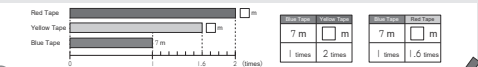
Math sentence $54 \div 15 = 3.6$

Answer 3.6 times

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4 - 11 Multiplication of Decimal Numbers
Times as Much with Decimal Numbers (2)

Example I have three pieces of tape. The blue tape is 7 m long. The yellow tape is 1.6 times longer than the blue tape. The red tape is 2 times longer than the blue tape.



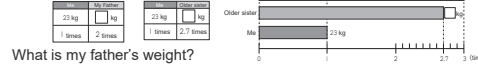
- ① How long is the red tape?
Math sentence $7 \times 2 = 14$ Answer 14 m
- ② How long is the yellow tape?
Math sentence $7 \times 1.6 = 11.2$ Answer 11.2 m

① I have three ribbons. The red ribbon is 12 m long. The yellow ribbon is 1.4 times longer than the red ribbon. The green ribbon is 2 times longer than the red ribbon.



- ① How long is the green ribbon?
Math sentence $12 \times 2 = 24$ Answer 24 m
- ② How long is the yellow ribbon?
Math sentence $12 \times 1.4 = 16.8$ Answer 16.8 m

② I weigh 23 kg. My father weighs 3 times more than me. My older brother weighs 2.7 times more than me.



- ① What is my father's weight?
Math sentence $23 \times 3 = 69$ Answer 69 kg
- ② What is my older brother's weight?
Math sentence $23 \times 2.7 = 62.1$ Answer 62.1 kg

③ There are 32 grade-5 students at my school. At another school, there are 3.5 times more as grade-5 students than at my school. How many grade-5 students are at the other school?



Complete the number line diagram and table.

(32) students	() students
(1) (3.5) times	() () times

Answer 112 students

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4 - 12 Multiplication of Decimal Numbers
Review

① Calculate the following multiplication problems by using the multiplication algorithm.

- ① $3.9 \times 2.3 = 8.97$
- ② $6.2 \times 4.7 = 29.14$
- ③ $3.67 \times 2.4 = 8.808$
- ④ $7.03 \times 1.9 = 13.357$
- ⑤ $5.55 \times 6.4 = 35.52$
- ⑥ $9.86 \times 6.5 = 64.09$
- ⑦ $8.54 \times 2.5 = 21.35$
- ⑧ $0.25 \times 3.2 = 0.8$
- ⑨ $9.16 \times 1.8 = 16.488$
- ⑩ $0.75 \times 0.8 = 0.6$
- ⑪ $8.37 \times 0.36 = 3.0132$
- ⑫ $0.35 \times 0.56 = 0.196$

② Which of the following will have a product that is less than 26?
(a) 26×0.95 (b) 26×1.03 (c) 26×2.5 (d) 26×0.9
Answer (a), (d)

③ Rewrite the following math sentences using the properties of operations. Then solve the problem.

- ① $16.35 \times 2.5 \times 4 = 16.35 \times (2.5 \times 4) = 16.35 \times 10 = 163.5$
- ② $2.4 \times 0.8 + 1.6 \times 0.8 = (2.4 + 1.6) \times 0.8 = 4.0 \times 0.8 = 3.2$
- ③ $7.6 \times 2.5 - 6.6 \times 2.5 = (7.6 - 6.6) \times 2.5 = 1.0 \times 2.5 = 2.5$

④ When a weight was hung on a 7.5 cm long spring, the spring stretched out 2.8 times its original length. How long is the spring with the weight attached?



Complete the number line diagram and table.

(7.5) cm	() cm
(1) (2.8) times	() () times

Answer 21 cm

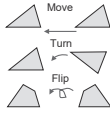
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5 - 1 Congruent Figures
Figures that Overlap Exactly

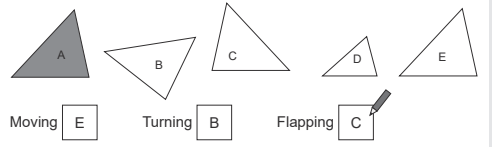
Instruction What is congruent?

- Two figures are said to be **congruent** when both overlap exactly after moving, turning, or flipping them.
- Congruent figures have the same shape and size.

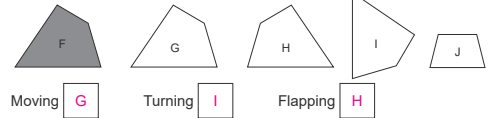
How can we find out congruency when you can't overlap two figures?



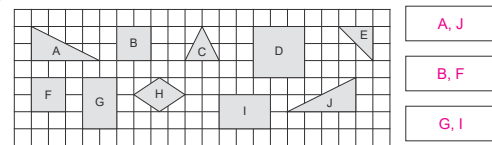
Example Find the congruent figures to figure A below and by how you can find them.



1 Find the congruent figures to figure F below and by how you can find them.



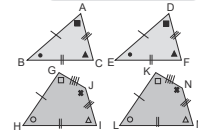
2 Find pairs of the congruent figures.



5 - 2 Congruent Figures
Properties of Congruent Figures

Instruction Properties of congruent figures The same symbol indicates the same length or angle.

- In congruent figures, the corresponding sides have equal lengths. For example, of the triangles, AB and DE, BC and EF, CA and FD.
- The corresponding angles have equal sizes. For example of the quadrilaterals, A and D, B and E, C and F.



Example The following triangles are congruent. Answer the following questions.

1 Which is the corresponding vertex to vertex C?
Vertex

2 Which is the corresponding angle to angle E?
Also, how many degrees is the size?
Angle Size

3 Which is the corresponding side to side DF?
Also, how many cm is it?
Side Length

The following quadrilaterals are congruent. Answer the following questions.

1 Which is the corresponding vertex to vertex E?
Vertex

2 Which is the corresponding angle to angle H?
How many degrees is it?
Angle Size

3 Which is the corresponding side to side FG?
How many cm long is it?
Side Length

5 - 3 Whole Numbers & Decimal Numbers
How to Draw Congruent Triangles

Instruction How to draw a triangle that is congruent to the triangle shown below.

If we determine where the three vertices are, we can draw a triangle.

We can measure the length and draw side BC. The problem is how we determine the position of vertex A.

On how to determine the position of vertex A, we have the following measurements.

1 Measure the length of the 3 sides, BC, AB, AC and then draw them.

Measure 6 cm and draw a circle.	Measure 5.3 cm and draw a circle.	A is determined and draw line AB, AC.
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2 Measure the length of the 2 sides and the angle in between, and then draw them.

Draw a line of 50°.	Measure 6 cm and draw a circle.	A is determined and draw line AC.
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3 Measure the length of the 1 side. Measure the angles formed by that side with the other two sides, and then draw them.

Draw a line of 50°.	Draw a line of 60°.	A is determined.
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Example Draw a triangle that is congruent to the triangle shown below.

1 Measure the length of the 3 sides and then draw it.

2 Draw a line of 70°. Measure 3 cm and draw a circle. A is determined and draw line AC.

3 Draw a line of 40°. Draw a line of 50°. A is determined.

4 Draw a triangle that is congruent to the triangle shown or described below.

1 2 3

5 6 7

8 9

5 - 4 Congruent Figures
How to Draw Congruent Quadrilaterals

Example 1 When you draw a congruent quadrilateral, which sides and angles you should measure?

Set A: Side BC, AB, AD, and DC
Set B: Side BC, angle B, AB, AD, and DC

Remember to measure the length of all 3 sides to draw a congruent triangle...

Why Set A is wrong?

The above two quadrilaterals have the same length of sides. Are they congruent? The size of the angle needs to be measured too.

1 When you draw a congruent quadrilateral, which set of the length of sides and or the size of angles you should measure?

Set A: Side CD, AB, BC, and AD
Set B: Side CD, angle D, AD, AB, and BC

Example 2 Draw a quadrilateral that is congruent to the quadrilateral shown below.

1 **2**

1 Determine the position of vertex B and draw BCD like drawing a congruent triangle by measuring the length of the 3 sides.

Draw ABD like drawing a congruent triangle by measuring the length of the 3 sides.

2 Determine the position of vertex B and draw BCD like drawing a congruent triangle by Measure the size of angle C and the length of side CB.

Determine the position of vertex A and draw ABD like drawing a congruent triangle by measuring the size of angle ABD and the length of side BA.

Congruent quadrilaterals can be drawn by using congruent triangles if the quadrilateral is divided into two triangles on a diagonal.

2 Draw a quadrilateral that is congruent to the quadrilateral shown or described below.

1 **2**

5 - 5 Congruent Figures
Angle of Figures

Instruction Making patterns using congruent triangles and quadrilaterals. Looking at the marked part below.

3 gathered angles become a straight line.

For any triangle, the sum of the three angles is 180° .

Example 1 Find the size of the following labeled angles below.

1 Math sentence $180 - (60 + 90) = 30$ Answer $A = 30^\circ$

2 Math sentence $180 - 135 = 45$ Answer $B = 45^\circ$

1 Find the size of the following labeled angles below.

1 Math sentence $180 - (80 + 40) = 60$ Answer $A = 60^\circ$

2 Math sentence $180 - 55 = 125$ Answer $B = 125^\circ$

Example 2 Find the size of the following labeled angles below.

Math sentence $180 - (95 + 35) = 50$ $A = 50^\circ$
 $180 - (75 + 35) = 70$ $B = 70^\circ$

The sum of all the angles are $(50 + 95 + 35) + (70 + 75 + 35) = 360^\circ$. So, the sum of angles in a quadrilateral is 360° .

2 Find the size of the following labeled angles below. Even if you don't divide quadrilateral into two triangles, since the sum of angles in a quadrilateral is 360° , you can find the angle.

1 Math sentence $180 - (100 + 50) = 30$ Answer $A = 30^\circ, B = 50^\circ$

2 Math sentence $360 - (60 + 80 + 80) = 140$ Answer $B = 140^\circ$

Instruction A figure enclosed by five straight lines is called a pentagon. Find the sum of the five angles of a pentagon.

If you draw straight lines from one vertex to a vertex that is not adjacent. It can be divided into 3 triangles.

Therefore, the sum of the five angles of a pentagon is the sum of the three angles of three triangles.
 $180 \times 3 = 540$ 540°

For any pentagon, the sum of the three angles is 540° .

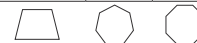
Example 2 A figure enclosed by six straight lines is called a hexagon. Find the sum of the six angles of a hexagon.

Math sentence $180 \times 4 = 720$ Answer 720°

A figure that is enclosed only by straight lines, such as triangles, quadrilaterals, pentagons, hexagons is called a polygon.
 Each straight line connects any two vertices that are not adjacent is called a diagonal.

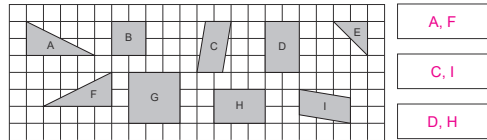
2 Complete the table below.

	Triangle	Quadrilateral	Heptagon	Octagon
Number of triangles	1	2	5	6
Sum of the angles	180°	360°	900°	1080°



5 - 6 Congruent Figures **Review**

1 Find pairs of the congruent figures.



2 The following figures are congruent. Answer the following questions.

1 (a) Which is the corresponding vertex to vertex F? Vertex **B**

(b) Which is the corresponding angle to angle E? How many degrees is it? Angle **A** Size **80°**

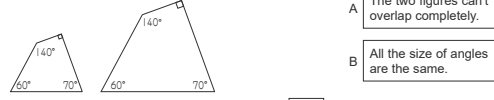
(c) Which is the corresponding side to side EF? How many cm long is it? Side **AB** Length **5.5 cm**

2 (a) Which is the corresponding vertex to vertex E? Vertex **D**

(b) Which is the corresponding angle to angle H? How many degrees is it? Angle **C** Size **80°**

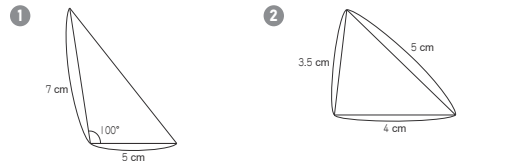
(c) Which is the corresponding side to side FG? How many cm long is it? Side **AB** Length **2.5 cm**

3 The following quadrilaterals are not congruent. Choose the appropriate reason from the followings.

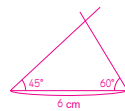


Answer **A**

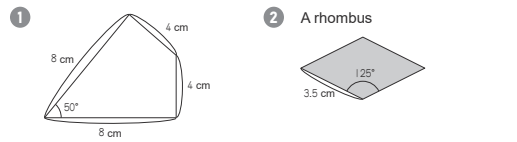
4 Draw a triangle that is congruent to the triangle shown or described below.



3 Triangle with two angles of 45° and 60°. The side in between with 6 cm.



5 Draw a quadrilateral that is congruent to the quadrilateral shown or described below.



6 - 1 Division of Decimal Numbers **Dividing by Decimal Numbers (1)**

Example A 2.6 m long iron pipe weighs 78 kg. How much does 1 m of the pipe weigh?

Math sentence: $78 \div 2.6 = 30$ Answer: 30 kg

1 A 2.4 m long copper pipe weighs 156 kg. How much does 1 m of the pipe weigh?

Math sentence: $156 \div 2.4 = 1560 \div 24 = 65$

Answer: 65 kg

2 A 3.6 m long ribbon costs 540 zeds. How much does 1 m of ribbon cost? ("zed(s)" is the fictional currency unit.)

Math sentence: $540 \div 3.6 = 5400 \div 36 = 150$

Answer: 150 zeds

6 - 2 Division of Decimal Numbers **Dividing by Decimal Numbers (2)**

Example Find the following quotients based on $221 \div 65 = 3.4$.

1 $22.1 \div 6.5 = 3.4$
2 $221 \div 65 = 34$
3 $22.1 \div 65 = 0.34$

We should remember these rules. It is very interesting! Especially, it is careful that the divisor becomes $\frac{1}{10}$, the quotient will be 10 times.

1 Find the quotient of each of the following based on $238 \div 17 = 14$.

1 $23.8 \div 1.7 = 14$ 2 $238 \div 1.7 = 140$
3 $23.8 \div 17 = 1.4$ 4 $2.38 \div 17 = 0.14$

2 Find the quotient of each of the following based on $896 \div 28 = 32$.

1 $89.6 \div 2.8 = 32$ 2 $896 \div 2.8 = 320$
3 $89.6 \div 28 = 3.2$ 4 $8.96 \div 28 = 0.32$

3 Find the quotient of each of the following based on $477 \div 159 = 3$.

1 $47.7 \div 15.9 = 3$ 2 $477 \div 15.9 = 30$
3 $47.7 \div 159 = 0.3$ 4 $477 \div 1.59 = 300$

6 - 3 Division of Decimal Numbers
Division of Decimal Numbers (1)

Example Calculate $4.2 \div 3.5$ by using the division algorithm.

Move the decimal point of the divisor **one place** to the right to make it a whole number (This means that the divisor becomes 10 times).

Move the decimal point of the dividend to the right the same number of places (**one place**) (This means that the dividend also become 10 times).

Divide by the whole number divisor. We can add a 0 at the end of the dividend.

The decimal point of the quotient will be in the same position as the decimal point of the dividend after it was moved to the right.

According to the division rule, the quotient is same even when calculating after multiplying both divisor and dividend by 10.

Calculate the following division problems by using the division algorithm.

1 $5.6 \div 1.6$ 2 $6.5 \div 2.6$ 3 $7.8 \div 6.5$ 4 $5.1 \div 3.4$

5 $3.6 \div 2.4$ 6 $4.2 \div 1.5$ 7 $9.1 \div 2.6$ 8 $7.7 \div 1.4$

9 $36.5 \div 2.5$ 10 $89.3 \div 3.8$ 11 $65.1 \div 4.2$ 12 $57.6 \div 4.5$

5	1.5	6	2.8	7	3.5	8	5.5
9	14.6	10	23.5	11	15.5	12	12.8

6 - 4 Division of Decimal Numbers
Division of Decimal Numbers (2)

Example Calculate $3.45 \div 1.5$ by using the division algorithm.

Move the decimal point of the divisor **one place** to the right to make it a whole number (This means that the divisor becomes 10 times).

Move the decimal point of the dividend to the right the same number of places (**one place**) (This means that the dividend also become 10 times).

Divide by the whole number divisor. The decimal point of the quotient will be in the same position as the decimal point of the dividend after it was moved to the right.

Based on the division rule, we think of $34.5 \div 15$, which has the same quotient.

Calculate the following division problems by using the division algorithm.

1 $6.76 \div 1.3$ 2 $8.51 \div 2.3$ 3 $7.05 \div 4.7$ 4 $9.88 \div 3.8$

5 $4.83 \div 2.3$ 6 $9.18 \div 5.1$ 7 $8.06 \div 6.2$ 8 $9.94 \div 7.1$

9 $5.28 \div 1.6$ 10 $8.12 \div 2.8$ 11 $13.56 \div 2.4$ 12 $20.59 \div 5.8$

5	2.1	6	1.8	7	1.3	8	1.4
9	3.3	10	2.9	11	5.65	12	3.55

6 - 5 Division of Decimal Numbers
Division of Decimal Numbers (3)

Example Calculate $0.63 \div 1.8$ by using the division algorithm.

Move the decimal point of the divisor **one place** to the right to make it a whole number (This means that the divisor becomes 10 times).

Move the decimal point of the dividend to the right the same number of places (**one place**) (This means that the dividend also become 10 times).

Divide by the whole number divisor. We can add a "0" at the end of divided when it is necessary.

The decimal point of the quotient will be in the same position as the decimal point of the dividend after it was moved to the right.

Based on the division rule, we think of $6.30 \div 18$, which has the same quotient.

Calculate the following division problems by using the division algorithm.

1 $0.84 \div 3.5$ 2 $0.42 \div 2.8$ 3 $2.34 \div 3.6$ 4 $8.33 \div 9.8$

5 $1.11 \div 1.5$ 6 $2.21 \div 2.6$ 7 $1.68 \div 4.8$ 8 $3.51 \div 6.5$

9 $3.77 \div 6.5$ 10 $3.24 \div 7.2$ 11 $0.28 \div 3.5$ 12 $0.36 \div 7.2$

5	0.74	6	0.85	7	0.35	8	0.54
9	0.58	10	0.45	11	0.08	12	0.05

6 - 6 Division of Decimal Numbers
Division of Decimal Numbers (4)

Example Calculate $8.547 \div 2.31$ by using the division algorithm.

Move the decimal point of the divisor **two places** to the right to make it a whole number (This means that the divisor becomes 100 times).

Move the decimal point of the dividend to the right the same number of places (**two places**) (This means that the dividend also become 100 times). Think of "854.7 ÷ 231."

Divide by the whole number divisor. The decimal point of the quotient will be in the same position as the decimal point of the dividend after it was moved to the right.

According to the division rule, the quotient is same even when calculating after multiplying both divisor and dividend by 100.

Calculate the following division problems by using the division algorithm.

1 $9.963 \div 3.69$ 2 $3.585 \div 2.39$ 3 $5.024 \div 1.57$ 4 $7.488 \div 4.16$

5 $5.248 \div 3.28$ 6 $9.672 \div 4.03$ 7 $9.184 \div 2.24$ 8 $8.512 \div 1.52$

9 $7.854 \div 1.87$ 10 $8.976 \div 2.64$ 11 $8.568 \div 4.76$ 12 $7.539 \div 3.59$

5	1.6	6	2.4	7	4.1	8	5.6
9	4.2	10	3.4	11	1.8	12	2.1

6 - 7 Division of Decimal Numbers

Division of Decimal Numbers (5)

Example Calculate $7.8 \div 3.25$ by using the division algorithm.

Move the decimal point of the divisor **two places** to the right to make it a whole number (This means that the divisor becomes 100 times).

Move the decimal point of the dividend to the right the same number of places (**two places**) (This means that the dividend also becomes 100 times). Think of "780 ÷ 325."

Divide by the whole number divisor. We can add a "0" at the end of the divided if it is necessary. The decimal point of the quotient will be in the same position as the decimal point of the dividend after it was moved to the right.

Based on the division rule, we think of $780.0 \div 325$, which has the same quotient.

Calculate the following division problems by using the division algorithm.

- $4.6 \div 1.84$
- $1.6 \div 0.25$
- $6.2 \div 2.48$
- $4.2 \div 5.25$
- $6.8 \div 4.25$
- $1.4 \div 1.75$
- $5.5 \div 1.25$
- $5.4 \div 2.25$
- $8.5 \div 1.25$
- $9.2 \div 3.68$
- $6.9 \div 1.84$
- $9.2 \div 7.36$

5	1.6	6	0.8	7	4.4	8	2.4
9	6.8	10	2.5	11	3.75	12	1.25

6 - 8 Division of Decimal Numbers

Division of Decimal Numbers (6)

Example Calculate $3 \div 7.5$ by using the division algorithm.

Move the decimal point of the divisor **one place** to the right to make it a whole number (This means that the divisor becomes 10 times).

Move the decimal point of the dividend to the right the same number of places (**one place**) (This means that the dividend also becomes 10 times) and write a "0" in the dividend.

Divide by the whole number divisor. We can add a "0" at the end of divided when it is necessary. The decimal point of the quotient will be in the same position as the decimal point of the dividend after it was moved to the right.

Based on the division rule, we think of $30 \div 75$, which has the same quotient.

Calculate the following division problems by using the division algorithm.

- $6 \div 2.4$
- $4 \div 2.5$
- $7 \div 2.8$
- $12 \div 1.25$
- $4 \div 1.6$
- $42 \div 5.6$
- $8 \div 2.5$
- $6 \div 7.5$
- $63 \div 8.4$
- $28 \div 2.5$
- $84 \div 1.12$
- $54 \div 22.5$

5	2.5	6	7.5	7	3.2	8	0.8
9	7.5	10	11.2	11	75	12	2.4

6 - 9 Division of Decimal Numbers

Size of the Quotients

Example I have a 1.2 m long plastic bar that weighs 24 kg and a 0.8 m long metal bar that weighs 24 kg. How much does 1 m of each bar weigh?

Plastic bar
Weight: 24 kg, Length: 1.2 m
Math sentence: $24 \div 1.2 = 20$
Answer: 20 kg

Metal bar
Weight: 24 kg, Length: 0.8 m
Math sentence: $24 \div 0.8 = 30$
Answer: 30 kg

When dividing by decimal numbers less than 1, the quotient will be greater than the dividend.

If a 2 m long bar weighs 24 kg, we can find 12 kg per m by using division ($24 \div 2$). When the lengths of bar are 1.2 m and 0.8 m, we also can use division.

- I have 1.2 m long iron wire that weighs 36 kg and a 0.9 m long copper wire that weighs 36 kg. How much does 1 m of each wire weigh?

Iron wire
Math sentence: $36 \div 1.2 = 30$
Answer: 30 kg

Copper wire
Math sentence: $36 \div 0.9 = 40$
Answer: 40 kg

Complete the number line diagram and table.

- Which of the following will have a quotient that is greater than 8?
(a) $8 \div 1.5$ (b) $8 \div 0.02$ (c) $8 \div 0.64$ (d) $8 \div 5$ Answer: (b), (c)
- Calculate the following division problems by using the algorithm.

- $2.7 \div 0.5$
- $7.98 \div 0.6$
- $0.342 \div 0.45$
- $0.9 \div 0.72$

Are all the quotients larger than the dividends?

6 - 10 Division of Decimal Numbers

Remainder with Division of Decimal Numbers

Example A 2.5 m ribbon is cut into 0.7 m pieces. How many 0.7 m pieces of ribbon are there? How long is the remaining piece?

Note that the decimal point of the quotient and the decimal point of the remainder are different!

In division of decimal numbers, the decimal point of the remainder will be in the same place that decimal point of the dividend was in before it was moved.

Math sentence: $2.5 \div 0.7 = 3 \text{ R } 0.4$
Answer: 3 people can get and 0.4 m will be left over.

Check the answer: (Divisor \times Quotient + Remainder = Dividend)
 $0.7 \times 3 + 0.4 = 2.5$

- Find the whole number quotient and the remainder by using the algorithm.

- $4.9 \div 2.3$
- $6.8 \div 1.5$
- $17.5 \div 9.6$
- $25.8 \div 6.4$
- $340 \div 7.5$

- A 17.5 kg fertilizer is divided into 3.2 kg small packages. How many small packages can we make? How many is the remaining fertilizer?

Math sentence: $17.5 \div 3.2 = 5 \text{ R } 1.5$
Answer: We can make 5 packages and 1.5 kg will be left over.

- A 24.5 m rope is cut into 5.6 m pieces to make jump ropes. How many jump ropes can we make? How long is the remaining piece of rope?

Math sentence: $24.5 \div 5.6 = 4 \text{ R } 2.1$
Answer: We can make 4 jump ropes and 2.1 m will be left over.

Complete the number line diagram.

6 - 11 Division of Decimal Numbers
Rounding the Quotients

Example 1.4 L of sand weighs 2.6 kg. How much does 1 L of this sand weigh? Round the quotient to the second highest place.

Weight $\frac{\square}{1.4}$ 2.6 (kg)

Amount $\frac{1}{1.4}$ 1.4 (L)

Math sentence: $2.6 \div 1.4 = 1.857$

Answer: Approximately 1.9 kg

To round to the second highest place, we have to look at the numeral in the third highest place.

According to the previous learning, The remainder is "0.010".

1 Calculate the following. Round the quotient to the second highest place.

- 1 $5.2 \div 6.8$ 2 $4.1 \div 6.8$ 3 $7.5 \div 4.2$ 4 $4.32 \div 7.8$ 5 $7 \div 8.9$
- 1 0.76 2 0.60 3 1.78 4 0.55 5 0.78

2 1.8 m of hose weighs 1.2 kg. How much does 1 m of this hose weigh? Round the quotient to the second highest place.

Math sentence: $1.2 \div 1.8 = 0.666$

Answer: Approximately 0.67 kg

3 3.9 m² of iron plate weighs 4.8 kg. How much does 1 m² of this iron plate weigh? Round the quotient to the second highest place.

Math sentence: $3.9 \div 4.8 = 0.8125$

Answer: Approximately 0.81 kg

6 - 12 Division of Decimal Numbers
Division and Times as Much with Decimal Numbers (1)

Example My water bottle contains 2.7 L of water. My brother's water bottle contains 1.8 L of water. How many times more litres of water do I have compared to my brother?

Amount of water $\frac{1.8}{1.8}$ 2.7 (L)

Times $\frac{1}{1.8}$ 2 (times)

Math sentence: $2.7 \div 1.8 = 1.5$

Answer: 1.5 times

Even when we have decimal numbers, we can use division to find out how many times one quantity is compared to a base amount.

1 My water bottle contains 8.5 L of black tea. My friend's bottle contains 3.4 L of black tea. How many times more litres of black tea do I have compared to my friend?

Math sentence: $8.5 \div 3.4 = 2.5$

Answer: 2.5 times

2 A pony at my farm weighed 50 kg at birth. A half year later, it now weighs 190 kg. How many times heavier does the pony weigh now compared to when it was born?

Math sentence: $190 \div 50 = 3.8$

Answer: 3.8 times

3 The distance from the train station to my house is 2.1 km. The distance from the train station to my friend's house is 3.5 km. How many times is my house from the train station compared to my friend's house?

Math sentence: $2.1 \div 3.5 = 0.6$

Answer: 0.6 times

6 - 13 Division of Decimal Numbers
Division and Times as Much with Decimal Numbers (2)

Example A 10-day old puppy 630 g. This weight is 1.8 times heavier than when it was born. How much did the puppy weigh at birth?

Weight $\frac{\square}{1.8}$ 630 (g)

Times $\frac{1}{1.8}$ 1.8 (times)

Math sentence: $630 \div 1.8 = 350$

Answer: 350 g

We can also think of this problem as the multiplication sentence, $\square \times 1.8 = 630$.

1 A 15-day old kitten weighs 900 g. This weight is 3.6 times heavier than when it was born. How much did the kitten weigh at birth?

Math sentence: $900 \div 3.6 = 250$

Answer: 250 g

2 The area of Town A is 13.8 km². This is 0.6 times the area of Town B. What is the area of Town B?

Math sentence: $13.8 \div 0.6 = 23$

Answer: 23 km²

3 I have two coloured pencils. The red pencil is 9.5 cm long. It is 1.25 times longer than the blue pencil. How long is the blue pencil?

Math sentence: $9.5 \div 1.25 = 7.6$

Answer: 7.6 cm

6 - 14 Division of Decimal Numbers
Division and Times as Much with Decimal Numbers (3)

Example The table shows the price of a notebook and a dictionary in 1990 and 2010. Calculate how many times more the prices increased for each item. Which item increased more times.

	1990	2010
Notebook	120 zeds*	150 zeds
Dictionary	1500 zeds	1530 zeds

(*zed(s)* is the fictional currency unit.)

Notebook Price $\frac{120}{120}$ 150 (zeds)

Times $\frac{1}{120}$ 1.25 (times)

Dictionary Price $\frac{1500}{1500}$ 1530 (zeds)

Times $\frac{1}{1500}$ 1.02 (times)

The price of the notebook: Math sentence: $150 \div 120 = 1.25$

The price of the dictionary: Math sentence: $1530 \div 1500 = 1.02$

Answer: The notebook

We are comparing by finding out what the process would be if we consider the prices for each item in 1990 as 1.

The table shows the price of a tennis ball and a tennis racket in 2000 and 2010. Calculate how many times more the prices increased for each item? Which item increased more times?

	2000	2010
Tennis ball	300 zeds	420 zeds
Tennis racket	4500 zeds	5400 zeds

(*zed(s)* is the fictional currency unit.)

Tennis ball Price $\frac{300}{300}$ 420 (zeds)

Times $\frac{1}{300}$ 1.4 (times)

Tennis racket Price $\frac{4500}{4500}$ 5400 (zeds)

Times $\frac{1}{4500}$ 1.2 (times)

Answer: The tennis ball

Complete the number line diagram and table.

6 - 15 Division of Decimal Numbers

Review (1)

1 Find the quotient of each of the following based on $731 \div 43 = 17$.

- 1 $73.1 \div 4.3 = 17$ 2 $731 \div 4.3 = 170$
 3 $73.1 \div 43 = 1.7$ 4 $731 \div 0.43 = 1700$
 5 $7.31 \div 43 = 0.17$ 7 $7.31 \div 4.3 = 1.7$

2 Which of the following will have a quotient that is greater than the dividend?

- (a) $36 \div 1.5$ (b) $81 \div 0.9$ (c) $0.066 \div 1.1$ (d) $35.7 \div 0.85$

When dividing by decimal numbers less than 1, the quotient will be greater than the dividend.

Answer (b), (d)

3 Calculate the following division problems by using the division algorithm.

- 1 $5.88 \div 1.4$ 2 $9.52 \div 3.4$ 3 $9.45 \div 2.7$ 4 $4.93 \div 2.9$

$1.4 \overline{) 5.88}$ $\underline{56}$ 28 $\underline{28}$ 0	$3.4 \overline{) 9.52}$ $\underline{68}$ 272 $\underline{272}$ 0	$2.7 \overline{) 9.45}$ $\underline{54}$ 405 $\underline{405}$ 0	$2.9 \overline{) 4.93}$ $\underline{58}$ 293 $\underline{293}$ 0
--------------------------------------------------------------------------------	----------------------------------------------------------------------------------	----------------------------------------------------------------------------------	----------------------------------------------------------------------------------

- 5 $1.61 \div 4.6$ 6 $1.45 \div 5.8$ 7 $2.53 \div 4.6$ 8 $5.85 \div 7.8$
 9 $0.27 \div 1.8$ 10 $0.39 \div 1.5$ 11 $2.16 \div 4.5$ 12 $4.51 \div 5.5$
 13 $2.79 \div 6.2$ 14 $2.21 \div 3.4$ 15 $4.14 \div 1.8$ 16 $9.66 \div 2.1$

5	0.35	6	0.25	7	0.55	8	0.75
9	0.15	10	0.26	11	0.48	12	0.82
13	0.45	14	0.65	15	2.3	16	4.6

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4 Calculate the following division problems by using the division algorithm.

- 1 $5.7 \div 2.28$ 2 $1.1 \div 0.25$ 3 $4.2 \div 1.75$ 4 $3.4 \div 4.25$

$2.28 \overline{) 5.700}$ $\underline{456}$ 1140 $\underline{1140}$ 0	$0.25 \overline{) 1.100}$ $\underline{100}$ 1000 $\underline{1000}$ 0	$1.75 \overline{) 4.200}$ $\underline{350}$ 700 $\underline{700}$ 0	$4.25 \overline{) 3.400}$ $\underline{340}$ 000 $\underline{3400}$ 0
---------------------------------------------------------------------------------------	---------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------	--------------------------------------------------------------------------------------

- 5 $2.3 \div 0.92$ 6 $9.3 \div 12.4$ 7 $2.7 \div 2.25$ 8 $1.6 \div 0.25$
 9 $12 \div 1.6$ 10 $33 \div 7.5$ 11 $14 \div 2.5$ 12 $54 \div 7.2$
 13 $8.528 \div 3.28$ 14 $3.925 \div 1.57$ 15 $7.491 \div 2.27$ 16 $7.905 \div 4.65$

5	2.5	6	0.75	7	1.2	8	6.4
9	7.5	10	4.4	11	5.6	12	7.5
13	2.6	14	2.5	15	3.3	16	1.7

5 A 1.5 m long hose weighs 270 g. How much does 1 m of the hose weigh?

Math sentence: $270 \div 1.5 = 180$

Weight: $(270) \text{ g}$
 Length: $(1.5) \text{ m}$

Complete the number line diagrams and tables.

Answer 180 g

6 A car drive 7.5 km using 0.6 L of gasoline. How far can a car drive on 1 L of gasoline?

Math sentence: $7.5 \div 0.6 = 12.5$

Distance: $(7.5) \text{ km}$
 Gasoline: $(0.6) \text{ L}$

Answer 12.5 km

7 2.4 m of plastic stick weighs 10.8 kg. How much does 1 m of this plastic stick weigh?

Math sentence: $10.8 \div 2.4 = 4.5$

Weight: $(10.8) \text{ kg}$
 Length: $(2.4) \text{ m}$

Answer 4.5 kg



Complete the number line diagrams and tables.

6 - 16 Division of Decimal Numbers

Review (2)

1 Find the whole number quotient and the remainder by using the algorithm.

- 1 $5.8 \div 2.6$ 2 $6.5 \div 3.7$ 3 $4.7 \div 2.3$ 4 $7.2 \div 1.5$
 5 $28.2 \div 3.7$ 6 $45.3 \div 8.7$ 7 $51.6 \div 6.4$ 8 $88.6 \div 9.2$
- 1 **2 R 0.6** 2 **1 R 2.8** 3 **2 R 0.1** 4 **4 R 1.2**
 5 **7 R 2.3** 6 **5 R 1.8** 7 **8 R 0.4** 8 **9 R 5.8**

2 Calculate the following. Round the answer to the second highest place.

- 1 $3.1 \div 1.9$ 2 $4.2 \div 2.7$ 3 $5.8 \div 6.9$ 4 $7.1 \div 9.2$
 5 $6.54 \div 7.3$ 6 $2.89 \div 4.3$ 7 $2 \div 5.8$ 8 $5 \div 8.2$
- 1 **1.63** 2 **1.55** 3 **0.84** 4 **0.77**
 5 **0.895** 6 **0.672** 7 **0.34** 8 **0.602**

3 A 3.3 m ribbon is cut into 0.5 m pieces to provide children. How many 0.5 m long pieces of ribbon are there? How long is the remaining piece?

Math sentence: $3.3 \div 0.5 = 6 \text{ R } 0.3$

Answer: We can get 6 pieces and 0.3 m will be left.

4 There are two ribbons. The blue ribbon is 5.5 m long and red ribbon is 6.6 m long. How many times difference is the red ribbon compared to the blue ribbon?

Math sentence: $6.6 \div 5.5 = 1.2$

Answer: 1.2 times

5 I walked 8.5 km yesterday and 3.4 km today. How many times longer did I walk yesterday compared to today?

Math sentence: $8.5 \div 3.4 = 2.5$

Answer: 2.5 times

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7 - 1 Whole Numbers

Even and Odd Numbers (1)

Example A classroom of students is split into two teams. Each student draws a numbered card to determine which team they belong to.

Red Team: 1, 3, 5, 7, ...
 White Team: 2, 4, 6, 8, ...

Even numbers are divisible by 2 and odd numbers are not divisible by 2.

- 1 A student draws a card with a 10 on it. Which team does he belong to? **White team**
 2 A student draws a card with an 11 on it. Which team does she belong to? **Red team**
 3 What kind of numbers are on the red team? **Odd numbers**
 4 What kind of numbers are on the white team? **Even numbers**

A group of children is split into two teams. Each child draws a numbered card to determine which team they belong to.

BLUE: 1, 3, 5, ... RED: 2, 4, 6, ...

- 1 Which team does the child who draw a 16 card belong to? **Red team**
 2 Which team does the child who draw a 17 card belong to? **Blue team**
 3 Which team does the child who draw a 21 card belong to? **Blue team**
 4 What kind of numbers are on the blue team and the red team?
 Blue Team: **Odd numbers** Red Team: **Even numbers**

5 Circle the numbers on the following number line, which belong to the blue team.



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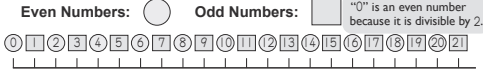
7 - 2 Whole Numbers
Even and Odd Numbers (2)

Instruction Whole numbers can be either odd numbers or even numbers.

Even Numbers: These numbers can be divided by 2 without a remainder.
 $0 \rightarrow 0 \div 2 = 0$
 $2 \rightarrow 2 \div 2 = 1$
 $4 \rightarrow 4 \div 2 = 2$
 $6 \rightarrow 6 \div 2 = 3$
 $8 \rightarrow 8 \div 2 = 4$

Odd Numbers: These numbers cannot be divided by 2 without a remainder. 0 is an even number.
 $1 \rightarrow 1 \div 2 = 0 \text{ R}1$
 $3 \rightarrow 3 \div 2 = 1 \text{ R}1$
 $5 \rightarrow 5 \div 2 = 2 \text{ R}1$
 $7 \rightarrow 7 \div 2 = 3 \text{ R}1$
 $9 \rightarrow 9 \div 2 = 4 \text{ R}1$

Even and odd numbers have an alternating pattern on the number line.



Example Categorize the following numbers as even or odd numbers.

27 42 87 342

Even Numbers: 42, 342 Odd Numbers: 27, 87

What digit should we look at to decide whether or not a number can be divided by 2 without a remainder? We can look at the digit in the ones place!!

1 Categorize the following numbers as even or odd numbers.

12 35 60 107 523 1268

Even Numbers: 12, 60, 1268 Odd Numbers: 35, 107, 523

2 Are the following even numbers or odd numbers?

1 9876 **2** 12345 **3** 736452 **4** 2938470 **5** 10000001

1 Even number **2** Odd number **3** Even number **4** Even number **5** Odd number

7 - 3 Whole Numbers
Multiples and Common Multiples (1)

Instruction A multiple is a general term for a number multiplied by a whole number.

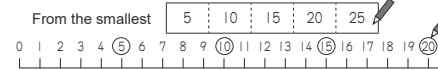
Multiples of 2: 2, 4, 6, 8, 10, 12, 14, ...
 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, ...
 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, ...
 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

The number 0 is not considered a multiple.
 We can find the multiples of 2 by doing $2 \times 1 = 2$, $2 \times 2 = 4$, $2 \times 3 = 6$, $2 \times 4 = 8$, $2 \times 5 = 10$, $2 \times 6 = 12$, $2 \times 7 = 14$, $2 \times 8 = 16$, $2 \times 9 = 18$, ...

Example Write five multiples of 5 starting from the smallest number and Circle these numbers showed on the number line.



Write five following multiples starting from the smallest number and circle these numbers showed on the number line < only for 1 and 2 >.

- Multiples of 6: 6, 12, 18, 24, 30
 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
- Multiples of 7: 7, 14, 21, 28, 35
 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
- Multiples of 8: 8, 16, 24, 32, 40
- Multiples of 9: 9, 18, 27, 36, 45
- Multiples of 10: 10, 20, 30, 40, 50

7 - 4 Whole Numbers
Multiples and Common Multiples (2)

Instruction Multiples of 2 and multiples of 3 can be shown as follows:

Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, ...
 Multiples of 3: 3, 6, 9, 12, 15, 18, 21, ...

6, 12 and 18 are numbers found in both groups. These numbers are called **common multiples** of 2 and 3. 6 is the smallest common multiple number of 2 and 3. This is called the **least common multiple**.

Example Write the first ten multiples of 4 and the first 10 multiples of 5. Find three common multiples of 4 and 5. Find the least common multiples of 4 and 5.

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40
 Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
 Common multiples of 4 and 5: 20, 40, 60 Least common multiple of 4 and 5: 20

Continue finding the multiples of 4 and 5. We can find 3 common multiples.

1 Write the first ten multiples of 2 and the first ten multiples of 4. Find three common multiples of 2 and 4. Find the least common multiple of 2 and 4.

Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
 Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40
 Common multiples of 2 and 4: 4, 8, 12 Least common multiple of 2 and 4: 4

2 Write the first ten multiples of 6 and the first ten multiples of 8. Find three common multiples of 6 and 8. Find the least common multiple of 6 and 8.

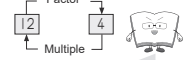
Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
 Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80
 Common multiples of 6 and 8: 24, 48, 72 Least common multiple of 6 and 8: 24

Continue finding the multiples of 6 and 8. We can find 3 common multiples.

7 - 5 Whole Numbers
Factors and Common Factors (1)

Instruction A factor is a number that can divide the number in question evenly with no remainders.

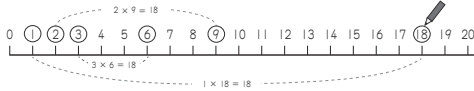
Factors of 8: 1, 2, 4, 8
 Factors of 12: 1, 2, 3, 4, 6, 12
 Factors of 15: 1, 3, 5, 15
 Factors of 7: 1, 7
 Factors of 13: 1, 13



Factors and multiples are related to each other. 4 is a factor of 12. 12 is a multiple of 4.

Prime numbers have only two factors: 1 and the number itself.

Example Circle the factors of 18 on the number line.



Circle the following numbers on the number line.

- Factors of 6: 0 1 2 3 4 5 6 7 8 9 10
- Factors of 10: 0 1 2 3 4 5 6 7 8 9 10
- Factors of 14: 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
- Factors of 20: 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
- Factors of 24: 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

7 - 6 Whole Numbers **Factors and Common Factors (2)**

Instruction Factors of 8 and factors of 12 are as follows;

Factors of 8 : 1, 2, 4, 8
 Factors of 12 : 1, 2, 3, 4, 6, 12

Factors such as 1, 2 and 4 that are factors of both 8 and 12 are called **common factors of 8 and 12**.
 The largest common factor is called the **greatest common factor**.

Example Write the factors of 15 and the factors of 18. Then, write the common factors and the greatest common factor.

Factors of 15 : 1, 3, 5, 15
 Factors of 18 : 1, 2, 3, 6, 9, 18
 Common factors of 15 and 18 : 1, 3
 Greatest common factor of 15 and 18 : 3

- Write the factors of 12 and the factors of 16. Then write the common factors and the greatest common factor.
 Factors of 12 : 1, 2, 3, 4, 6, 12
 Factors of 16 : 1, 2, 4, 8, 16
 Common factors of 12 and 16 : 1, 2, 4
 Greatest common factor of 12 and 16 : 4
- Write the factors of 18, the factors of 27, and the factors of 36. Then write the common factors and the greatest common factor.
 Factors of 18 : 1, 2, 3, 6, 9, 18
 Factors of 27 : 1, 3, 9, 27
 Factors of 36 : 1, 2, 3, 4, 6, 9, 12, 18, 36
 Common factors of 18, 27 and 36 : 1, 3, 9
 Greatest common factor of 18, 27 and 36 : 9

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7 - 7 Whole Numbers **Application of Common Multiples and Factors**

Example A square was made by placing 6 cm wide and 8 cm long papers edge to edge. What is the length of one side of the smallest square that can be made?

A square has the same length on all four sides. We think about the common multiples of 6 and 8, especially the least common multiples. This should be the length of the sides of the smallest square.

Multiples of 6 : 6, 12, 18, 24, 30, 36, ...
 Multiples of 8 : 8, 16, 24, 32, 40, 48, ...
 Least common multiples of 6 and 8 : 24
Answer 24 cm

- Rectangular tiles 3 cm wide and 5 cm long are placed edge to edge to make a square. What is the length of one side of the smallest square that can be made? How many tiles are needed to make the square?
 Multiples of 3 : 3, 6, 9, 12, 15, 18, 21, 24, ...
 Multiples of 5 : 5, 10, 15, 20, 25, 30, ...
 Least common multiples of 3 and 5 : 15
 Width: $15 \div 3 = 5$ Length: $15 \div 5 = 3$ $5 \times 3 = 15$
Answer The length of one side is 15 cm and 15 tiles are needed.
- A box with a height of 5 cm and another box with a height of 7 cm are stacked separately. How many cm are the heights of both boxes the same? How many boxes with 5 cm and 7 cm are there at that time?
 Multiples of 5 : 5, 10, 15, 20, 25, 30, 35, 40, ...
 Multiples of 7 : 7, 14, 21, 28, 35, 42, ...
 Least common multiples of 5 and 7 : 35
 Box with a height of 5 cm: $35 \div 5 = 7$
 Box with a height of 7 cm: $35 \div 7 = 5$
Answer When the height is 35 cm, both boxes have the same height. At that time, 7 boxes with a height 5 cm and 5 boxes with a height of 7 cm.

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7 - 8 Whole Numbers **Review**

- Categorize the following numbers as even or odd numbers.
 8, 15, 63, 100, 398, 2839
 Even Numbers : 8, 100, 398
 Odd Numbers : 15, 63, 2839
- Write the least common multiple of the numbers in each ().
 ① (2, 7) : 14
 ② (4, 10) : 20
 ③ (3, 5, 6) : 30
 We can find it by listing up each multiples of 3, 5, and 6.
 Multiples of 3 : 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
 Multiples of 5 : 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
 Multiples of 6 : 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
- Write the greatest common factor of the numbers in each ().
 ① (32, 40) : 8
 ② (27, 81) : 27
 ③ (4, 12, 18) : 2
 We can find it by listing up each factor of 4, 12, and 18.
 Factors of 4 : 1, 2, 4
 Factors of 12 : 1, 2, 3, 4, 6, 12
 Factors of 18 : 1, 2, 3, 6, 9, 18

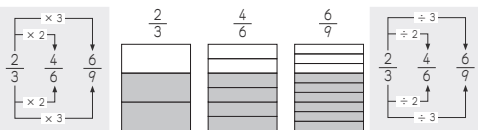
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- (8, 16, 20) : 4 ⑤ (12, 36, 60) : 12
- Which numbers are the prime numbers among the following?
 61, 71, 81, 91, 101, 111
 61, 71, 101
 $81 \div 3 = 27, 81 \div 9 = 9, 91 \div 7 = 13, 91 \div 13 = 7,$
 $111 \div 3 = 37, 111 \div 37 = 3$
- The smallest possible square is made by placing 10 cm wide and 12 cm long rectangular tiles edge to edge. What is the length of one side of the square? How many tiles are needed to make the square?
 Multiples of 10 : 10, 20, 30, 40, 50, 60, 70, ...
 Multiples of 12 : 12, 24, 36, 48, 60, 72, ...
 Least common multiples of 10 and 12 : 60
 Width: $60 \div 10 = 6$ Length: $60 \div 12 = 5$ $6 \times 5 = 30$
Answer The length of sides is 60 cm and 30 tiles are needed.
- We want to cut out squares that are the same size from a piece of paper that is 18 cm wide and 24 cm long with no paper scraps remaining. What size are the squares? How many squares will we have?
 Factors of 18 : 1, 2, 3, 6, 9, 18
 Factors of 24 : 1, 2, 3, 4, 6, 8, 12, 24
 Greatest common factor of 18 and 24 : 6
 Width: $18 \div 6 = 3$ Length: $24 \div 6 = 4$ $3 \times 4 = 12$
Answer The length of sides of the squares is 6 cm and we have 12 squares.

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8 - 1 Addition and Subtraction of Fractions
Fractions of the Same Size

Instruction $\frac{2}{3}$, $\frac{4}{6}$ and $\frac{6}{9}$ are the same size.



The size of a fraction does not change if we multiply or divide both the denominator and the numerator by the same number.

Example Write the appropriate numbers in the .

1 $\frac{3}{4} = \frac{6}{8}$

2 $\frac{1}{5} = \frac{3}{15}$

Regarding (1), we can multiply both the denominator and the numerator by 2.

Write the appropriate numbers in the .

1 $\frac{1}{3} = \frac{2}{6}$

2 $\frac{5}{6} = \frac{10}{12}$

3 $\frac{6}{7} = \frac{18}{21}$

4 $\frac{3}{8} = \frac{9}{24}$

5 $\frac{2}{9} = \frac{6}{27}$

6 $\frac{3}{10} = \frac{12}{40}$

7 $\frac{1}{4} = \frac{3}{12}$

8 $\frac{4}{5} = \frac{24}{30}$

9 $\frac{1}{2} = \frac{9}{18}$

10 $\frac{10}{16} = \frac{5}{8}$

11 $\frac{2}{24} = \frac{1}{12}$

12 $\frac{27}{30} = \frac{9}{10}$

13 $\frac{32}{18} = \frac{16}{9}$

14 $\frac{15}{21} = \frac{5}{7}$

15 $\frac{20}{24} = \frac{5}{6}$

8 - 2 Addition and Subtraction of Fractions
Simplifying the Fraction

Instruction Multiples of the numerator and denominator by the same number are all equal to each other.

For example $\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}, \frac{7}{28}, \frac{8}{32}, \dots$

To **simplify the fraction**, divide the numerator and denominator by the greatest common factor.

When we divide the numerator and denominator by common factor repeatedly such as "÷ 2", and "÷ 3", we can also simplify the fraction.

$\frac{36}{4} = \frac{9}{1} = 9$

$\frac{36}{12} = \frac{3}{1} = 3$

Example Simplify the following fractions.

1 $\frac{7}{21} = \frac{1}{3}$

2 $\frac{4}{10} = \frac{2}{5}$

We can divide both the denominator and the numerator by the greatest common factor, cannot we?

Simplify the following fractions.

1 $\frac{3}{9} = \frac{1}{3}$

2 $\frac{12}{15} = \frac{4}{5}$

3 $\frac{24}{16} = \frac{3}{2}$

4 $\frac{15}{25} = \frac{3}{5}$

5 $\frac{8}{10} = \frac{4}{5}$

6 $\frac{18}{24} = \frac{3}{4}$

7 $\frac{14}{35} = \frac{2}{5}$

8 $\frac{16}{36} = \frac{4}{9}$

9 $\frac{56}{49} = \frac{8}{7}$

10 $2 \frac{14}{18} = 2 \frac{7}{9}$

11 $3 \frac{9}{24} = 3 \frac{3}{8}$

12 $1 \frac{3}{18} = 1 \frac{1}{6}$

13 $4 \frac{9}{30} = 4 \frac{3}{10}$

14 $1 \frac{21}{36} = 1 \frac{7}{12}$

15 $2 \frac{30}{48} = 2 \frac{5}{8}$

You can also simplify the fraction part of a mixed number.

8 - 3 Addition and Subtraction of Fractions
Finding a Common Denominator

Instruction When comparing two fractions, they must have the same denominator. If they do not, a common denominator must be found. The numerator must be recalculated so that it is the equivalent of the original fraction.

When comparing the size of fractions between $\frac{3}{4}$ and $\frac{4}{5}$, the least common denominator is 20.

$\frac{3}{4} = \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}, \frac{24}{32}, \frac{27}{36}, \frac{30}{40}, \dots$

$\frac{4}{5} = \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \frac{20}{25}, \frac{24}{30}, \frac{28}{35}, \frac{32}{40}, \frac{36}{45}, \frac{40}{50}, \dots$

Example 1 Compare the following fractions by finding a common denominator.

1. Find the **least common denominator** by thinking about multiples of 3 and 5.
Multiples of 3: 3, 6, 9, 12, 15, 18, 21, ...
Multiples of 5: 5, 10, 15, 20, 25, 30, ...
The least common denominator can be found by multiplying the numbers of both denominators (3 × 5).

2. **Convert to fractions** with the least common denominator.

3. Compare two fractions with the same denominator.

$\frac{2}{3} > \frac{3}{5}$

1 Compare the following fractions by finding a common denominator.

1 $\frac{3}{4}, \frac{5}{7} \Rightarrow \frac{21}{28} > \frac{20}{28}$

2 $\frac{9}{5}, \frac{11}{6} \Rightarrow \frac{54}{30} < \frac{55}{30}$

3 $\frac{2}{3}, \frac{3}{4} \Rightarrow \frac{8}{12} < \frac{9}{12}$

4 $\frac{11}{7}, \frac{11}{8} \Rightarrow \frac{88}{56} > \frac{77}{56}$

5 $\frac{3}{5}, \frac{4}{7} \Rightarrow \frac{21}{35} > \frac{20}{35}$

6 $\frac{3}{8}, \frac{4}{9} \Rightarrow \frac{27}{72} < \frac{32}{72}$

Example 2 Compare the following fractions by finding a common denominator.

1. Find the **least common denominator** by thinking about multiples of 3 and 9.
Multiples of 3: 3, 6, 9, 12, 15, 18, 21, ...
Multiples of 9: 9, 18, 27, 36, 45, 54, ...
The least common denominator is the denominator of the larger fraction. In this case, it is 9.

2. **Convert to one fraction** with the least common denominator.

3. Compare two fractions with the same denominator.

$\frac{2}{3} < \frac{7}{9}$

2 Compare the following fractions by finding a common denominator.

1 $\frac{1}{2}, \frac{5}{6} \Rightarrow \frac{3}{6} < \frac{5}{6}$

2 $\frac{5}{6}, \frac{13}{18} \Rightarrow \frac{15}{18} > \frac{13}{18}$

3 $\frac{3}{4}, \frac{7}{12} \Rightarrow \frac{9}{12} > \frac{7}{12}$

4 $\frac{11}{15}, \frac{4}{5} \Rightarrow \frac{11}{15} < \frac{12}{15}$

Example 3 Compare the following fractions by finding a common denominator.

1. Find the **least common denominator** by thinking about multiples of 6 and 8.
Multiples of 6: 6, 12, 18, 24, 30, 36, ...
Multiples of 8: 8, 16, 24, 32, 40, 48, ...
The least common denominator is between the larger denominator of the fraction (8) and the number multiplying both denominators of fractions (6 × 8).

2. **Convert to one fraction** with the least common denominator.

3. Compare two fractions with the same denominator.

$\frac{5}{6} > \frac{7}{8}$

3 Compare the following fractions by finding a common denominator.

1 $\frac{3}{4}, \frac{5}{6} \Rightarrow \frac{9}{12} < \frac{10}{12}$

2 $\frac{5}{10}, \frac{13}{25} \Rightarrow \frac{25}{50} < \frac{26}{50}$

3 $\frac{11}{12}, \frac{5}{9} \Rightarrow \frac{33}{36} > \frac{20}{36}$

4 $\frac{7}{9}, \frac{13}{15} \Rightarrow \frac{35}{45} < \frac{39}{45}$

8 - 4 Addition and Subtraction of Fractions
Addition of Fractions (1)

Example My mother made $\frac{1}{2}$ L and $\frac{1}{3}$ L of orange juice. How much orange juice is there altogether?

The math sentence is $\frac{1}{2} + \frac{1}{3}$.

To calculate this, we must find a common denominator of 2 and 3. The common denominator is 6. Therefore, the math sentence can be changed to $\frac{3}{6} + \frac{2}{6}$.

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

$$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

Math sentence $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

Answer $\frac{5}{6}$ L

1 A small carton of milk contains $\frac{1}{5}$ L of milk and a large carton of milk contains $\frac{1}{2}$ L of milk. How much milk is there altogether?

Math sentence $\frac{1}{5} + \frac{1}{2} = \frac{2}{10} + \frac{5}{10} = \frac{7}{10}$ Answer $\frac{7}{10}$ L

2 Calculate the following by finding a common denominator.

- | | |
|-------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|
| 1 $\frac{1}{4} + \frac{1}{3} = \frac{3}{12} + \frac{4}{12} = \frac{7}{12}$ | 2 $\frac{2}{3} + \frac{3}{7} = \frac{14}{21} + \frac{9}{21} = \frac{23}{21}$ |
| 3 $\frac{2}{5} + \frac{1}{6} = \frac{12}{30} + \frac{5}{30} = \frac{17}{30}$ | 4 $\frac{1}{4} + \frac{2}{5} = \frac{5}{20} + \frac{8}{20} = \frac{13}{20}$ |
| 5 $\frac{4}{7} + \frac{1}{2} = \frac{8}{14} + \frac{7}{14} = \frac{15}{14}$ | 6 $\frac{4}{3} + \frac{6}{5} = \frac{20}{15} + \frac{18}{15} = \frac{38}{15}$ |
| 7 $\frac{3}{8} + \frac{2}{3} = \frac{9}{24} + \frac{16}{24} = \frac{25}{24}$ | 8 $\frac{2}{5} + \frac{1}{4} = \frac{8}{20} + \frac{5}{20} = \frac{13}{20}$ |
| 9 $\frac{1}{2} + \frac{2}{9} = \frac{9}{18} + \frac{4}{18} = \frac{13}{18}$ | 10 $\frac{2}{9} + \frac{3}{5} = \frac{10}{45} + \frac{27}{45} = \frac{37}{45}$ |
| 11 $\frac{1}{6} + \frac{1}{7} = \frac{7}{42} + \frac{6}{42} = \frac{13}{42}$ | 12 $\frac{3}{8} + \frac{3}{5} = \frac{15}{40} + \frac{24}{40} = \frac{39}{40}$ |

8 - 5 Addition and Subtraction of Fractions
Addition of Fractions (2)

Example Calculate $\frac{5}{6} + \frac{2}{3}$.

To calculate this, we must find a common denominator of 6 and 3. The common denominator is 6. Therefore, the math sentence can be changed to $\frac{5}{6} + \frac{4}{6}$.

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

$$\frac{5}{6} + \frac{2}{3} = \frac{5}{6} + \frac{4}{6} = \frac{9}{6} = \frac{3}{2}$$

It is fine to show the answer as a mixed number.

If the answer can be simplified, we should do it. In the above case, we simplify $\frac{9}{6}$ as $\frac{3}{2}$.

Calculate the following by finding a common denominator.

- | | |
|---------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------|
| 1 $\frac{1}{6} + \frac{1}{12} = \frac{2}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$ | 2 $\frac{4}{15} + \frac{2}{5} = \frac{4}{15} + \frac{6}{15} = \frac{10}{15} = \frac{2}{3}$ |
| 3 $\frac{3}{4} + \frac{5}{12} = \frac{9}{12} + \frac{5}{12} = \frac{14}{12} = \frac{7}{6}$ | 4 $\frac{5}{6} + \frac{3}{2} = \frac{5}{6} + \frac{9}{6} = \frac{14}{6} = \frac{7}{3}$ |
| 5 $\frac{11}{36} + \frac{1}{9} = \frac{11}{36} + \frac{4}{36} = \frac{15}{36} = \frac{5}{12}$ | 6 $\frac{1}{13} + \frac{5}{26} = \frac{2}{26} + \frac{5}{26} = \frac{7}{26}$ |
| 7 $\frac{3}{8} + \frac{1}{24} = \frac{9}{24} + \frac{1}{24} = \frac{10}{24} = \frac{5}{12}$ | 8 $\frac{1}{9} + \frac{1}{18} = \frac{2}{18} + \frac{1}{18} = \frac{3}{18} = \frac{1}{6}$ |
| 9 $\frac{5}{12} + \frac{13}{30} = \frac{25}{60} + \frac{26}{60} = \frac{51}{60} = \frac{17}{20}$ | 10 $\frac{11}{30} + \frac{41}{45} = \frac{33}{90} + \frac{82}{90} = \frac{115}{90} = \frac{23}{18}$ |
| 11 $\frac{1}{15} + \frac{5}{6} = \frac{2}{30} + \frac{25}{30} = \frac{27}{30} = \frac{9}{10}$ | 12 $\frac{13}{10} + \frac{13}{15} = \frac{39}{30} + \frac{26}{30} = \frac{65}{30} = \frac{13}{6}$ |

8 - 6 Addition and Subtraction of Fractions
Addition of Fractions (3)

Example 1 Calculate $1\frac{2}{3} + \frac{5}{6}$.

$$1\frac{2}{3} + \frac{5}{6} = 1\frac{4}{6} + \frac{5}{6} = 1\frac{9}{6} = 2\frac{1}{2}$$

It is better to simplify the answer and change it to an appropriate mixed number.

A common denominator must be found in order to calculate this problem. We do not touch the whole number and move to 1 to the answer. After adding two fractions, we must simplify the answer if possible.

In addition, when the part of the fraction of the answer is an improper fraction, we must change it to an appropriate mixed number.

1 Calculate the following by finding a common denominator.

- | | |
|--------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------|
| 1 $1\frac{1}{5} + \frac{1}{3} = 1\frac{3}{15} + \frac{5}{15} = 1\frac{8}{15}$ | 2 $1\frac{1}{6} + \frac{1}{12} = 1\frac{2}{12} + \frac{1}{12} = 1\frac{3}{12} = 1\frac{1}{4}$ |
| 3 $2\frac{1}{2} + \frac{1}{6} = 2\frac{3}{6} + \frac{1}{6} = 2\frac{4}{6} = 2\frac{2}{3}$ | 4 $2\frac{5}{6} + \frac{7}{15} = 2\frac{25}{30} + \frac{14}{30} = 2\frac{39}{30} = 3\frac{13}{10}$ |

Example 2 Calculate $1\frac{2}{3} + 2\frac{5}{6}$.

$$1\frac{2}{3} + 2\frac{5}{6} = 1\frac{4}{6} + 2\frac{5}{6} = 3\frac{9}{6} = 4\frac{1}{2}$$

It is necessary to simplify the answer and change it to an appropriate mixed number.

Use the common denominator of 6 to add the fractions of both $\frac{2}{3}$ and $\frac{5}{6}$. Adding the two fractions $\frac{4}{6}$ and $\frac{5}{6}$ together equals to $\frac{9}{6}$.

Add the whole numbers together to get the answer $3\frac{9}{6}$.

By using the greatest common factor, $\frac{9}{6}$ is simplified to $\frac{3}{2}$. Finally simplify into the appropriate mixed number of $4\frac{1}{2}$.

2 Calculate the following by finding a common denominator.

- | | |
|----------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------|
| 1 $1\frac{3}{5} + 2\frac{2}{3} = 1\frac{6}{15} + 2\frac{10}{15} = 3\frac{16}{15} = 4\frac{1}{15}$ | 2 $2\frac{5}{6} + 1\frac{1}{2} = 2\frac{5}{6} + 1\frac{3}{6} = 3\frac{8}{6} = 4\frac{1}{3}$ |
| 3 $2\frac{3}{4} + 3\frac{5}{12} = 2\frac{9}{12} + 3\frac{5}{12} = 5\frac{14}{12} = 6\frac{1}{6}$ | 4 $1\frac{7}{15} + 2\frac{5}{6} = 1\frac{14}{30} + 2\frac{25}{30} = 3\frac{39}{30} = 4\frac{13}{10}$ |

Don't forget to make the most appropriate mixed numbers.

8 - 7 Addition and Subtraction of Fractions
Subtraction of Fractions (1)

Example A container has $\frac{3}{4}$ L of tea in it. $\frac{2}{3}$ L is poured out. How much tea is left?

The math sentence is $\frac{3}{4} - \frac{2}{3}$.

To calculate this, we must find a common denominator of 4 and 3. The common denominator is 12. Therefore, the math sentence can be changed to $\frac{9}{12} - \frac{8}{12}$.

Which fraction is greater?

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Math sentence $\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$

Answer $\frac{1}{12}$ L

1 The red ribbon is $\frac{1}{2}$ m long. The blue ribbon is $\frac{3}{5}$ m long. Which ribbon is longer? How much longer is one ribbon than the other?

Math sentence $\frac{3}{5} - \frac{1}{2} = \frac{6}{10} - \frac{5}{10} = \frac{1}{10}$ Answer longer than the red ribbon.

2 Calculate the following by finding a common denominator.

- | | |
|---------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|
| 1 $\frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$ | 2 $\frac{5}{7} - \frac{2}{5} = \frac{25}{35} - \frac{14}{35} = \frac{11}{35}$ |
| 3 $\frac{1}{3} - \frac{1}{8} = \frac{8}{24} - \frac{3}{24} = \frac{5}{24}$ | 4 $\frac{4}{7} - \frac{1}{3} = \frac{12}{21} - \frac{7}{21} = \frac{5}{21}$ |
| 5 $\frac{5}{8} - \frac{2}{5} = \frac{25}{40} - \frac{16}{40} = \frac{9}{40}$ | 6 $\frac{5}{6} - \frac{3}{7} = \frac{35}{42} - \frac{18}{42} = \frac{17}{42}$ |
| 7 $\frac{5}{9} - \frac{1}{4} = \frac{20}{36} - \frac{9}{36} = \frac{11}{36}$ | 8 $\frac{4}{5} - \frac{2}{3} = \frac{12}{15} - \frac{10}{15} = \frac{2}{15}$ |
| 9 $\frac{7}{10} - \frac{5}{9} = \frac{63}{90} - \frac{50}{90} = \frac{13}{90}$ | 10 $\frac{5}{8} - \frac{2}{7} = \frac{35}{56} - \frac{16}{56} = \frac{19}{56}$ |

8 - 8 Addition and Subtraction of Fractions
Subtraction of Fractions (2)

Example Calculate $\frac{2}{3} - \frac{1}{6}$.

To calculate this, we must find a common denominator of 3 and 6. The common denominator is 6. So, the math sentence will be $\frac{4}{6} - \frac{1}{6}$.

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

$$\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

Simplify the fraction by finding the greatest common factor. In this case it is 3.

We can simplify $\frac{3}{6}$ to $\frac{1}{2}$.

Calculate the following by finding a common denominator and simplify the answer by finding the greatest common factor.

- 1 $\frac{2}{3} - \frac{5}{12} = \frac{8}{12} - \frac{5}{12} = \frac{3}{12} = \frac{1}{4}$
- 2 $\frac{2}{3} - \frac{7}{15} = \frac{10}{15} - \frac{7}{15} = \frac{3}{15} = \frac{1}{5}$
- 3 $\frac{5}{6} - \frac{5}{24} = \frac{20}{24} - \frac{5}{24} = \frac{15}{24} = \frac{5}{8}$
- 4 $\frac{3}{2} - \frac{7}{10} = \frac{15}{10} - \frac{7}{10} = \frac{8}{10} = \frac{4}{5}$
- 5 $\frac{25}{24} - \frac{5}{12} = \frac{25}{24} - \frac{10}{24} = \frac{15}{24} = \frac{5}{8}$
- 6 $\frac{11}{13} - \frac{7}{39} = \frac{33}{39} - \frac{7}{39} = \frac{26}{39} = \frac{2}{3}$
- 7 $\frac{15}{28} - \frac{1}{4} = \frac{15}{28} - \frac{7}{28} = \frac{8}{28} = \frac{2}{7}$
- 8 $\frac{3}{5} - \frac{7}{20} = \frac{12}{20} - \frac{7}{20} = \frac{5}{20} = \frac{1}{4}$
- 9 $\frac{27}{30} - \frac{3}{4} = \frac{54}{60} - \frac{45}{60} = \frac{9}{60} = \frac{3}{20}$
- 10 $\frac{7}{12} - \frac{5}{15} = \frac{35}{60} - \frac{20}{60} = \frac{15}{60} = \frac{1}{4}$
- 11 $\frac{11}{10} - \frac{14}{15} = \frac{33}{30} - \frac{28}{30} = \frac{5}{30} = \frac{1}{6}$
- 12 $\frac{5}{6} - \frac{3}{10} = \frac{25}{30} - \frac{9}{30} = \frac{16}{30} = \frac{8}{15}$

8 - 9 Addition and Subtraction of Fractions
Subtraction of Fractions (3)

Example Calculate $1\frac{1}{4} - \frac{1}{2}$.

$$1\frac{1}{4} - \frac{1}{2} = 1\frac{1}{4} - \frac{2}{4}$$

$$= \frac{5}{4} - \frac{2}{4}$$

$$= \frac{3}{4}$$

Step 1: Find common denominators.
Step 2: Convert to improper fractions if needed.

In order to subtract fractions, both fractions must have the same denominator.

4 is the common denominator.

However, we cannot subtract $\frac{2}{4}$ from $1\frac{1}{4}$. Therefore, the mixed number $1\frac{1}{4}$ must be converted into an improper fraction: $\frac{5}{4}$.

Then we can do $\frac{5}{4} - \frac{2}{4}$.

Calculate the following.

- 1 $1\frac{1}{4} - \frac{2}{3} = 1\frac{3}{12} - \frac{8}{12} = \frac{7}{12}$
- 2 $1\frac{1}{3} - \frac{3}{5} = 1\frac{5}{15} - \frac{9}{15} = \frac{6}{15} = \frac{2}{5}$
- 3 $1\frac{1}{7} - \frac{3}{5} = 1\frac{5}{35} - \frac{12}{35} = \frac{19}{35}$
- 4 $1\frac{1}{6} - \frac{3}{5} = 1\frac{5}{30} - \frac{18}{30} = \frac{17}{30}$
- 5 $1\frac{2}{5} - \frac{3}{4} = 1\frac{8}{20} - \frac{15}{20} = \frac{13}{20}$
- 6 $3\frac{1}{6} - \frac{5}{7} = 3\frac{7}{42} - \frac{30}{42} = 2\frac{49}{42} = 2\frac{7}{6} = 3\frac{1}{6}$
- 7 $2\frac{2}{9} - \frac{2}{3} = 2\frac{2}{9} - \frac{6}{9} = \frac{11}{9} = 1\frac{2}{9}$
- 8 $2\frac{1}{12} - \frac{1}{6} = 2\frac{1}{12} - \frac{2}{12} = \frac{11}{12}$
- 9 $3\frac{2}{3} - \frac{7}{9} = 3\frac{4}{9} - \frac{7}{9} = 2\frac{15}{9} = 2\frac{5}{3} = 3\frac{2}{3}$
- 10 $2\frac{3}{8} - \frac{9}{10} = 2\frac{15}{40} - \frac{36}{40} = 1\frac{19}{40}$
- 11 $1\frac{1}{4} - \frac{5}{6} = 1\frac{3}{12} - \frac{10}{12} = \frac{5}{12}$
- 12 $3\frac{3}{10} - \frac{13}{15} = 3\frac{9}{30} - \frac{26}{30} = 2\frac{39}{30} = 2\frac{13}{10}$

8 - 10 Addition and Subtraction of Fractions
Subtraction of Fractions (4)

Example Calculate $2\frac{1}{4} - 1\frac{2}{3}$.

$$2\frac{1}{4} - 1\frac{2}{3} = 2\frac{3}{12} - 1\frac{8}{12}$$

$$= 1\frac{15}{12} - 1\frac{8}{12}$$

$$= \frac{7}{12}$$



It is important to the whole number can be changed into fraction if necessary, and the whole numbers can be calculate by themselves.

To calculate this, we must find the common denominator of 4 and 3.

However, it is impossible to subtract $\frac{8}{12}$ from $\frac{3}{12}$. Both mixed numbers must be converted to improper fractions in order to subtract them.

$2\frac{3}{12}$ becomes $1\frac{15}{12}$.

The whole numbers, 1 and 1 can be calculate $1 - 1$.

Calculate the following.

- 1 $2\frac{1}{5} - 1\frac{1}{3} = 2\frac{2}{15} - 1\frac{5}{15} = \frac{18}{15} - \frac{5}{15} = \frac{13}{15}$
- 2 $2\frac{2}{3} - 1\frac{3}{4} = 2\frac{8}{12} - 1\frac{9}{12} = \frac{20}{12} - \frac{9}{12} = \frac{11}{6}$
- 3 $3\frac{1}{7} - 1\frac{3}{4} = 3\frac{4}{28} - 1\frac{21}{28} = 2\frac{32}{28} - \frac{21}{28} = \frac{11}{28}$
- 4 $3\frac{1}{4} - 1\frac{4}{5} = 3\frac{5}{20} - 1\frac{16}{20} = 2\frac{25}{20} - \frac{16}{20} = \frac{9}{20}$
- 5 $5\frac{2}{5} - 3\frac{2}{3} = 5\frac{6}{15} - 3\frac{10}{15} = 2\frac{16}{15} - \frac{10}{15} = \frac{11}{15}$
- 6 $4\frac{1}{6} - 3\frac{2}{5} = 4\frac{5}{30} - 3\frac{12}{30} = 3\frac{35}{30} - \frac{12}{30} = \frac{23}{30}$
- 7 $3\frac{2}{15} - 1\frac{2}{3} = 3\frac{2}{15} - 1\frac{10}{15} = 2\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$
- 8 $4\frac{1}{8} - 1\frac{1}{2} = 4\frac{1}{8} - 1\frac{4}{8} = 3\frac{9}{8} - \frac{4}{8} = 2\frac{5}{8}$
- 9 $2\frac{1}{6} - 1\frac{7}{12} = 2\frac{2}{12} - 1\frac{7}{12} = 1\frac{2}{12} - \frac{7}{12} = \frac{7}{12}$
- 10 $3\frac{1}{6} - 1\frac{8}{15} = 3\frac{5}{30} - 1\frac{16}{30} = 2\frac{35}{30} - \frac{16}{30} = \frac{19}{30}$
- 11 $4\frac{1}{6} - 2\frac{7}{8} = 4\frac{4}{24} - 2\frac{21}{24} = 3\frac{28}{24} - \frac{21}{24} = 1\frac{7}{24}$
- 12 $4\frac{7}{10} - 2\frac{3}{4} = 4\frac{14}{20} - 2\frac{15}{20} = 3\frac{34}{20} - \frac{15}{20} = \frac{19}{20}$

8 - 11 Addition and Subtraction of Fractions
Time and Fractions

Instruction We can express time by using fractions as follows:



15 minutes

$$\frac{15}{60} = \frac{1}{4}$$

of an hour



30 minutes

$$\frac{30}{60} = \frac{1}{2}$$

of an hour



45 minutes

$$\frac{45}{60} = \frac{3}{4}$$

of an hour



5 minutes

$$\frac{5}{60} = \frac{1}{12}$$

of an hour

Because 1 hour is 60 minutes, we can express time by using a fraction whose denominator is 60.

Example Write the appropriate fractions in the .

1 10 minutes = $\frac{10}{60} = \frac{1}{6}$ of an hour

2 30 seconds = $\frac{30}{60} = \frac{1}{2}$ of a minute

Because 1 minute is 60 seconds, we can express time by using a fraction whose denominator is 60. We can do the same with minutes to hours.

Write the appropriate fractions in the .

1 20 minutes = $\frac{20}{60} = \frac{1}{3}$ of an hour

2 50 minutes = $\frac{50}{60} = \frac{5}{6}$ of an hour

3 90 minutes = $\frac{90}{60} = \frac{3}{2}$ of an hour

4 100 minutes = $\frac{100}{60} = \frac{5}{3}$ of an hour

5 1 minutes = $\frac{1}{60}$ of an hour

6 45 seconds = $\frac{45}{60} = \frac{3}{4}$ of a minute

7 25 seconds = $\frac{25}{60} = \frac{5}{12}$ of a minute

8 1 seconds = $\frac{1}{60}$ of a minute

8 - 12

Addition and Subtraction of Fractions

Review

1 Compare the following fractions and write the appropriate inequal sign in the \square .

① $\frac{5}{21} < \frac{2}{7}$ ② $\frac{5}{6} > \frac{7}{9}$ ③ $\frac{8}{9} > \frac{13}{15}$ ④ $\frac{3}{8} < \frac{5}{12}$

2 Calculate the following and simplify the answer.

① $\frac{1}{6} + \frac{2}{7} = \frac{7}{42} + \frac{12}{42} = \frac{19}{42}$ ② $\frac{5}{9} + \frac{1}{5} = \frac{25}{45} + \frac{9}{45} = \frac{34}{45}$

③ $\frac{3}{5} + \frac{1}{15} = \frac{6}{15} + \frac{1}{15} = \frac{7}{15}$ ④ $\frac{1}{6} + \frac{3}{10} = \frac{5}{30} + \frac{9}{30} = \frac{14}{30} = \frac{7}{15}$

⑤ $\frac{2}{3} - \frac{1}{5} = \frac{10}{15} - \frac{3}{15} = \frac{7}{15}$ ⑥ $\frac{3}{4} - \frac{2}{5} = \frac{15}{20} - \frac{8}{20} = \frac{7}{20}$

⑦ $\frac{19}{28} - \frac{1}{4} = \frac{19}{28} - \frac{7}{28} = \frac{12}{28} = \frac{3}{7}$ ⑧ $\frac{7}{9} - \frac{1}{6} = \frac{14}{18} - \frac{3}{18} = \frac{11}{18}$

⑨ $3\frac{4}{7} + 2\frac{2}{3} = 3\frac{12}{21} + 2\frac{14}{21} = 5\frac{26}{21} = 6\frac{5}{21}$ ⑩ $1\frac{5}{6} + 1\frac{11}{18} = 1\frac{15}{18} + 1\frac{11}{18} = 2\frac{26}{18} = 2\frac{13}{9}$

⑪ $1\frac{2}{15} - \frac{3}{10} = 1\frac{4}{30} - \frac{9}{30} = \frac{34}{30} - \frac{9}{30} = \frac{25}{30} = \frac{5}{6}$ ⑫ $3\frac{7}{8} - 1\frac{9}{10} = 3\frac{35}{40} - 1\frac{36}{40} = 2\frac{75}{40} - 1\frac{36}{40} = 1\frac{39}{40}$

3 Write the appropriate fractions in the \square .

① 10 minutes = $\frac{10}{60} = \frac{1}{6}$ of an hour ② 40 minutes = $\frac{40}{60} = \frac{2}{3}$ of an hour

③ 30 seconds = $\frac{30}{60} = \frac{1}{2}$ of a minute ④ 5 seconds = $\frac{5}{60} = \frac{1}{12}$ of a minute

4 Write the appropriate numbers from 1 to 9 in the \square . The same number cannot be used twice.

① $\frac{\square}{6} + \frac{\square}{24} = \frac{5}{24}$ ② $\frac{32}{45} - \frac{\square}{15} = \frac{8}{45}$ ③ $3\frac{\square}{13} + 1\frac{7}{26} = 4\frac{9}{26}$

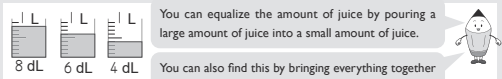
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9 - 1

Average

Meaning of Average

Example 1 The juices are in a container. Divide them so that the three juices are equal.



You can equalize the amount of juice by pouring a large amount of juice into a small amount of juice.

You can also find this by bringing everything together and then dividing it. Let's calculate with this method!

1 What is the total quantity of juice in the three containers?

Math sentence $8 + 6 + 4 = 18$ Answer 18 dL

2 If you divide all the juice into three equal parts, how many deciliters do you get?

Math sentence $18 \div 3 = 6$ Answer 6 dL

3 Find the math sentence and answer for the case where the juice in the three containers is the same. (In other words, find the average of three containers.)

Math sentence $(8 + 6 + 4) \div 3 = 6$

When you take several quantities and equalize them all to the same quantity, the quantity you get is called the **average**.

The following table shows the number of books that a girl read from September to December. What is the average number of books she read in one month?

Month	September	October	November	December
Number of books	2	3	2	5

1 What is the total number of books she read in the four months?

Math sentence $2 + 3 + 2 + 5 = 12$ Answer 12 books

2 If you divide all the books read into four months, how many books per month did she read?

Math sentence $12 \div 4 = 3$ Answer 3 books

3 Find the math sentence and answer for the case where the books in the four months is the same.

Math sentence $(2 + 3 + 2 + 5) \div 4 = 3$ Answer 3 books

89

9 - 2

Average

Calculation of average

Example 1 Every morning we run and here is a table showing how many times A and B ran round the school grounds last week. B was ill and had to take a day off, so he only ran for four days. Who did the best job?

Day	Day 1	Day 2	Day 3	Day 4	Day 5
A	5	3	5	4	2
B	5	4	7	4	

We cannot compare the total number because B is absent for 1 day. So let's compare the average to find who did best job.

1 Calculate the average of A.

Math sentence $(5 + 3 + 5 + 4 + 2) \div 5 = 3.8$

Answer 3.8 times

Total quantity \div Number of days

2 Calculate the average of B.

Math sentence $(5 + 4 + 7 + 4) \div 4 = 5$

Sometimes averages can be expressed as decimals.

Answer 5 times

Total quantity \div Number of days

3 Who did the best job? Answer B

We can compare the average even total number is not same!

The following table shows the math test scores for group 1 and group 2.

Person	A	B	C
Score (point)	90	67	71

※ Maximum score is 100

Person	D	E	F	G
Score (point)	88	62	78	74

1 Calculate the average of math test score of group 1.

Math sentence $(90 + 67 + 71) \div 3 = 76$ Answer 76 points

2 Calculate the average of math test score of group 2.

Math sentence $(88 + 62 + 78 + 74) \div 4 = 75.5$ Answer 75.5 points

3 Which group has better math test score? Answer $Group 1$

90

9 - 3

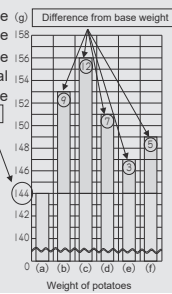
Average

Provisional average

Example I weighed 6 potatoes. The table and graph show the results. The calculation can be complicated, so we take the smallest, 144, as base weight (Provisional average) to make it easier to calculate the average number.

1 Find the difference of weight of each potato from base weight.

Number	(a)	(b)	(c)	(d)	(e)	(f)
Weight (g)	144	153	156	151	147	149
Difference (g)	0	9	12	7	3	5



2 Calculate the average of difference of the weights determined above.

Math sentence $(0 + 9 + 12 + 7 + 3 + 5) \div 6 = 6$

Answer (Average of difference of the weights) 6 g

3 Calculating the average of weights by adding average of difference to the provisional average.

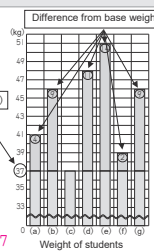
You can find the average by adding the average of difference to the base average.

Math sentence $144 + 6 = 150$ Answer (Average of the weight) 150 g

I weighed 7 people and the table and graph show the results. We take the smallest, 37, as base weight (provisional average).

1 Find the difference of weight of each student from base weight.

Number	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Weight (g)	41	46	37	48	51	39	46
Difference (g)	4	9	0	11	14	2	9



2 Calculate the average of difference of the weights determined above.

Math sentence $(4 + 9 + 0 + 11 + 14 + 2 + 9) \div 7 = 7$

3 Calculating the average of weights by adding average of difference to the provisional average.

Math sentence $37 + 7 = 44$ Answer (Average of the weight) 44 Kg

91

9 - 4

Average

Problem (1)

Example You want to measure the approximate length of Eastern school building using your friends' stride.

- 1 The table shows information on one of your friends. He took 10 steps 3 times. How many m is his stride? Find the average of 10 steps.

Trial	Distance of 10 steps
1	6.21 m
2	6.25 m
3	6.23 m

The length of one step is not always the same, so we take several steps and decide it as an average.

Math sentence $(6.21 + 6.25 + 6.23) \div 3 = 6.23$ Answer 6.23 m

- 2 Find the length of one step.

Math sentence $6.23 \div 10 = 0.623$

To find one step, you just take ten steps to one tenth!

- 3 He measured the length of eastern school building by his steps. It was 30 steps. Estimate the length of the building.

Math sentence $0.623 \times 30 = 18.69$ Answer 18.69 m

The table shows the walking time of 1000 m of your friend.

- 1 What is her average record? Write the math sentence and find the answer.

Trial	Record (minutes)
1	13.6
2	13.4
3	20.4
4	13.8

Outlier (Failure)

When you want to get an average, you may want to exclude outliers from the calculation. Firstly, check why does the outlier come and if the cause was failure, you can exclude it.

Math sentence $(13.6 + 13.4 + 13.8) \div 3 = 13.6$ Answer 13.6 minutes

- 2 Estimate your friend's record of 500 m.

Math sentence $13.6 \div 2 = 6.8$ Answer 6.8 minutes

- 3 If your friend will run 2000 m, estimate her record of 2000 m.

Math sentence $13.6 \times 2 = 27.2$ Answer 27.2 minutes

92

9 - 5

Average

Problem (2)

Example 1 Your sister finished a 140-page book in just eight days. How many pages did she read a day on average?

Math sentence $140 \div 8 = 17.5$ Answer 17.5 pages

Example 2 A bus could carry a maximum of 55 passengers and up to 4500 kg. What is the maximum weight per person in kilograms? (Rounding down the answer to the nearest one.)

Math sentence $4500 \div 55 = 81.8181\cdots$ Answer 81 kg

- 1 Your brother finished 260-page novel in just 13 days. How many pages did she read a day on average?

Math sentence $260 \div 13 = 20$ Answer 20 pages

- 2 A ship could carry maximum of 98 passenger and up to 8500 kg. What is the maximum weight per person in kilograms? (Rounding down the answer to the nearest one.)

Math sentence $8500 \div 98 = 86.7346\cdots$ Answer 86 kg

- 3 There is 48 g of sweets at total. We put the sweets in the bags and the average weight of the sweets in a bag is 1.6g. How many bags do we made?

Math sentence $48 \div 1.6 = 30$ Answer 30 bags

- 4 If the apple weighs 200 g on average. How many apples are there in the box of apples which weigh 5 kg?

Math sentence $5\text{kg} = 5000\text{g}$ $5000 \div 200 = 25$ Answer 25 apples

93

9 - 6

Average

Review

- 1 The table below shows the number of children who were absent from your class on the last 5 days. On average, how many students were absent each day?

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of students	5	2	1	3	4

Math sentence $(5 + 2 + 1 + 3 + 4) \div 5 = 3$ Answer 3 students

- 2 My friend walked 700 steps around the school. If your friend's stride is about 0.65 m, about how many m do you think are around the school?

Math sentence $700 \times 0.65 = 455$ Answer 455 m

- 3 The table below shows one student's record of four long jumps. What is the average length of this student's record?

Trial	First	Second	Third	Fourth
Record	2.36 m	0.42 m	2.46 m	2.38 m

Math sentence **Second is outlier so we do not calculate.**
 $(2.36 + 2.46 + 2.38) \div 3 = 2.4$ Answer 2.4 m

- 4 The children will be divided into two groups, A and B, to prepare their hospitality dishes. The number of people in each group and the total number of dishes they make are shown in the table below. Which group makes more food per person? Compare the average number of dishes made by each person.

Trial	Number of children	Number of dishes made
Group A	15	135
Group B	12	114

- 1 How many dishes will each person in Group A make?

Math sentence $135 \div 15 = 9$ Answer 9 dishes

94

- 2 How many dishes will each person in Group B make?

Math sentence $114 \div 12 = 9.5$ Answer 9.5 dishes

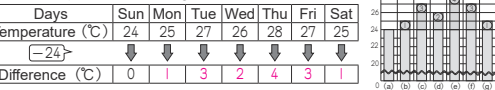
- 3 Which group will make more on average per person?

Answer Group B

- 5 The table below shows the maximum temperatures recorded for one week. We take the smallest, 24, as base temperature (provisional average).

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Temperature (°C)	24	25	27	26	28	27	25
Difference (°C)	0	1	3	2	4	3	1

- 1 Find the average maximum temperature by writing down the difference from the provisional average.



- 2 Calculate the provisional average of maximum temperature determined above.

Math sentence $(0 + 1 + 3 + 2 + 4 + 3 + 1) \div 7 = 2$

- 3 Calculate the average of maximum temperature by adding the provisional average.

Math sentence $24 + 2 = 26$ Answer (Average of temperature) 26 °C

- 6 If one small orange weighs 80 g on average,

1 How many kg does 100 small orange weight? Answer 8 kg

- 2 How many small oranges weigh 4 kg?

Math sentence $4\text{ kg} = 4000\text{ g}$ $4000 \div 80 = 50$ Answer 50 small oranges

- 7 There is a 3000 g carton of potatoes. The average weight of the potatoes in the bag is 153 g. How many potatoes are there approximately in the bag? (Round the answer to the nearest one.)

Math sentence $3000 \div 153 = 19.607\cdots$ Answer 20 potatoes

95

10-1 Amount per Unit (1)

Example 1 We keep chickens in two chicken coops (a), (b). (a) is 2 m² and has 12 chickens, (b) is 3 m² and 15 chickens.

	m ²	Chicken
(a)	2	12
(b)	3	15

Which is more crowded? Let's compare the number of chickens per 1 m².

Let's compare the average of number of chicken per m². A quantity express in this way is called amount per unit.

Math sentence
(a) $12 \div 2 = 6$
No. of chickens (m²) Amount per unit
Answer (Chicken per m²)
6 chicken per m²

Math sentence
(b) $15 \div 3 = 5$
Answer (Chicken per m²)
5 chicken per m²

Answer (a)

The youth group went on a trip. Which room is most crowded? (a) is 10 m² and fits 3 people, (b) is 8 m² and fits 4 people.

	Room (a)	Room (b)
Size (m ²)	10	8
Number of people	3	2

The Number of people per area is called **population density**.

Which is more crowded, (a) or (b)? Let's compare the number of youths per m².

Math sentence
(a) $3 \div 10 = 0.3$
Answer (people per m²)
0.3 people per m²

Math sentence
(b) $2 \div 8 = 0.25$
Answer (people per m²)
0.25 people per m²

Answer (b)

96

10-2 Amount per Unit (2)

Example There are East town and West town. We want to know which town is crowded.

1 What is the number of people per 1 km² (population density) in East town?

	Population	Area (km ²)
East	9720	30
West	8920	20

Math sentence
East town $9720 \div 30 = 324$
Answer (people per 1 km²)
324 people per km²

2 What is the number of people per 1 km² (population density) in West town?

Math sentence
West town $8920 \div 20 = 446$
Answer (people per 1 km²)
446 people per km²

3 Which town is more crowded?
Answer **West town**

The table on the right shows the population and area of Village A and B.

	Population	Area (km ²)
A	1840	10
B	2780	20

1 What is the number of people per km² (population density) in Village A?

Math sentence
Village A $1840 \div 10 = 184$
Answer (people per 1 km²)
184 people per km²

2 What is the number of people per km² (population density) in Village B?

Math sentence
Village B $2780 \div 20 = 139$
Answer (people per 1 km²)
139 people per km²

3 Which village is more crowded?
Answer **Village A**

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10-3 Amount per Unit (3)

Example You and your friend harvested potatoes at each house.

	Potato (kg)	Area (m ²)
You	100	50
Friend	200	80

1 Which field had a better harvest? Calculate the amount of potatoes per m² for each field.

Math sentence
You $100 \div 50 = 2$
Answer (kg per m²)
2 kg per m²

Math sentence
Friend $200 \div 80 = 2.5$
Answer (kg per m²)
2.5 kg per m²

2 Which field had a better harvest?
Answer **Friend**

There are 2 cars, A and B. Compare A and B in terms of the amount of gasoline they used and the distance they can travel.

	Distance (km)	Gasoline (L)
A	700	35
B	800	50

1 Calculate the distance each car can travel on 1 L of gasoline.

Math sentence
A $700 \div 35 = 20$
Answer (km per L)
20 km per L

Math sentence
B $800 \div 50 = 16$
Answer (km per L)
16 km per L

2 Which car travel longer?
Answer **A**

98

10-4 Amount per Unit (4)

Example Find the weight per m for piece of wire with a weight of 200 g for 5 m.

1 What is the weight per m of this wire?

Math sentence
 $200 \div 5 = 40$
Answer (Weight per 1 m)
40 g per m

2 What is the weight of 9 m of this wire?

Now that we know the weight per m, we can use multiplication to find out the weight of the length we need.

Math sentence
 $40 \times 9 = 360$
Answer **360 g**

There is a car that runs 360 km on 20 L of gasoline.

1 What is the distance traveled per L of gasoline?

Math sentence
 $360 \div 20 = 18$
Answer (km per L)
18 km per L

2 How far does this car go with 9 L of gasoline?

Math sentence
 $18 \times 9 = 162$
Answer **162 km**

99

10 - 5 Amount per Unit **Speed**

Example Here is a table of the times that you and your friends ran.

1 How many m per second did you run?

	Distance (m)	Seconds
You	80	20
Friend	100	16

Distance: 0 to 80 (m) ÷ (20) (seconds)

Math sentence: $80 \div 20 = 4$ Answer: 4 m per second

2 How many m per second did your friend run?

Distance: 0 to 100 (m) ÷ (16) (seconds)

Math sentence: $100 \div 16 = 6.25$ Answer: 6.25 m per second

3 Who's faster, you or your friend? Answer: friend

The travel distance per unit of time is called the **speed**.
The math sentence to find speed is **(Speed) = (Distance) ÷ (Time)**

Here is a table of the times that your sister and your brother walked.

1 How many m per minute did you run?

	Distance (m)	Minutes
	700	10
	900	15

Distance: 0 to 700 (m) ÷ (10) (minutes)

Math sentence: $700 \div 10 = 70$ Answer: 70 m per minute

2 How many m per minutes did your brother walk?

Distance: 0 to 900 (m) ÷ (15) (minutes)

Math sentence: $900 \div 15 = 60$ Answer: 60 m per minute

3 Who's faster, your sister or your brother? Answer: Sister

100

10 - 6 Amount per Unit **Distance**

Example There is a car that runs at 50 km per hour (speed). How many km can it travel in 2 hours? How many km can it travel in 3 hours?

Distance: 0 to 100 (km) × 2 (hours)

Math sentence: $50 \times 2 = 100$ Answer: 100 km

Distance: 0 to 150 (km) × 3 (hours)

Math sentence: $50 \times 3 = 150$ Answer: 150 km

The math sentence to find speed is **(Distance) = (Speed) × (Time)**

A bird can fly at 850 m per minute. How many m can it travel in 2 minutes? How many m can it travel in 4 minutes?

Distance: 0 to 1700 (m) × 2 (minutes)

Math sentence: $850 \times 2 = 1700$ Answer: 1700 m

Distance: 0 to 3400 (m) × 4 (minutes)

Math sentence: $850 \times 4 = 3400$ Answer: 3400 m

A child walks 70 m per minute. How many m can he travel in 50 minutes? How many m can he travel in 90 minutes.

Distance: 0 to 3500 (m) × 50 (minutes)

Math sentence: $70 \times 50 = 3500$ Answer: 3500 m

Distance: 0 to 6300 (m) × 90 (minutes)

Math sentence: $70 \times 90 = 6300$ Answer: 6300 m

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10 - 7 Amount per Unit **Time**

Example A cyclist travels at 300 m per minutes. How many minutes does it take the cyclist to go 6300 m?

Distance: 0 to 6300 (m) ÷ (300) (minutes)

Math sentence: $300 \times \square = 6300$

If you consider the relationship between the overall distance and the unit distance, you can see the corresponding relationship between the journey times.

Distance: 0 to 6300 (m) ÷ (300) (minutes)

Math sentence: $\square = 6300 \div 300 = 21$ Answer: 21 minutes

The math sentence to find time is **(Time) = (Distance) ÷ (Speed)**

1 A ship travels at 38 km per hour. How many hours does it take to travel 228 km?

Distance: 0 to 228 (km) ÷ (38) (hours)

Math sentence: $\square = 228 \div 38 = 6$ Answer: 6 hours

2 An airplane travels at 18 km per minute. How many minutes does it take to travel 90 km?

Distance: 0 to 90 (km) ÷ (18) (minutes)

Math sentence: $\square = 90 \div 18 = 5$ Answer: 5 minutes

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10 - 8 Amount per Unit **Speed of work**

Example There are two machines, A and B. A pumps out 240 L of water in 8 minutes. B pumps out 300 L of water in 12 minutes. Which machine pumps out more water per minutes?

1 Find the each machine.

A: Water: 0 to 240 (L) ÷ (8) (minutes)

Math sentence: $240 \div 8 = 30$ Answer: 30 L per minute

B: Water: 0 to 300 (L) ÷ (12) (minutes)

Math sentence: $300 \div 12 = 25$ Answer: 25 m

2 Which machine is faster? Answer: A

There are two printer, A and B. A prints out 300 sheets of paper in 4 minutes. B prints out 360 sheets of paper in 12 minutes. Which printer prints faster?

1 Find each speed.

A: Paper: 0 to 300 (sheets) ÷ (4) (minutes)

Math sentence: $300 \div 4 = 75$ Answer: 75 sheets per minute

B: Paper: 0 to 360 (sheets) ÷ (12) (minutes)

Math sentence: $360 \div 12 = 30$ Answer: 30 sheets per minute

2 Which printer print faster? Answer: A

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10 - 9 Amount per Unit

Review

1 The table on the right shows the population and area of State A and State B.

	Population	Area (km ²)
A	810000	3000
B	850000	5000

1 What is the number of people per 1 km² (population density) in State A?

Population Area

Math sentence: $810000 \div 3000 = 270$ Answer (people per km²): **270 people per km²**

2 What is the number of people per 1 km² (population density) in State B?

Population Area

Math sentence: $850000 \div 5000 = 170$ Answer (people per km²): **170 people per km²**

3 Which state is more crowded?

Answer: **A state**

2 Potatoes were harvested. A total weight of 43.2 kg was harvested from 6 m² field A, and 62.1 kg was harvested from 9 m² field B.

1 What is the weight per m² of field A?

Weight Area

Math sentence: $43.2 \div 6 = 7.2$ Answer (kg per m²): **7.2 kg per m²**

2 What is the weight per m² of this field B?

Weight Area

Math sentence: $62.1 \div 9 = 6.9$ Answer (kg per m²): **6.9 kg per m²**

3 Which field harvested more potatoes?

Answer: **Field A**

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3 It took you 7 minutes to ride your bicycle to the school which is 1400 m away. What was your speed you rode at?

Math sentence: $1400 \div 7 = 200$

Answer: **200 m per minute**

4 An elephant runs 13 km per hour. How many km can it travel in 4 hours?

Math sentence: $13 \times 4 = 54$

Answer: **54 km**

5 How many minutes does it take a person running at 300 m per minute to run 5600 m?

Math sentence: $5600 \div 300 = 18$

Answer: **18 minutes**

6 It took 3 minutes for a car traveling at 30 km per hour to cross the bridge.

1 How many m per minute is 30 km per hour?

1 km = 1000 m. So, firstly, calculate the km per minutes.

30 km is **30000** m, so 30 km per hour means **30000** m per hour. 1 hour is 60 minutes, so divide the distance by 60.



Math sentence: $30000 \div 60 = 500$

Answer: **500 m per minute**

2 What is the length of this bridge?

Math sentence: $500 \times 3 = 1500$

Answer: **1500 m**

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11 - 1

Relationship between Fractions, Decimal Numbers and Whole Numbers

Division and Fractions

Instruction When we share 2 L of juice among 3 people equally, each person get $\frac{2}{3}$ L of juice.

Diagram showing 2 L of juice divided into 3 equal parts.

Math sentence: $2 \div 3 = \frac{2}{3}$

Sometimes when a whole number is divided by another whole number, the quotient is a fraction.

Example 1 Express the following quotients as fractions.

1 $5 \div 4 = \frac{5}{4}$ 2 $6 \div 7 = \frac{6}{7}$

1 Express the following quotients as fractions. Simplify the quotients if possible.

1 $3 \div 4 = \frac{3}{4}$ 2 $5 \div 12 = \frac{5}{12}$ 3 $11 \div 17 = \frac{11}{17}$
 4 $9 \div 2 = \frac{9}{2}$ 5 $3 \div 12 = \frac{1}{4}$ 6 $12 \div 8 = \frac{3}{2}$

Example 2 Express the following fractions as division sentences.

1 $\frac{1}{9} = 1 \div 9$ 2 $\frac{5}{6} = 5 \div 6$

2 Express the following fractions as division sentences.

1 $\frac{1}{4} = 1 \div 4$ 2 $\frac{3}{11} = 3 \div 11$ 3 $\frac{7}{2} = 7 \div 2$
 4 $\frac{2}{5} = 2 \div 5$ 5 $\frac{8}{7} = 8 \div 7$ 6 $\frac{13}{6} = 13 \div 6$

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11 - 2

Relationship between Fractions, Decimal Numbers and Whole Numbers

Fractions and Times as Much

Example There is a piece of red tape 4 m long. There is a piece of blue tape 3 m long. How many times longer is the red tape than the blue tape?

Diagram showing a number line from 0 to 4 m, with a 3 m segment marked.

Math sentence: $4 \div 3 = \frac{4}{3}$

Answer: $\frac{4}{3}$ times

1 There is a yellow rope and a green rope. The length of the yellow rope is 8 m and the length of the green rope is 5 m. How many times longer is the yellow rope than the green rope?

Math sentence: $8 \div 5 = \frac{8}{5}$

Answer: $\frac{8}{5}$ times

2 A small bucket holds 3 L of water and a large bucket holds 7 L of water. How many times more litres does the large bucket hold than the small bucket?

Math sentence: $7 \div 3 = \frac{7}{3}$

Answer: $\frac{7}{3}$ times

3 My adult dog weighs 11 kg and my puppy weighs 6 kg. How many times heavier does my adult dog weigh than my puppy?

Math sentence: $11 \div 6 = \frac{11}{6}$

Answer: $\frac{11}{6}$ times

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11 - 3 Relationship between Fractions, Decimal Numbers and Whole Numbers

Fractions and Decimal Numbers (1)

Example If a 3 m ribbon is shared equally among 5 children, how long will each piece of ribbon be? Express the answer as a fraction and as a decimal number.

This is divisible. Therefore, the answer could be expressed by using either fraction or decimal number.

Math sentence: $3 \div 5 = \frac{3}{5}$ or 0.6

Answer: $\frac{3}{5}$ m or 0.6 m

$\frac{3}{5}$ and 0.6 are the same amount expressed in two different ways.

1 If a 14 m ribbon is shared equally among 5 students, how long will each piece of ribbon be? Express the answer as a fraction and as a decimal number.

Math sentence: $14 \div 5 = \frac{14}{5}$ or 2.8

Answer: $\frac{14}{5}$ m or 2.8 m

- 2 Express the following as decimal numbers.
- 1 $\frac{1}{4} = 1 \div 4 = 0.25$ 2 $\frac{1}{10} = 1 \div 10 = 0.1$
- 3 $\frac{5}{8} = 5 \div 8 = 0.625$ 4 $\frac{23}{5} = 23 \div 5 = 4.6$
- 5 $1\frac{1}{2} = 1 + 1 \div 2 = 1.5$ 6 $3\frac{2}{5} = 3 + 2 \div 5 = 3.4$
- 3 Express the following fractions as decimal numbers rounded to the hundredths place.
- 1 $\frac{1}{3} = 1 \div 3 = 0.333\ldots = 0.33$ 2 $\frac{5}{6} = 5 \div 6 = 0.833\ldots = 0.83$
- 3 $\frac{9}{7} = 9 \div 7 = 1.285\ldots = 1.29$ 4 $\frac{7}{12} = 7 \div 12 = 0.583\ldots = 0.58$

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11 - 4 Relationship between Fractions, Decimal Numbers and Whole Numbers

Fractions and Decimal Numbers (2)

Example 1 Write 0.3, 0.05, and 1.23 as fractions.

1 $0.3 = \frac{3}{10}$ 2 $0.05 = \frac{5}{100}$ 3 $1.23 = \frac{123}{100}$

Decimal numbers can be written as fractions with denominators of 10, 100, 1000 and so on.

1 Write the following decimal numbers as fractions.

1 $0.4 = \frac{4}{10}$ 2 $1.7 = \frac{17}{10}$ 3 $0.08 = \frac{8}{100}$

4 $0.63 = \frac{63}{100}$ 5 $1.09 = \frac{109}{100}$ 6 $2.87 = \frac{287}{100}$

Example 2 Write 7, 15 and 123 as fractions.

1 $7 = 7 \div 1 = \frac{7}{1}$ 2 $15 = 15 \div 1 = \frac{15}{1}$ 3 $123 = 123 \div 1 = \frac{123}{1}$

Whole numbers can be written as fractions with denominators of 1. Even if the denominator is not 1, the denominator become 1 when simplifying the fraction.

2 Write the following whole numbers as fractions.

1 $9 = \frac{9}{1}$ 2 $23 = \frac{23}{1}$ 3 $415 = \frac{415}{1}$ 4 $700 = \frac{700}{1}$

Example 3 Which of the following fractions can be written as whole numbers?

(a) $\frac{6}{2} = \frac{3}{1} = 3$ (b) $\frac{20}{6} = \frac{10}{3}$ (c) $\frac{40}{8} = \frac{5}{1} = 5$ (d) $\frac{55}{10} = \frac{11}{2}$

If we simplify each fraction, we can find which can be written as whole number.

Answer: (a), (c)

3 Which of the following fractions can be written as whole numbers?

(a) $\frac{5}{1} = 5$ (b) $\frac{8}{3}$ (c) $\frac{28}{4} = 7$ (d) $\frac{31}{6}$ (e) $\frac{147}{7} = 21$ Answer: (a), (c), (e)

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11 - 5 Relationship between Fractions, Decimal Numbers and Whole Numbers

Calculations of Fractions and Decimal Numbers

Example 1 Which is greater, $\frac{5}{6}$ or 0.8?

To compare numbers, both numbers must be either fractions or decimals.

Method 1: Comparing by fractions
We change 0.8 to a fraction.
 $0.8 = \frac{8}{10} = \frac{4}{5}$
We find a common denominator of 30.
 $\frac{5}{6} = \frac{25}{30}$
 $0.8 = \frac{4}{5} = \frac{24}{30}$ $\frac{5}{6} > 0.8$

Method 2: Comparing by decimal numbers
We change $\frac{5}{6}$ to a decimal number.
 $\frac{5}{6} = 5 \div 6 = 0.8333\ldots$
 $\frac{5}{6} > 0.8$

1 Which number is greater? Write the inequality sign in the \square .

1 $0.7 > \frac{2}{3}$ 2 $\frac{4}{15} < 0.27$ 3 $1.85 < 1\frac{19}{20}$ 4 $\frac{9}{4} > 2.2$

Example 2 Calculating the following problems.

1 $\frac{3}{4} + 0.6 = 0.75 + 0.6 = 1.35$ or $\frac{3}{4} + \frac{6}{10} = \frac{15}{20} + \frac{12}{20} = \frac{27}{20}$ or $1\frac{7}{20}$

2 $\frac{3}{4} - 0.6 = 0.75 - 0.6 = 0.15$ or $\frac{3}{4} - \frac{6}{10} = \frac{15}{20} - \frac{12}{20} = \frac{3}{20}$

2 Calculate the following addition and subtraction problems.

1 $1.5 + \frac{1}{5} = 1.5 + 0.2 = 1.7$ or $\frac{15}{10} + \frac{2}{10} = \frac{17}{10}$ or $1\frac{7}{10}$

2 $\frac{1}{7} + 0.9 = \frac{1}{7} + \frac{9}{10} = \frac{10}{70} + \frac{63}{70} = \frac{73}{70}$ or $1\frac{3}{70}$

3 $1.6 - \frac{4}{5} = 1.6 - 0.8 = 0.8$ or $\frac{16}{10} - \frac{8}{10} = \frac{8}{10} = \frac{4}{5}$

4 $\frac{24}{25} - 0.84 = 0.96 - 0.84 = 0.12$ or $\frac{24}{25} - \frac{84}{100} = \frac{96}{100} - \frac{84}{100} = \frac{12}{100} = \frac{3}{25}$

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11 - 6 Relationship between Fractions, Decimal Numbers and Whole Numbers

Review

1 Express the following quotients as fractions. Simplify the quotients if possible.

1 $7 \div 9 = \frac{7}{9}$ 2 $6 \div 8 = \frac{3}{4}$ 3 $28 \div 12 = \frac{7}{3}$

2 Express the following as decimal numbers.

1 $\frac{2}{5} = 2 \div 5 = 0.4$ 2 $\frac{7}{10} = 7 \div 10 = 0.7$

3 $\frac{15}{6} = 15 \div 6 = 2.5$ 4 $\frac{9}{8} = 9 \div 8 = 1.125$

3 Write the following decimal numbers as fractions.

1 $0.9 = \frac{9}{10}$ 2 $3.1 = \frac{31}{10}$ 3 $0.17 = \frac{17}{100}$

4 $0.09 = \frac{9}{100}$ 5 $5.27 = \frac{527}{100}$ 6 $2.01 = \frac{201}{100}$

4 Which number is greater? Write the inequality sign in the \square .

1 $0.23 < \frac{1}{4}$ 2 $1.6 > \frac{7}{5}$ 3 $2.4 < 2\frac{1}{2}$

5 Calculate the following.

1 $2.7 + \frac{3}{4} = 2.7 + 0.75 = 3.45$ or $\frac{27}{10} + \frac{3}{4} = \frac{54}{20} + \frac{15}{20} = \frac{69}{20}$ or $3\frac{9}{20}$

2 $0.12 + \frac{1}{3} = \frac{12}{100} + \frac{1}{3} = \frac{36}{300} + \frac{100}{300} = \frac{136}{300} = \frac{34}{75}$

3 $\frac{5}{8} - 0.52 = 0.625 - 0.52 = 0.105$ or $\frac{5}{8} - \frac{52}{100} = \frac{125}{200} - \frac{104}{200} = \frac{21}{200}$

4 $0.4 - \frac{1}{7} = \frac{4}{10} - \frac{1}{7} = \frac{28}{70} - \frac{10}{70} = \frac{18}{70} = \frac{9}{35}$

111

12-1 Ratio How to Express a Ratio

Example 1 Your sister and brother practice the shoot.

	Number of shots	Number of scores
(a) Your sister	10	8
(c) Your brother	12	9

1 Let's express the results with fraction

It can be expressed as a fraction as above.

Number of score	Compared quantity
Number of shots	Base quantity

Answer (a) $\frac{8}{10}$ (b) $\frac{9}{12}$

2 Let's compare the results with fraction.

It is easier to compare when the denominators are the same.

(a) $\frac{8}{10} = \frac{48}{60}$ (b) $\frac{9}{12} = \frac{45}{60}$

3 Who has the best shooting record?
Answer (a) your sister

You and your brother practice the shoot.

	Number of shots	Number of scores
(a) You	10	9
(b) Your brother	15	12

1 Let's express the results with fraction

Answer (a) $\frac{9}{10}$ (b) $\frac{12}{15}$

2 Let's compare the results with fraction.

(a) $\frac{9}{10} = \frac{27}{30}$ (b) $\frac{12}{15} = \frac{24}{30}$

3 Who has the best shooting record?
Answer (a) you

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12-2 Ratio Calculation of Ratio (1)

Example 1 Your sister and brother practice the shoot.

1 Let's calculate (a)'s scoring rate.

	Number of shots	Number of scores
(a) Your sister	20	16
(b) Your brother	30	27

Let's calculate the number of score per shoot (scoring rate), like amount per unit.

Number of shots: 0, 0.5, 1
Scoring rate: 0, 0.5, 1
Compared quantity: 16, 20
Base quantity: 20

Math sentence $16 \div 20 = 0.8$

1 Let's calculate (b)'s scoring rate.

Number of shots: 0, 0.5, 1
Scoring rate: 0, 0.5, 1
Compared quantity: 27, 30
Base quantity: 30

Math sentence $27 \div 30 = 0.9$

2 Who has the best shooting record? Answer (a) Your brother

If we consider the performance of a shot as a quantity based on the number of shot (base quantity) and the number of score of shots as a comparison (compared quantity), it can be expressed by the following math sentence.
Success of record = Number of score \div Number of shots = $\frac{\text{Number of score}}{\text{Number of shots}}$

Your sister and brother practice the shoot.

	Number of shots	Number of scores
(a) Your sister	50	35
(b) Your brother	40	32

1 Let's calculate (a)'s scoring rate.

Number of shots: 0, 0.5, 1
Scoring rate: 0, 0.5, 1
Compared quantity: 35, 50
Base quantity: 50

Math sentence $35 \div 50 = 0.7$

2 Let's calculate (b)'s scoring rate.

Number of shots: 0, 0.5, 1
Scoring rate: 0, 0.5, 1
Compared quantity: 32, 40
Base quantity: 40

Math sentence $32 \div 40 = 0.8$

3 Who has the best shooting record? Answer (b) your brother

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12-3 Ratio Calculation of Ratio (2)

Example 1 The following table shows the capacity of a bus and the number of passengers on a given day.

	Capacity	Passenger	Ratio
(a) Small bus	20	16	<input type="checkbox"/>
(b) Big bus	80	60	<input type="checkbox"/>

1 Calculate of ratio of congestion of small bus to find the answer.

The capacity (ratio of coaggregation) can be calculated as the number of passengers compared to the base capacity of 1.

(a) Compared quantity: 16, Base quantity: 20, Ratio: $\frac{16}{20} = 0.8$

Math sentence $16 \div 20 = 0.8$

2 Calculate of ratio of congestion of big bus to find the answer.

(b) Compared quantity: 60, Base quantity: 80, Ratio: $\frac{60}{80} = 0.75$

Math sentence $60 \div 80 = 0.75$

3 Which bus is more crowded?
Answer (a) is more crowded

A congestion ratio of 0.75 means that the ratio of the number of passengers to the capacity of the train is 0.75.

When the base quantity is considered as the unit, the size of the compared quantity is the ratio. And ratio can calculate as following.

Ratio = Compared quantity \div Base quantity

If compared quantity and base quantity are whole number, we can express as following: Ratio = $\frac{\text{Compared quantity}}{\text{Base quantity}}$

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The following table shows the capacity of a bus and the number of passengers on a given day.

	Capacity	Passenger	Ratio
(a) Small airplane	140	98	<input type="checkbox"/>
(b) Big airplane	360	270	<input type="checkbox"/>

1 Calculate of ratio of congestion of small airplane to find the answer.

(a) Compared quantity: 98, Base quantity: 140, Ratio: $\frac{98}{140} = 0.7$

Math sentence $98 \div 140 = 0.7$

2 Calculate of ratio of congestion of big bus to find the answer.

(b) Compared quantity: 270, Base quantity: 360, Ratio: $\frac{270}{360} = 0.75$

Math sentence $270 \div 360 = 0.75$

3 Which bus is more crowded?
Answer (b) is more crowded

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12-4 Ratio Percentage (1)

Example There are 30 passengers in the bus that has 50 seats.

1 Find the ratio of crowdedness of the bus.

Compared quantity: 30 (people)
Base quantity: 50 (people)
Math sentence: $30 \div 50 = 0.6$

Represent this ratio by making the base quantity as 100.

$30 \div 50 = \square \div 100$

Relative amount 0.01
1%

① 0% ② 1% ③ 100%

We often represent a ratio by making the base quantity as 100. This representation is called **percentage**. The ratio 0.01 which is a decimal number, is called **one percent** and is written as 1%. If we represent the ratio 1 as a percentage, it is 100%.

2 Represent the crowdedness of the bus as a percentage.

Compared quantity: 30 (people)
Base quantity: 50 (people)
Math sentence: $0.6 \times 100 = 60$
Answer: **60%**

There is a train with a capacity of 100 passengers per carriage. There are 74 passengers in the first car of the train.

1 Find the ratio of crowdedness of the bus.

Compared quantity: 74 (people)
Base quantity: 100 (people)
Math sentence: $74 \div 100 = 0.74$

2 Represent the crowdedness of the bus as a percentage.

Math sentence: $0.74 \times 100 = 74$
Answer: **74%**

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12-5 Ratio Percentage (2)

Example 1 Express a number expressed as a decimal as a percentage.

(a) 0.06 → **6%** (b) 0.2 → **20%**

Let's multiply by 100 to find the percentage.

2 Express a percentage as a percentage in decimal form.

(a) 7% → **0.07** (b) 35% → **0.35**

Let's divide by 100 to find the ratio.

1 Express a number expressed as a decimal as a percentage.

(a) 0.08 → **8%** (b) 0.6 → **60%** (c) 1.25 → **125%**

2 Express a percentage as a percentage in decimal form.

(a) 9% → **0.09** (b) 58% → **0.58** (c) 140% → **1.4**

Example A shirts costing 2000 zeds was sold for 1400 zeds. Express the percentage of the price you paid based on the regular price as a percentage.

Compared quantity: 1400 (zeds)
Base quantity: 2000 (zeds)
Math sentence: $1400 \div 2000 = 0.7$

2 Represent the regular price as a percentage.

Math sentence: $0.7 \times 100 = 70$
Answer: **70%**

There are 40 students in the whole class, of which 18 are boys. Express the percentage of boys in the class based on the total number of boys in the class.

Compared quantity: 18 (people)
Base quantity: 40 (people)
Math sentence: $18 \div 40 = 0.45$

2 Represent the total number of boys as a percentage.

Math sentence: $0.45 \times 100 = 45$
Answer: **45%**

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12-6 Ratio Finding Compared Quality (1)

Example 1 A shirt with a regular price of 2000 zeds is sold for 60% of the regular price. How much will I have to pay to get it? ("zed(s)" is the fictional currency unit.)

Compared quantity: 60% → 0.6
Base quantity: 2000 (zeds)
Math sentence: $2000 \times 0.6 = 1200$
Answer: **1200 zeds**

Example 2 I bought apples for 80 zeds each and decided to sell them for 120% of the purchase price. How much will you sell each apple for? ("zed(s)" is the fictional currency unit.)

Compared quantity: 120% → 1.2
Base quantity: 80 (zeds)
Math sentence: $80 \times 1.2 = 96$
Answer: **96 zeds**

1 A scarf with a regular price of 1200 zeds is sold for 70% of regular price. How much will I have to pay to get it? ("zed(s)" is the fictional currency unit.)

Compared quantity: 70% → 0.7
Base quantity: 1200 (zeds)
Math sentence: $1200 \times 0.7 = 840$
Answer: **840 zeds**

2 We made a cake which cost 300 zeds per piece. You decide to sell this cake at 150% of the price. How much will you sell one cake for? ("zed(s)" is the fictional currency unit.)

Compared quantity: 150% → 1.5
Base quantity: 300 (zeds)
Math sentence: $300 \times 1.5 = 450$
Answer: **450 zeds**

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12-6 Ratio Finding Compared Quality (2)

Example 1 In football shooting practice, I shot 10 times and scored 70% of the shoots. How many times did you score?

Compared quantity: 70% → 0.7
Base quantity: 10 (times)
Math sentence: $10 \times 0.7 = 7$
Answer: **7 shoots**

Example 2 I bought onions for 50 zeds each and decided to sell them for 130% of original price. What is the sale price? ("zed(s)" is the fictional currency unit.)

Compared quantity: 130% → 1.3
Base quantity: 50 (zeds)
Math sentence: $50 \times 1.3 = 65$
Answer: **65 zeds**

1 In football shooting practice, I shot 12 times and scored 75% of the shoots. How many times did you score?

Compared quantity: 75% → 0.75
Base quantity: 12 (times)
Math sentence: $12 \times 0.75 = 9$
Answer: **9 shoots**

2 I bought potatoes for 80 zeds each and decided to sell them for 125% of original price. What is the sale price? ("zed(s)" is the fictional currency unit.)

Compared quantity: 125% → 1.25
Base quantity: 80 (zeds)
Math sentence: $80 \times 1.25 = 100$
Answer: **100 zeds**

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12-7 Ratio Finding Base Quality (1)

Example We grow maize on our farmland. We grow maize on an area of 12 m², which is 60% of our total area. What is the total area of the farm?

Area Ratio: $\frac{12}{()} = \frac{0.6}{1}$

Compared quantity: 12 (m²)

Base quantity: () (m²)

60% → 0.6

To get base quantity, multiply the compared quantity with 0.6

Math sentence: $\square \times 0.6 = 12$

Answer: 20 m²

The track and field club has 24 interested students. This is 80% of available slots. How many slots are available?

Number Ratio: $\frac{24}{()} = \frac{0.8}{1}$

Compared quantity: 24 (people)

Base quantity: () (people)

80% → 0.8

To get base quantity, multiply the compared quantity with 0.8

Math sentence: $\square \times 0.8 = 24$

Answer: For 30 students

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12-7 Ratio Finding Base Quality (2)

Example 1 A shirt was sold for 60% of the regular price which is 3000 zeds. How much is the regular price? ("zed(s)" is the fictional currency unit.)

Price Ratio: $\frac{3000}{()} = \frac{0.6}{1}$

Compared quantity: 3000 (zeds)

Base quantity: () (zeds)

60% → 0.6

To get base quantity, divide compared quantity into 0.6

Math sentence: $\square \times 0.6 = 3000$

Math sentence: $3000 \div 0.6 = 5000$

Answer: 5000 zeds

Example 2 I bought tomatoes for 60 zeds each and it was for 120% of the original price. How much is the original price? ("zed(s)" is the fictional currency unit.)

Price Ratio: $\frac{60}{()} = \frac{1.2}{1}$

Compared quantity: 60 (zeds)

Base quantity: () (zeds)

120% → 1.2

To get base quantity, divide compared quantity into 1.2

Math sentence: $\square \times 1.2 = 60$

Math sentence: $60 \div 1.2 = 50$

Answer: 50 zeds

1 I bought a cake for 80% of regular price which is 1200 zeds. How much is the regular price of the cake? ("zed(s)" is the fictional currency unit.)

Price Ratio: $\frac{1200}{()} = \frac{0.8}{1}$

Compared quantity: 1200 (zeds)

Base quantity: () (zeds)

80% → 0.8

To get base quantity, divide compared quantity into 0.8

Math sentence: $\square \times 0.8 = 1200$

Math sentence: $1200 \div 0.8 = 1500$

Answer: 1500 zeds

2 You made a bread and decided to sell it with 120 zeds. This price was 250% of the cost of ingredients. What is the cost of ingredients? ("zed(s)" is the fictional currency unit.)

Price Ratio: $\frac{120}{()} = \frac{2.5}{1}$

Compared quantity: 120 (zeds)

Base quantity: () (zeds)

250% → 2.5

To get base quantity, divide compared quantity into 2.5

Math sentence: $\square \times 2.5 = 120$

Math sentence: $120 \div 2.5 = 48$

Answer: 48 zeds

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12-8 Ratio Problems (1)

Example 1 We grow tomatoes on our farmland. We grow tomatoes on an area of 16 m², which is 40% of our total area. What is the total area of the farm?

Area Ratio: $\frac{16}{()} = \frac{0.4}{1}$

Compared quantity: 16 (m²)

Base quantity: () (m²)

40% → 0.4

To get base quantity, divide compared quantity into 0.4

Math sentence: $16 \div 0.4 = 40$

Answer: 40 m²

Example 2 There were 3000 people who came to the grand park yesterday. That's 120% of yesterday's total. How many people came to the grand park yesterday?

Number Ratio: $\frac{3000}{()} = \frac{1.2}{1}$

Compared quantity: 3000 (people)

Base quantity: () (people)

120% → 1.2

To get base quantity, divide compared quantity into 1.2

Math sentence: $3000 \div 1.2 = 2500$

Answer: 2500 people

1 The science club has 42 interested students. This is 20% of total number of students. How many students are there?

Number Ratio: $\frac{42}{()} = \frac{0.2}{1}$

Compared quantity: 42 (people)

Base quantity: () (people)

20% → 0.2

To get base quantity, divide compared quantity into 0.2

Math sentence: $42 \div 0.2 = 210$

Answer: 210 students

There were 9000 people who came to the Stadium yesterday. That's 150% of yesterday's total. How many people came to the Stadium yesterday?

Number Ratio: $\frac{9000}{()} = \frac{1.5}{1}$

Compared quantity: 9000 (people)

Base quantity: () (people)

150% → 1.5

To get base quantity, divide compared quantity into 1.5

Math sentence: $9000 \div 1.5 = 6000$

Answer: 6000 people

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12-8 Ratio Problems (2)

Example Let's make math problems. The sentence below can be expressed using a figure like the one on the right.

20 students gather for the youth group assembly. 8 of them are girls. Girls make up 40% of the total number of students.

Area Ratio: $\frac{8}{20} = \frac{0.4}{1}$

- Make a math problem for finding ③.
- Make a math problem for finding ②.

The sentence below can be expressed using a figure like the one on the right.

There are a total of 500 children at your school. 260 of them are girls. Girls make up 52% of the total number of students.

Number Ratio: $\frac{260}{500} = \frac{0.52}{1}$

- Make a math problem for finding ③.
- Make a math problem for finding ②.

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12-9 Ratio Review

- 1** You shoot 20 times and score 14 times. What is the ratio of the score?
- 1** Base quantity is and compared quantity is .
- 2** Express the math sentence by words.
Ratio = $\frac{\text{Compared quantity}}{\text{Base quantity}}$
- 3** Express the math sentence by fraction.
Ratio = $\frac{\text{Compared quantity}}{\text{Base quantity}}$
- Math sentence $14 \div 20 = 0.7$ Answer 0.7
- 2** The cooking club has 25 available slots. The number of interested students is 0.8 times as many as the number of available slots. How many interested students are there?
-
- Math sentence $25 \times 0.8 = 20$ Answer 20 students
- 3** The reference book for science class costs 1800 zeds. This is 1.2 times as many as the price of a dictionary. What is the price of the dictionary? ("zed(s)" is the fictional currency unit.)
-
- Math sentence $1800 \div 1.2 = 1500$ Answer 1500 zeds
- 4** The bottle is 300 mL of juice drink. Of the total amount, 15% is fruit juice. How many mL of fruit juice are in the bottle?
- 1** Express 15% using decimal number. Answer 0.15
- 2** Write a math sentence to calculate the compared quantity.
-
- Math sentence $300 \times 0.15 = 45$ Answer 45 mL

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- 5** A dog that was born a week ago. It weighs 184g. The current weight is 160% of its weight a week ago. What was its weight at birth in g?
-
- Math sentence $184 \div 1.6 = 115$ Answer 115 g
- 6** My friend bought a marker for 30% off the regular price of 300 zeds. What is the sale price? ("zed(s)" is the fictional currency unit.)
- 1** 30% off means % of regular price.
-
- Math sentence $300 \times 0.7 = 210$ Answer 210 zeds
- 7** Your football team's record was 8 wins and 2 losses.
- 1** What is the percentage of games won to games played?
-
- Math sentence $8 \div 10 = 0.8$ Answer 80%
- 8** It is said that 86% of the composition of an apple is water. How much does an apple weigh if it contains 258 g of water?
-
- Math sentence $258 \div 0.86 = 300$ Answer 300 g
- 9** A book is priced at 1200 zeds before tax, what is the price after adding the 10% tax? ("zed(s)" is the fictional currency unit.)
- 1** 10% adding tax means % of regular price.
-
- Math sentence $1200 \times 1.1 = 1320$ Answer 1320 zeds

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13-1 Ratio and Graph Read Strip Charts

- Example 1** The chart below shows the number of cars passing by the school, by type.
- A graph that represents the part-whole relationship by rectangle-like bands is called **strip chart (graph)**. It is usually arranged in order of increasing number. There are several names of the graph.
-
- 1** What percentage of the total number of motobike do you have?
- 2** What percentage of the total number of bus do you have?
- 3** When we were investigating, there were 50 cars in total that passed by the school. How many trucks were there?
- Percentage of truck is $50 \times 0.12 = 6$
- We surveyed Year 5 students about what they would like to be in the future and the results are shown in the table below.
-
- 1** What percentage of the total number of sports players do you have?
- 2** What percentage of the total number of teachers do you have?
- 3** There were 50 students in total. How many students who want to be a business oner?
- Business oner is $50 \times 0.24 = 12$

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13-2 Ratio and Graph Read Pie Charts

- Example 1** The graph below represents the ratio of favorite sports of grade 5 students.
-
- 1** What percentage of the total is the ratio of football?
- 2** What percentage of the total is the ratio of track and field?
- 3** There were 50 students in total. How many students who like volleyball?
- Volleyball is $50 \times 0.18 = 9$
- A graph that is drawn as a circle is called **pie chart (graph)**. There are several name of the graph.
- The pie chart below represents the rates of energy consumed per household. Answer the following question.
- *Kerosene is a light fuel
-
- 1** What percentage of the total energy consumption is electricity? About how many % is it?
- 2** What percentage of the total is the ratio of LP Gas?
- 3** About how many times as much as natural gas is consumption as LP gas?
- Natural gas

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13 - 3 Ratio and Graph **Draw Strip Charts**

Example The table on the right shows the number of books sold by a bookshop in a month.

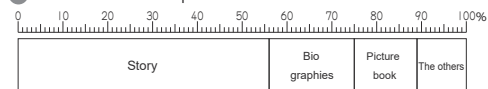
Type	Number of books	Percentage (%)
Story	48	(a) 56%
Biographies	16	(b) 19%
Picture book	12	(c) 14%
The others	9	(d) 11%
Total	85	(e) 100%

- Find the proportions that apply to (a)~(e) in the table on the right. (Percentages should be rounded to the nearest whole number.)



$$\text{Percentage} = \frac{\text{Part}}{\text{Total}}$$

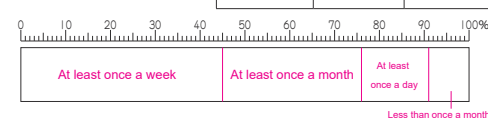
- Let's show it on a strip chart



The table on the right shows the number of times 80 students read a book in one week. Find the proportions that apply to (a)~(d) in the table on the right. (Percentages should be rounded to the nearest whole number).

Number of times	Number of students	Percentage (%)
At least once a day	12	(a) 15%
At least once a week	36	(b) 45%
At least once a month	25	(c) 31%
Less than once a month	7	(d) 9%
Total	80	100%

- Let's show it on a strip chart



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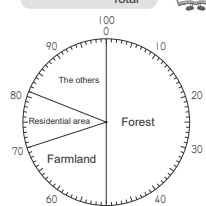
13 - 4 Ratio and Graph **Draw Pie Charts**

Example The table shows the land use in a town.

- Find the proportions that apply to (a)~(e) in the table on the right. (Percentages should be rounded to the nearest whole number.)

Type	Area (km ²)	Percentage (%)
Forest	40	(a) 50%
Farmland	16	(b) 20%
Residential area	9	(c) 11%
The others	15	(d) 19%
Total	80	(e) 100%

$$\text{Percentage} = \frac{\text{Part}}{\text{Total}}$$



- Let's show it on a pie chart.

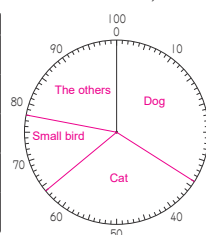
Even if some numbers are less than the others, the others are placed last.



In a group, student answered which animal they wanted to own.

- Find the proportions that apply to (a)~(e) in the table on the right. (Percentages should be rounded to the nearest whole number.)

Animal	Number	Percentage (%)
Dog	17	(a) 34%
Cat	15	(b) 30%
Small bird	7	(c) 14%
The others	11	(d) 22%
Total	50	(e) 100%

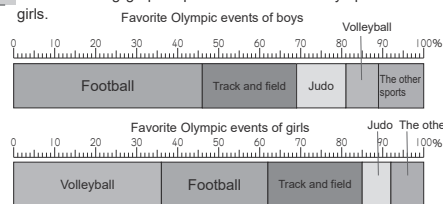


- Let's show it on a pie chart.

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13 - 5 Ratio and Graph **Review (1)**

- The following graph represent the favorite Olympic event of boys and girls.



- What percentage of the total is the ratio of volleyball by each boys and girls?

boys **8%** Girls **36%**

- What percentage of the total is other sports in each graph?

boys **11%** Girls **8%**

- Which sport has the smallest difference between the proportion of boys and girls?

Track and Field

- The pie chart shows the ratio of amount of banana harvested by region in 2021.

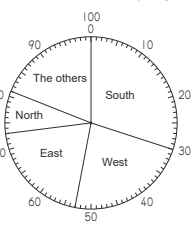
- What percentage of the total is South region?

30%

- The national amount of banana in this year was 600000 t. How many tons was approximate amount of banana in East region?

$$600000 \times 0.2 = 120000$$

120000t



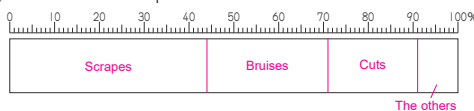
130

13 - 6 Ratio and Graph **Review (2)**

- The following tables shows the number of injury reports of the hospital per month.

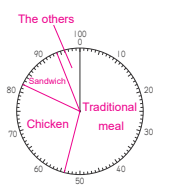
Type	Number of people	Percentage (%)
Scrapes	40	(a) 44%
Bruises	24	(b) 27%
Cuts	18	(c) 20%
The others	8	(d) 9%
Total	90	(e) 100%

- Let's show it on a strip chart



- The following table shows the number of lunch menu for 200 families in the school event day. Find the proportions that apply to (a)~(d) in the table on the right. (Percentages should be rounded to the nearest whole number.) Show it on a pie chart.

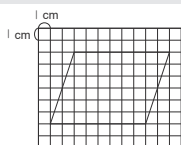
menu	Number of people	Percentage (%)
Traditional meal	108	(a) 54%
Chicken	56	(b) 28%
Sandwich	24	(c) 12%
The others	12	(d) 6%
Total	200	100%



131

14-1 Area of Quadrilaterals and Triangles
Area of Parallelograms (1)

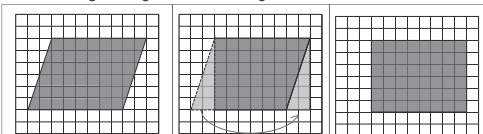
Instruction Find the area of the parallelogram below.



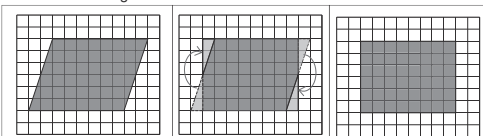
We learned how to find the area of a rectangle in the previous grade.

The area of a parallelogram can be found by changing the figure into a rectangle.

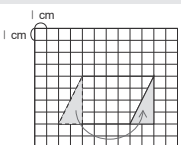
We can change the figure into a rectangle as follows:



We can also change it as follows.



Example Find the area of the parallelogram below.



1 What is the length and width of the rectangle we can make from this parallelogram?

Length **6 cm** Width **4 cm**

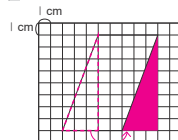
We cut and move a part of parallelogram to make rectangle. Length is longer than width.

2 How many cm^2 is the area?

Math sentence $6 \times 4 = 24$

Answer **24 cm^2**

1 Find the area of the parallelogram below.



1 How many cm^2 in lengths and widths is the rectangle is equal to the area of this parallelogram?

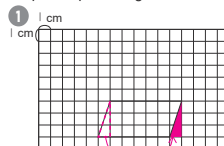
Length **8 cm** Width **5 cm**

2 How many cm^2 is the area?

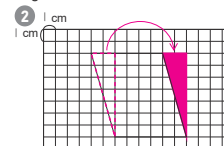
Math sentence $8 \times 5 = 40$

Answer **40 cm^2**

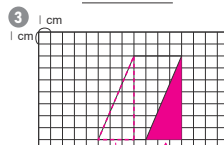
2 Find the area of the following parallelograms by cutting and moving a part of parallelogram to make rectangle.



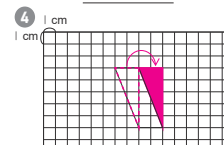
Math sentence $6 \times 3 = 18$
Answer **18 cm^2**



Math sentence $7 \times 6 = 42$
Answer **42 cm^2**



Math sentence $7 \times 4 = 28$
Answer **28 cm^2**

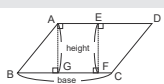


Math sentence $5 \times 2 = 10$
Answer **10 cm^2**

14-2 Area of Quadrilaterals and Triangles
Area of Parallelograms (2)

Instruction Area of Parallelogram

- Base is a side of polygon, particularly one oriented perpendicular to the direction in which height is measured.
- Draw straight line AG, EF, and other lines which are perpendicular to the base of BC. The length of these straight lines is called the height against the base of BC.



Even if you don't change the figure into a rectangle, you can find the area by calling "length" is base and "width" is height.

(Area of Parallelogram) = (Base) \times (Height)

Example Regarding the parallelogram below, answer the following questions.

1 How long does the base and height? Base **8 cm** Height **7 cm**

2 Find the area of the following parallelogram.
Math sentence $8 \times 7 = 56$
Answer **56 cm^2**

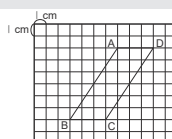
1 Find the length of the base and height, and the area of the following parallelograms.

1 Base **9 cm** Height **6 cm**
Math sentence $9 \times 6 = 54$
Answer **54 cm^2**
Height is perpendicular to the base.

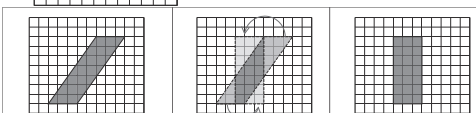
2 Base **10 cm** Height **8 cm**
Math sentence $10 \times 8 = 80$
Answer **80 cm^2**
Base does not come to the bottom of the figure all the time.

14-3 Area of Quadrilaterals and Triangles
Area of Parallelograms (3)

Example Find the area of the parallelogram shown below when BC is the base.



To find the height, change the figure into a rectangle.



- When the side BC is the base, the distance between the two lines is the height of parallelogram ABCD.

1 Find the area of the following parallelograms.

1 Base **5 cm** Height **9 cm**
Math sentence $5 \times 9 = 45$
Answer **45 cm^2**

2 Base **3.2 cm** Height **4 cm**
Math sentence $4 \times 3.2 = 12.8$
Answer **12.8 cm^2**

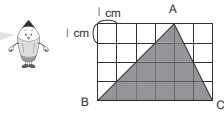
2 Find the area of parallelograms A, B, and C.

Parallelogram A **32 cm^2**
Parallelogram B **32 cm^2**
Parallelogram C **32 cm^2**

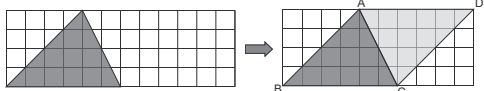
14-4 Area of Quadrilaterals and Triangles
Area of Triangles (1)

Instruction How to find the area of the triangle on the right.

I wonder we can find the area by measures we already know.



Idea 1: By putting another triangle to make a parallelogram

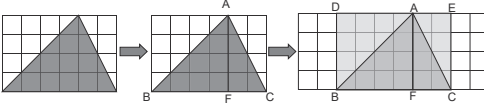


If you put the same triangle ABC, you can make parallelogram ABCD. You already know how to find the area of a parallelogram. So...

Then, you can find the area and halve it.

Math sentence: $6 \times 4 \div 2$
 Base of parallelogram ABCD: 6
 Height of parallelogram ABCD: 4

Idea 2: By dividing the triangle to make a rectangle

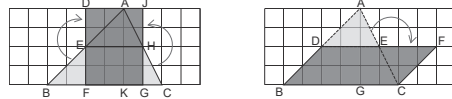


If you divide the triangle into two ABF and AFC, then you put the same size of these triangles, you can make a rectangle, DBCE. You already know how to find the area of a rectangle. So...

Then, you can find the area of the rectangle and halve it.

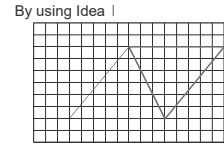
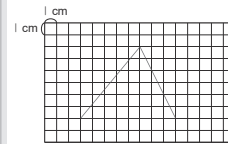
Math sentence: $6 \times 4 \div 2$
 Length of rectangle DBCE: 6
 Width of rectangle DBCE: 4

Alternatively, you can cut some part of the triangle and move to make rectangle or parallelogram.

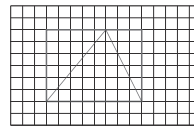


What are the similarities and differences among the ideas on how to find the area of the triangle?
 If you can change a triangle into a rectangle or parallelogram, you can find the area.

Example Find the area of the following triangle.

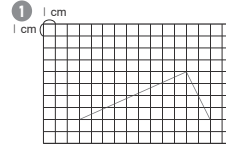


By using Idea 2

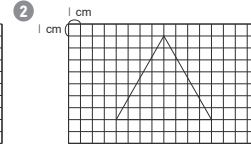


Math sentence: $8 \times 6 \div 2 = 24$
 Answer: 24 cm^2

Find the area of the following triangles.



Math sentence: $11 \times 4 \div 2 = 22$
 Answer: 22 cm^2

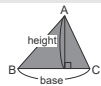


Math sentence: $8 \times 7 \div 2 = 28$
 Answer: 28 cm^2

14-5 Area of Quadrilaterals and Triangles
Area of Triangles (2)

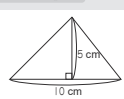
Instruction Area of Triangle

- When side BC is the base of the triangle, the height is the length of the perpendicular line that extends from vertex A to base BC.



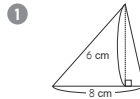
(Area of Triangle) = (Base) \times (Height) \div 2

Example Find the area of the following triangle.

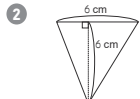


Math sentence: $10 \times 5 \div 2 = 25$
 Answer: 25 cm^2

Find the area of the following triangles.

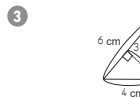


Math sentence: $8 \times 6 \div 2 = 24$
 Answer: 24 cm^2



Math sentence: $6 \times 6 \div 2 = 18$
 Answer: 18 cm^2

The base is not always at the bottom.



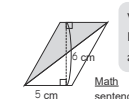
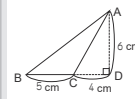
Math sentence: $6 \times 3 \div 2 = 9$
 Answer: 9 cm^2



Math sentence: $10 \times 4 \div 2 = 20$
 Answer: 36 cm^2

14-6 Area of Quadrilaterals and Triangles
Area of Triangles (3)

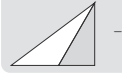
Example Find the area of the triangle shown below when BC is the base.



Where is the height of a triangle?
 By putting another triangle to make a parallelogram.

Math sentence: $5 \times 6 \div 2 = 15$
 Answer: 15 cm^2

How about subtracting triangle ACD from triangle ABD?



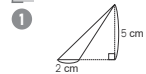
Math sentence: $(5+4) \times 6 \div 2 = 27$
 $4 \times 6 \div 2 = 12$
 $27 - 12 = 15$
 Answer: 15 cm^2

The both ideas can find the same area. So the height of triangle ABC is 6 cm.

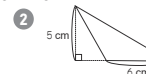
- When the side BC is the base, the distance between the two lines is the height of the triangle ABC.



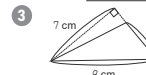
Find the area of the following triangles.



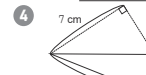
Math sentence: $2 \times 5 \div 2 = 5$
 Answer: 5 cm^2



Math sentence: $6 \times 5 \div 2 = 15$
 Answer: 15 cm^2



Math sentence: $4 \times 7 \div 2 = 14$
 Answer: 14 cm^2

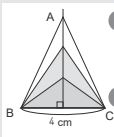


Math sentence: $6 \times 7 \div 2 = 42$
 Answer: 42 cm^2

14-7 Area of Quadrilaterals and Triangles
Area of Triangles (4)

Example A triangle has the base of 4 cm. The height is increased from 1 cm to 2 cm. Answer the following questions.

1 Find the area of the triangle when the height is 1 cm.
 Math sentence: $4 \times 1 \div 2 = 2$
 Answer: 2 cm^2



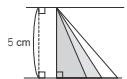
2 How much does the area increase every time the height is increased by 1 cm? Complete the table below.

Height (cm)	1	2	3	4	5	6	7
Area (cm ²)	2	4	6	8	10	12	14

- 3** When the height changes from 2 cm to 4 cm, how many times does the area increase?
 Times: **Twice**
- 4** Write a math sentence to find the area of a triangle with a height of \triangle cm and an area of \circ cm².
 $4 \times \triangle \div 2 = \circ$

A right angle triangle has the height of 5 cm. The base is increased from 1 cm to 2 cm and so on. Answer the following questions.

1 When the base is 1 cm, find the area of the triangle.
 Math sentence: $1 \times 5 \div 2 = 2.5$
 Answer: 2.5 cm^2



2 How much does the area increase every time the height is increased by 1 cm? Complete the table below.

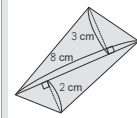
Base (cm)	1	2	3	4	5	6	7
Area (cm ²)	2.5	5	7.5	10	12.5	15	17.5

3 When the base changes from 4 cm to 12 cm, how many times does the area increase?
 Times: **Three times**

4 Write a math sentence to find the area of a triangle with a base of \square cm and an area of \circ cm².
 $\square \times 5 \div 2 = \circ$

14-8 Area of Quadrilaterals and Triangles
How to Find the Area of Various Figures

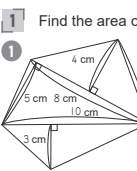
Example Find the area of the following figures.



Math sentence: $8 \times 3 \div 2 = 12$ $12 + 8 = 20$
 $8 \times 2 \div 2 = 8$
 Answer: 20 cm^2

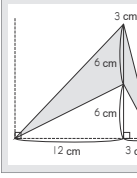
Dividing them into two triangles

1 Find the area of the following figures.



Math sentence: $8 \times 4 \div 2 = 16$
 $8 \times 5 \div 2 = 20$ $16 + 20 + 15 = 51$
 $10 \times 3 \div 2 = 15$
 Answer: 51 cm^2

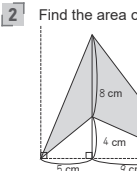
Example 2 Find the area of the following figures.



Math sentence: $(6 \times 12 \div 2) + (6 \times 3 \div 2) = 36 + 9 = 45$
 Alternatively, $(15 \times 12 \div 2) - (15 \times 6 \div 2) = 90 - 45 = 45$
 Answer: 45 cm^2

Alternatively, subtract two triangles

2 Find the area of the following figures.



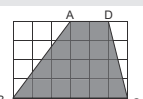
Math sentence: $(8 \times 5 \div 2) + (8 \times 9 \div 2) = 20 + 36 = 56$
 Alternatively, $(14 \times 12 \div 2) - (14 \times 4 \div 2) = 84 - 28 = 56$
 Answer: 56 cm^2

The area of a quadrilateral or a pentagon can be found by dividing them into triangles.

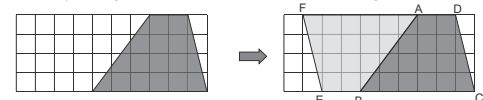
14-9 Area of Quadrilaterals and Triangles
Area of Trapezoids (1)

Instruction How to find the area of the trapezoid on the right.

Let's think about it. I wonder we can find like rectangles and triangles.



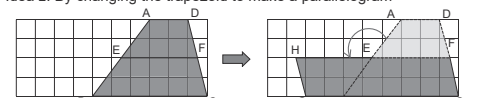
Idea 1: By putting another trapezoid to make a parallelogram



If you put the same trapezoid ABCD, you can make parallelogram FECD. Now you can find the area of a trapezoid. So...

Then, you can find the area and halve it.
 Math sentence: $8 \times 4 \div 2$
 Base of parallelogram FECD: 8
 Height of parallelogram FECD: 4

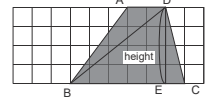
Idea 2: By changing the trapezoid to make a parallelogram



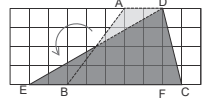
If you divide the trapezoid into two AEFD and EBCF, then you turn and move AEFD, you can make a parallelogram, HGCF. You already know how to find the area of a parallelogram. So...

Then, you can find the area of the parallelogram.
 Math sentence: 8×2
 Base of parallelogram HGCF: 8
 Height of parallelogram HGCF: 2

Alternatively, you can find the area as follows.



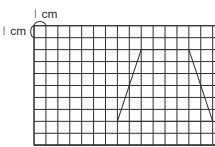
Since the area is sum of triangle ABD and triangle DBC, and then the height is DE,
 $(AD \times DE) \div 2 + (BC \times DE) \div 2$
 $(2 \times 4) \div 2 + (6 \times 4) \div 2$



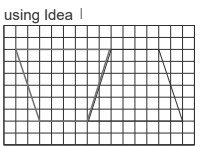
Since the base of the triangle is AD + BC and the height is DF,
 $(AD + BC) \times DF \div 2$
 $(2 + 6) \times 4 \div 2$

What are the similarities and differences among the ideas on how to find the area of the triangle?
 If you can change a triangle into a rectangle or parallelogram, you can find the area.

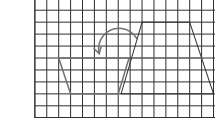
Example Find the area of the following triangle.



By using Idea 1

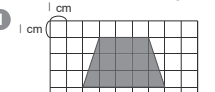


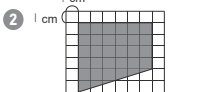
By using Idea 2



Math sentence: $12 \times 6 \div 2 = 36$
 Answer: 36 cm^2

Find the area of the following trapezoids.

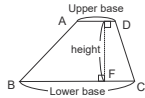
1 
 Math sentence: $(3 + 5) \times 3 \div 2 = 12$
 Answer: 12 cm^2

2 
 Math sentence: $(4 + 6) \times 6 \div 2 = 30$
 Answer: 30 cm^2

14-10 Area of Quadrilaterals and Triangles
Area of Trapezoids (2)

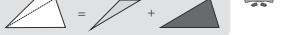
Instruction Area of Trapezoid

- The two parallel sides of the trapezoid are called upper base and lower base, and the distance between them is called the height.
- If you know the upper base, the lower base, and height, the area of a trapezoid can be found.

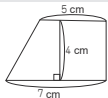


$$(\text{Area of Trapezoid}) = (\text{Upper base} + \text{Lower base}) \times (\text{Height}) \div 2$$

You can also find the area by dividing the trapezoid into triangles.



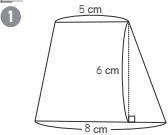
Example Find the area of the following trapezoid.



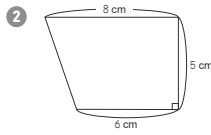
Math sentence $(5 + 7) \times 4 \div 2 = 24$

Answer 24 cm^2

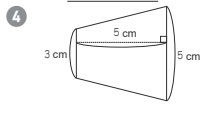
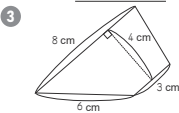
1 Find the area of the following trapezoids.



Math sentence $(5 + 8) \times 6 \div 2 = 39$
Answer 39 cm^2

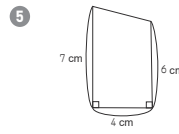


Math sentence $(8 + 6) \times 5 \div 2 = 35$
Answer 35 cm^2

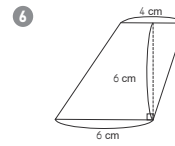


Math sentence $(3 + 8) \times 4 \div 2 = 22$
Answer 22 cm^2

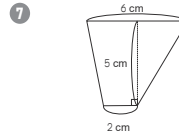
Math sentence $(3 + 5) \times 5 \div 2 = 20$
Answer 20 cm^2



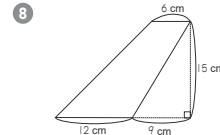
Math sentence $(6 + 7) \times 4 \div 2 = 26$
Answer 26 cm^2



Math sentence $(4 + 6) \times 6 \div 2 = 30$
Answer 30 cm^2

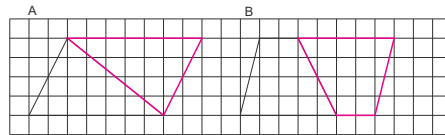


Math sentence $(6 + 2) \times 5 \div 2 = 20$
Answer 20 cm^2



Math sentence $(6 + 12) \times 15 \div 2 = 135$
Answer 135 cm^2

2 Compare the areas of the following figures by adding another figure to calculate easily.



The area of figure A: 14 cm^2

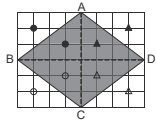
The area of figure B: 14 cm^2

The area of the figure A is **the same as figure B**

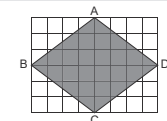
14-11 Area of Quadrilaterals and Triangles
Area of Rhombuses (1)

Instruction How to find the area of the rhombus on the right.

Idea 1: Dividing the rhombus and find the area

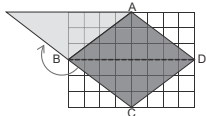


Half of the area of a rectangle

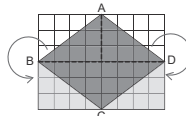


Dividing a rhombus into two triangles

Idea 2: Changing the rhombus into a known figure and find the area

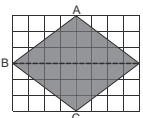
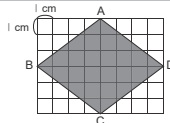


Changing it into a parallelogram base \times height

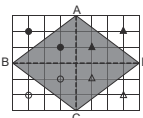


Changing it into a rectangle length \times width

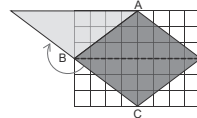
1 Find the area of the rhombus on the right. Match the diagrams and explanations.



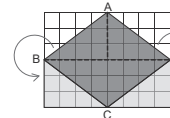
Dividing a rhombus into two triangles, $(8 \times 3 \div 2) \times 2$



Changing it into a rectangle length \times width, $8 \times (6 \div 2)$

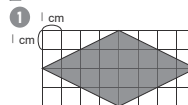


Changing it into a parallelogram base \times height, $8 \times (6 \div 2)$

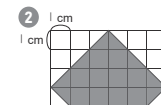


Since it is half of the area of a rectangle, $(8 \times 6) \div 2$

2 Find the area of the following rhombuses.



Math sentence $(8 \times 4) \div 2 = 16$
Answer 16 cm^2



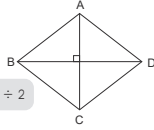
Math sentence $(6 \times 6) \div 2 = 18$
Answer 18 cm^2

Can we find the area of a rhombus by using a formula?

14-12 Area of Quadrilaterals and Triangles
Area of Rhombuses (2)

Instruction Area of Rhombus

- If you know the length of the two diagonals, the area of rhombus can be found.



$$(\text{Area of Rhombus}) = (\text{Diagonal}) \times (\text{Diagonal}) \div 2$$

Example 1 Find the area of the following rhombus.

1
 Math sentence $6 \times 10 \div 2 = 30$
 Answer 30 cm^2

2
 Math sentence $(2.5 \times 2) \times (6 \times 2) \div 2 = 30$
 Answer 30 cm^2

Find the area of the following rhombuses.

1
 Math sentence $12 \times 4 \div 2 = 24$
 Answer 24 cm^2

2
 Math sentence $(4 \times 2) \times (5 \times 2) \div 2 = 40$
 Answer 40 cm^2

3
 Math sentence $(4 \times 2) \times (3 \times 2) \div 2 = 24$
 Answer 24 cm^2

4
 Math sentence $(3 \times 2) \times 3 \div 2 = 9$
 Answer 9 cm^2

Example 2 Find the area of the following figure.

1
 Math sentence $6 \times 9 \div 2 = 27$
 Answer 27 cm^2

The area of the figure equals to the half area of the rectangle EFGH.

2 Find the area of the following figures.

1
 Math sentence $10 \times 7 \div 2 = 35$
 Answer 35 cm^2

2
 Math sentence $9 \times 6 \div 2 = 27$
 Answer 27 cm^2

14-13 Area of Quadrilaterals and Triangles
Review

1 Where is the height against each coloured base?

1
 EC (CE)

2
 JH (HJ)

2 Which pairs of figures have the same area?

A, B

3 Find the area of the following figures.

1
 Math sentence $3 \times 5 = 15$
 Answer 15 cm^2

2
 Math sentence $5 \times 4 = 20$
 Answer 20 cm^2

3
 Math sentence $6.4 \times 4 \div 2 = 12.8$
 Answer 12.8 cm^2

4
 Math sentence $5 \times 6 \div 2 = 15$
 Answer 15 cm^2

5
 Math sentence $(4 + 5) \times 5 \div 2 = 22.5$
 Answer 22.5 cm^2

6
 Math sentence $(6 + 7) \times 4 \div 2 = 26$
 Answer 26 cm^2

7
 Math sentence $3.6 \times 6 \div 2 = 10.8$
 Answer 10.8 cm^2

8
 Math sentence $5 \times 8 \div 2 = 20$
 Answer 20 cm^2

4 Find the area of the following figures.

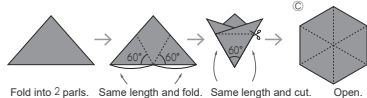
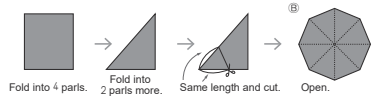
1
 Math sentence $(16 \times 8 \div 2) + (16 \times 10 \div 2) = 16 \times 18 \div 2 = 16 \times 9 = 144$
 Answer 144 cm^2

2
 Math sentence $(8 \times 5 \div 2) + (8 \times 9 \div 2) = 8 \times 14 \div 2 = 8 \times 7 = 56$
 Alternatively, $(14 \times 12 \div 2) - (14 \times 4 \div 2) = 84 - 28 = 56$
 Answer 56 cm^2

15-1 Regular Polygons and Circles

Regular Polygons

Instruction Polygons Use square paper to make figures.



- A polygon is made of straight lines, and the shape is "closed" (all the lines connect up).
- All sides are equal in length and all angles equal in size is called a **regular polygon**.

How many sides and angles are there in each?

Also, how about the length of sides and sizes of angles?

The table below shows typical kinds of polygons.

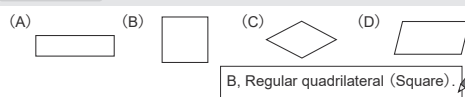
Name of figure	Triangle	Quadrilateral	Pentagon
Number of sides	3	4	5
Example figures			

A regular polygon has all the sides, and the angles are equal in length and in size.

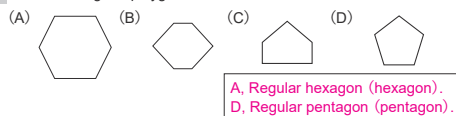
Name of figure	Hexagon	Heptagon	Octagon
Number of sides	6	7	8
Example figures			

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Example 1 Find the regular polygons and write the names.



1 Find the regular polygons and write the names.



Example 2 The following figure is a polygon whose length of sides and size of angles are equal. Answer the following questions.

1 What is the name of the polygon?
Regular pentagon (pentagon)

2 Write down the sides where the length is the same as side AB.
BC, CD, DE, EA

3 Write down the angles where the size is the same as angle A.
B, C, D, E

2 The following figure is a polygon whose length of sides and size of angles are equal. Answer the following questions.

1 What is the name of the polygon?
Regular hexagon (hexagon)

2 Write down the sides where the length is the same as side AF.
AB, BC, CD, DE, EF

3 Write down the angles where the size is the same as angle B.
A, C, D, E, F

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15-2 Regular Polygons and Circles

Regular Octagons and Regular Hexagon

Example 1 Investigate the regular octagon shown below, the diagonals connect opposite vertices and intersect at point O. Answer the following questions.

1 How many degrees is angle AOB?
Math sentence: $360 \div 8 = 45$ Answer: angle AOB = 45°

2 What kind of triangle is formed by the diagonals? Write the reason.
Name of triangle: Isosceles triangle Reason: The point O is the centre of the circle and the length from O is the same.

3 How many degrees is angle BAH?
Since the triangles formed by the diagonals are isosceles triangle, angle OAB = OBA. Since triangle OAB and OHA are congruent, angle OAH = OAB. $\text{AOB} + \text{OAB} + \text{OBA} = 180^\circ$.
Since $\text{BAH} = \text{OAB} + \text{OBA}$, $180^\circ - \text{AOB} = \text{BAH}$.
Math sentence: $180 - 45 = 135$ Answer: angle BAH = 135°

1 In the regular hexagon shown below, the diagonals connect opposite vertices and intersect at point O. Answer the following questions.

1 How many degrees is angle A?
Since angle A is one sixth of 360° ,
Math sentence: $360 \div 6 = 60$ Answer: angle A = 60°

2 What kind of triangle is formed by the diagonals?
Since angle A is 60° and the length from point O is the same.
Equilateral triangle

3 How many degrees is angle B?
The triangles formed by the diagonals are equilateral triangle.
Math sentence: $180 - 60 = 120$ Answer: angle B = 120°

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Example 2 Draw a regular hexagon by using a circle.

1. Draw a circle.

2. Mark the length same as radius on the circumference.

3. Connect the intersections with straight lines using a ruler.

2 Draw a regular octagon by using a circle.

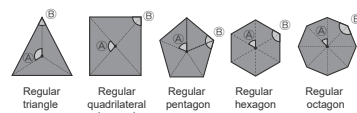
1. Draw a circle.

2. Divide the angle around the centre of a circle into 8 equal angles, 45° each.

3. Connect the intersections with straight lines using a ruler.

3 Summarize the number of sides and the size of angles of regular polygons.

	Regular triangle	Regular quadrilateral (square)	Regular pentagon	Regular hexagon	Regular octagon
Number of sides	3	4	5	6	8
Size of angle A	120°	90°	72°	60°	45°
Size of angle B	60°	90°	108°	120°	135°



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15-3 Regular Polygons and Circles **Circumference**

Instruction Circumference

- The perimeter of a circle is called **circumference**. A line that bends like a circumference is called a **curve**.
- The circumference is longer than 3 times the diameter and shorter than 4 times the diameter.

Is this true with any circle?
Regardless of the circle's size, (circumference) ÷ (diameter) is always the same number.

- The number we get from (circumference) ÷ (diameter) is called the ratio of circumference, 3.1415...
- The ratio of circumference, 3.1415... It is a number that continues infinitely, but normally is used as 3.14. (Ratio of circumference) = (circumference) ÷ (diameter)

If you know a diameter and ratio of the circumference, 3.14, you can find the circumference.

Example 1 Find the circumference of the circle below

Math sentence $3 \times 3.14 = 9.42$

Answer 9.42 cm

Example 2 When you draw a circle with a diameter of 6 m on the playground, how many m will the circumference of the circle be?

Math sentence $6 \times 3.14 = 18.84$

Answer 18.84 m

1 Find the circumference of the circle below

(1) **Math sentence** $4 \times 3.14 = 12.56$ **Answer** 12.56 cm

(2) **Math sentence** $5 \times 3.14 = 15.7$ **Answer** 15.7 cm

2 When you draw a circle with a diameter of 9 m on the playground, how many m will the circumference of the circle be?

Math sentence $9 \times 3.14 = 28.26$

Answer 28.26 m

3 A circle with a radius of 10 m was drawn. How many meters is the circumference?

Math sentence $10 \times 2 \times 3.14 = 62.8$

Answer 62.8 m

(Diameter) = (Radius) × 2

15-4 Regular Polygons and Circles **Calculation of the Circumference**

Example 1 Find the diameter of a circle with circumferences of 18.84 cm.

Math sentence $18.84 \div 3.14 = 6$

Answer 6 cm

1 Find the diameter of a circle with circumferences of 40.82 cm.

Math sentence $40.82 \div 3.14 = 13$

Answer 13 cm

Example 2 A girl measured the length around a rounded pond and found it was 78 m. How many m is the diameter of the pond? Round the number to the nearest one.

Math sentence $78 \div 3.14 = 24.8407643$

Answer Approximately 25 m

2 A boy measured the length around a round pond. It was 200 m. How many m is the diameter of the pond? Round the number to the nearest one.

Math sentence $200 \div 3.14 = 63.6942675$

Answer Approximately 64 m

Example 3 Find the radius of the following figures.

a circle with a circumference of 21.98 cm.

Math sentence $21.98 \div 3.14 = 7$
 $7 \div 2 = 3.5$

Answer 3.5 cm

(Radius) = (Diameter) ÷ 2

3 Find the radius of the following figures.

1 a circle with a circumference of 25.12 cm.

Math sentence $25.12 \div 3.14 = 8$
 $8 \div 2 = 4$

Answer 4 cm

2 a circle with a circumference of 62.8 cm.

Math sentence $62.8 \div 3.14 = 20$
 $20 \div 2 = 10$

Answer 10 cm

Example 4 Find the length around the figure below.

Math sentence $(5 \times 2 \times 3.14 \div 2) + (5 \times 2) = 25.7$

Answer 25.7 cm

The figure is a half circle with a 5 cm radius.

There are two kinds of lengths, curved and straight lines.

4 Find the length around the figure below.

(1) **Math sentence** $(8 \times 2 \times 3.14 \div 4) + (8 \times 2) = 28.56$

Answer 28.56 cm

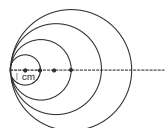
(2) **Math sentence** $(6 \times 2 \times 3.14 \div 2) + (6 \times 3.14 \div 2) \times 2 = (18.84) + (9.42) \times 2 = 37.68$

Answer 37.68 cm

The curved line is a quarter of the circumference with 8 cm radius.

15 - 5 Regular Polygons and Circles
Relationship between Circumference and Diameter

Example Investigate how the circumference changes when the diameter changes.



When the diameter increases, the size of the circle is larger...

1 Write a math sentence to calculate the circumference, C , if the diameter is d cm.

$$C = d \times 3.14$$

Circumference = Diameter × Pi

2 As d changes from 1 to 4, what are the corresponding values for C ? Complete the table below.

Diameter d (cm)	1	2	3	4
Circumference C (cm)	3.14	6.28	9.42	12.56

3 When the diameter increases by 1 cm, how many cm does the circumference increase?

Math sentence: $6.28 - 3.14 = 3.14$

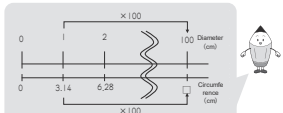
Find the difference the length of the circumference when diameter is 1 cm and 2 cm.

Answer: 3.14 cm

4 Calculate the circumference when the diameter is 100 cm.

Math sentence: $100 \times 3.14 = 314$

Answer: 314 cm



How does the circumference change when the radius changes?

1 Write a math sentence to calculate the circumference, C , if the radius is r cm.

$$C = 2r \times 3.14$$

Circumference = Radius × 2 × Pi

2 As r changes from 1 to 4, what are the corresponding values for C ? Complete the table below.

Radius r (cm)	1	2	3	4	5
Circumference C (cm)	6.28	12.56	18.84	25.12	31.4

3 When the radius increases by 1 cm, how many cm does the circumference increase?

Math sentence: $12.56 - 6.28 = 6.28$

Answer: 6.28 cm

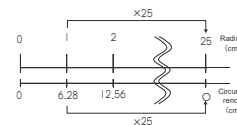
4 How many times does the circumference increase when you double the radius?

Twice

5 Calculate the circumference when the radius is 25 cm.

Math sentence: $25 \times 2 \times 3.14 = 157$

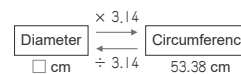
Answer: 157 cm



6 Find the diameter of a circle with a circumference of 53.38 cm.

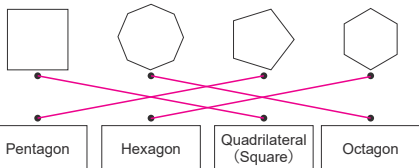
Math sentence: $53.38 \div 3.14 = 17$

Answer: 17 cm



15 - 6 Regular Polygons and Circles
Review

1 Match the figures of the regular polygons to their names.



2 In the regular pentagon shown below, the diagonals connect opposite vertices and intersect at point O. Answer the following questions.



How many degrees is angle A and angle B?

Since angle A is one fifth of 360° ,

Math sentence: $360 \div 5 = 72$ Answer: angle A = 72°

Since the triangles formed by the diagonals are equilateral triangle,

Math sentence: $(180 - 72) \div 2 = 54$ Answer: angle B = 54°

3 Find the circumferences of the following figures.

1 a circle with a diameter of 30 cm. 2 a circle with a radius of 7.5 cm.

Math sentence: $30 \times 3.14 = 94.2$

Answer: 94.2 cm

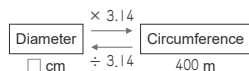
Math sentence: $15 \times 3.14 = 47.1$

Answer: 47.1 cm

4 A girl measured the length around a rounded forest and found it was 400 m. How many m is the diameter of the forest? Round the number to the nearest one.

Math sentence: $400 \div 3.14 = 127.388535$

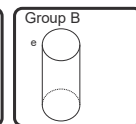
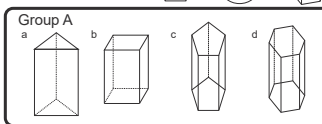
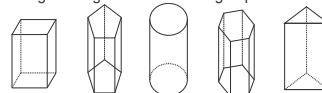
Answer: Approximately 127 m



16 - 1 Prisms and Cylinders
Various Solids and Prisms

Instruction Various Solids

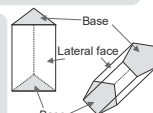
Sort the following solid figures into the two groups as follows.



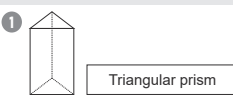
These are sorted by looking at the bottom of the shape.

Also, how about shapes of surface? Figures in Group A are enclosed by plane. Figure in Group B has curved surface.

- The two parallel congruent circles of a prism are called base, and the rectangular or square faces around the bases are called lateral faces.
- When the bases are triangles, quadrilaterals, pentagons, these prisms are called **triangular prism**, **quadrangular prism**, **pentagonal prism**, respectively.



Example 1 Write the name of the following solids.



1 Write the name of the following solids.



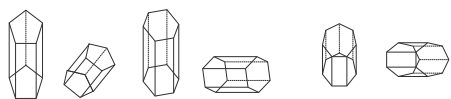
Example 2 Complete the table below.

	Triangular prism	Quadrangular prism
Shape of bases	Triangle	Quadrilateral
Shape of lateral faces	Rectangle or square	Rectangle or square
Number of faces	$2 + 3 = 5$	$2 + 4 = 6$
Number of vertices	$3 \times 2 = 6$	$4 \times 2 = 8$
Number of edges	$3 \times 2 + 3 = 9$	$4 \times 2 + 4 = 12$

There is a relation between the number of faces, vertices, edges and shape of bases.

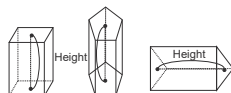
2 Complete the table below.

	Pentagonal prism	Hexagonal prism	Heptagonal prism
Shape of bases	Pentagon	Hexagon	Heptagon
Shape of lateral faces	Rectangle or square	Rectangle or square	Rectangle or square
Number of faces	$2 + 5 = 7$	$2 + 6 = 8$	$2 + 7 = 9$
Number of vertices	$5 \times 2 = 10$	$6 \times 2 = 12$	$7 \times 2 = 14$
Number of edges	$5 \times 2 + 5 = 15$	$6 \times 2 + 6 = 18$	$7 \times 2 + 7 = 21$



Instruction Height of Prisms

- The length of the line that is perpendicular to the two bases of a prism is called the **height of the prism**.



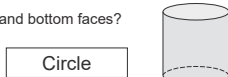
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16-2 Prisms and Cylinders

Cylinders

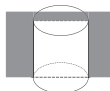
Example Answer the following questions on the solid below.

1 What kind of shape are the top and bottom faces?



Circle

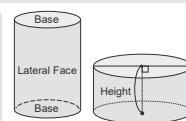
2 When you cut the figure as follows, what kind of shape can you see?



Rectangle

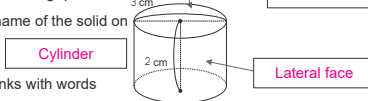
Instruction Cylinder

- The two parallel congruent circles of a cylinder are base and the curved face around the bases is called the lateral face.
- The length of the line that is perpendicular to the two bases of a cylinder is called the **height of the cylinder**.



1 Answer the following questions.

1 What is the name of the solid on the right?



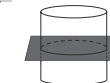
Cylinder

2 Fill in the blanks with words

3 How many cm of the height?

2 cm

2 When you cut the cylinder as follows, what kind of shape you can see?



Circle

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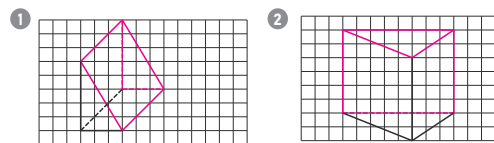
16-3 Prisms and Cylinders

Sketch and Net of a Prism

Example 1 Finish drawing the sketch of the triangular prism as shown below.

1. Draw three edges from one vertex.	2. Draw the visible edges.	3. Draw the invisible edges using a dotted line.

1 Finish drawing the sketch of the triangular prisms as shown below.



Example 2 Finish drawing the net of the triangular prism on the right.

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1. Draw lateral faces.	2. Draw a base.	3. Draw the other base opposite side.

2 Finish drawing the net of the triangular prism on the right.

3 The below shows a net of a triangular prism. Answer the following questions.

1 Which face is the base of the prism? A, E

2 How many cm of the height? 5 cm

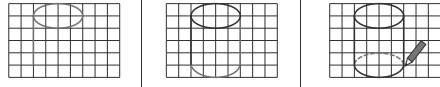
3 Circle the vertices that match up with vertex F.

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16 - 4 Prisms and Cylinders
Sketch and Net of a Cylinder

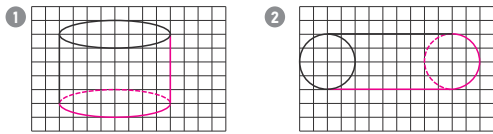
Example 1 Finish drawing the sketch of the cylinder as shown below.

1. Draw a bottom base and an edge from one vertex.
2. Draw the visible parts.
3. Draw the invisible parts using a dotted line.

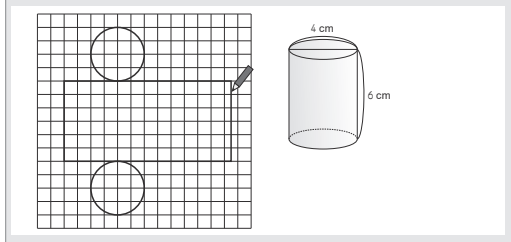


Connect dots by handwriting without a compass.

1 Finish drawing the sketch of the cylinders as shown below.



Example 2 Finish drawing the net of the cylinder on the right.

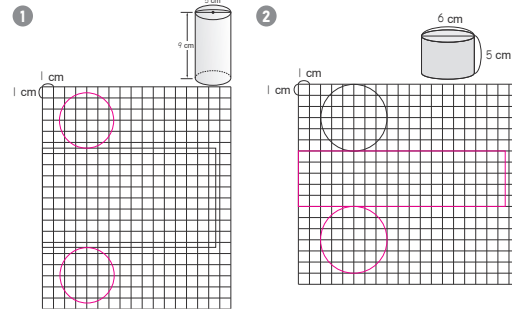


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1. Draw a base.
2. Draw the lateral face.
3. Draw the other base opposite side.

The length of the lateral face is circumference, (diameter) \times 3.14

2 Finish drawing the net of the cylinder.



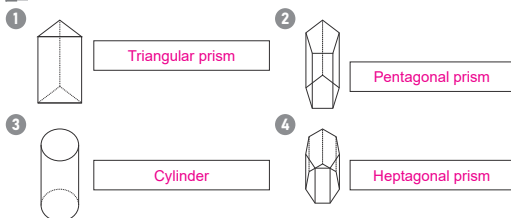
3 The below shows a net of a cylinder. Answer the following questions.

- 1 Which face is the base of the prism? **A, E**
- 2 How many cm of the line AB? **Math sentence** $4 \times 3.14 = 12.56$
Answer 12.56 cm

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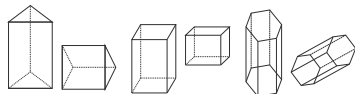
16 - 5 Prisms and Cylinders
Review

1 Write the name of the following solids.

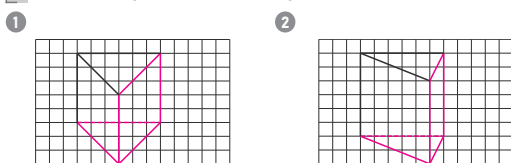


2 Complete the table below.

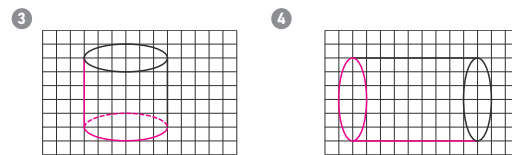
	Triangular prism	Quadrangular prism	Hexagonal prism
Shape of bases	Triangle	Quadrilateral	Hexagon
Shape of lateral faces	Rectangle or square	Rectangle or square	Rectangle or square
Number of faces	$2 + 3 = 5$	$2 + 4 = 6$	$2 + 6 = 8$
Number of vertices	$3 \times 2 = 6$	$4 \times 2 = 8$	$6 \times 2 = 12$
Number of edges	$3 \times 2 + 3 = 9$	$4 \times 2 + 4 = 12$	$6 \times 2 + 6 = 18$



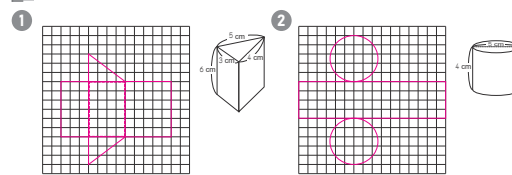
3 Finish drawing the sketch of the triangular prism as shown below.



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4 Finish drawing the net of the prism and cylinder below.



5 The below shows a net of a triangular prism. Answer the following questions.

- 1 How many cm of the height? **4 cm**
- 2 Circle the vertices that match up with vertex H.

6 The below shows a net of a triangular prism. Answer the following questions.

- 1 Which face is the base of the prism? **A, C**
- 2 How many cm of the height? **3 cm**
- 3 Circle the vertices that match up with vertex L.

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Number & Operation
Entire Grade-5 Review (1)

1 Write the correct numbers in the .

1 $35.678 = 30 + 5 + 0.6 + 0.07 + 0.008$
 $= 10 \times \boxed{3} + 1 \times \boxed{5} + 0.1 \times \boxed{6} + 0.01 \times \boxed{7} + 0.001 \times \boxed{8}$

2 $10 \times 5 + 1 \times 7 + 0.1 \times 2 + 0.01 \times 8 = \boxed{57.28}$

3 7.04 times 10 is 4 7.04 times 100 is

5 0.8 times $\frac{1}{10}$ is 6 0.8 times $\frac{1}{100}$ is

2 Write the least common multiple of the numbers in each ().

1 (4, 7) 2 (8, 18) 3 (12, 14)

3 Write the greatest common factors of the numbers in each ().

1 (12, 15) 2 (10, 18) 3 (27, 36)

4 Categorize the following numbers as even, odd, and prime numbers.
 2 3 5 9 13 14 15 18 19 20

Even numbers Odd numbers

Prime numbers

5 Write the correct number in the .

1 $\frac{3}{7} = \frac{\boxed{6}}{\boxed{14}} = \frac{9}{\boxed{21}}$ 2 $\frac{5}{9} = \frac{\boxed{15}}{\boxed{27}} = \frac{40}{\boxed{72}}$

3 $5 \div 6 = \frac{5}{\boxed{6}}$ 4 $\frac{4}{9} = 4 \div \boxed{9}$

6 Calculate the following problems by using the algorithm.

1 2.8×5.4 2 6.6×4.5 3 0.28×0.79 4 2.64×4.2

1	15.12	2	29.70	3	0.2212	4	11.088
5	4.5	6	0.125	7	3.5	8	140

7 Calculate the following by finding a common denominator.

1 $\frac{1}{5} + \frac{2}{7}$ 2 $\frac{5}{24} + \frac{5}{8}$ 3 $1\frac{5}{6} + \frac{3}{4}$ 4 $1\frac{3}{10} + 3\frac{8}{15}$

5 $\frac{5}{8} - \frac{9}{20}$ 6 $\frac{7}{12} - \frac{1}{3}$ 7 $2\frac{1}{3} - \frac{5}{8}$ 8 $3\frac{1}{9} - 1\frac{2}{3}$

1	$\frac{17}{35}$	2	$\frac{5}{6}$	3	$2\frac{7}{12}$	4	$4\frac{5}{6}$
5	$\frac{7}{40}$	6	$\frac{1}{4}$	7	$1\frac{17}{24}$	8	$1\frac{4}{9}$

8 There is a 1 m copper bar that weighs 16.4 kg. How many kg does 5.5 m of this copper bar weigh?

Math sentence $16.4 \times 5.5 = 90.20$

Weight $(16.4) \times (\square) = (\square)$ kg
 Length $(1) \times (\square) = (\square)$ m

Answer

9 There is a 0.7 m iron bar that weighs 8.82 kg. How many kg does 1 m of this iron bar weigh?

Math sentence $8.82 \div 0.7 = 12.6$

Weight $(8.82) \div (\square) = (\square)$ kg
 Length $(0.7) \div (\square) = (\square)$ m

Answer

10 A small bottle holds 4 L of juice and a large bottle holds 9 L of juice. How many times more L does the large bottle hold than the small bottle?

Math sentence $9 \div 4 = \frac{9}{4}$ or 2.25

Amount $(4) \div (\square) = (\square)$ L
 Times $(9) \div (\square) = (\square)$ times

Answer

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Geometry
Entire Grade-5 Review (2)

1 The following quadrilaterals are congruent. Answer the following questions.

1 Which is the corresponding vertex to vertex E?
 Vertex

2 Which is the corresponding angle to angle G?
 Angle Size

3 Which is the corresponding side to side EF?
 Side Length

2 Find the size of the following labeled angles below.

1 2

3 Regarding the following solids, answer the following questions.

1 Which has curved surface?

2 Write each name of shape of base.
 A B C D

4 Find the area of the following figures.

1 2

5 Find the volume of the following figures.

1 2

6 Find the circumferences of the following figures.

1 a circle with diameter of 13 cm. 2 a circle with radius of 3.5 cm.

1 2

7 A girl measured the length around a rounded pond and found it was 350 m. How many m of the diameter of the pond? Round the first decimal place.

Math sentence $350 \div 3.14 = 111.464968$

Answer

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Change & Relation
Entire Grade-5 Review (3)

1 The table shows the number of lost items on the market last week. On average, how many items were lost each day?

Days	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Number of lost items	5	4	1	3	6	7	9

Math sentence
 $(5 + 4 + 1 + 3 + 6 + 7 + 9) \div 7 = 5$ Answer 5 items

2 There are 40 passenger in a bus that has 50 seat. What is the crowdedness of this bus?

Passenger Ratio Math sentence $40 \div 50 = 0.8$
Answer 80%

3 Car A runs 840 km on 30 L of gasoline and car B runs 750 km on 25 L of gasoline. Which car runs more per L of gasoline, A or B?

Math sentence
Car A $840 \div 30 = 28$ Answer 28 km per L
Car B $750 \div 25 = 30$ Answer 30 km per L
Answer Car B

4 It takes 30 minutes to walk from home to school. If you walk at a speed of 120 m per minute, how many km will it take you to get from home to school?

Math sentence
Distance $120 \times 30 = 3600$
 $3600 \text{ m} = 3.6 \text{ km}$ Answer 3.6 km

5 The distance you can go in 4 hours hiking is 21 km. how many km do you cover per hour?

Distance Math sentence $21 \div 4 = 5.25$
Hours Answer 5.25 km

6 Find the number of \square by writing it in math sentence.

(a) 42 m is $\square\%$ of 56 m.
Length Ratio Math sentence $42 \div 56 = 0.75$
Answer 75%

(b) 80% of 4.5 kg is \square kg.
Math sentence $4.5 \times 0.8 = 3.6$ Answer 3.6 kg

(c) 25% of 2000 zeds is \square zeds.
Math sentence $2000 \times 0.25 = 500$ Answer 500 zeds

(d) 40% of \square L is 48 L.
Weight Area Math sentence $48 \div 0.4 = 19.2$
Answer 19.2 kg

7 The table below shows the number of books per type based on 250 books in book shelf. Find the proportions in the table below and draw the graph on the right.

Animal	Percentage	Number of books
Story	<u>34%</u>	85
Biography	<u>24%</u>	60
Picture book	<u>20%</u>	50
The others	<u>22%</u>	55
Total	<u>100%</u>	250

Data Utilization
Entire Grade-5 Review (4)

1 The graph on the right shows planted area in crops in a town.

1 Complete the table below.

Name of Crops	Maize	Cassava	Rice	The others
Ratio (%)	<u>25</u>	<u>18</u>	<u>17</u>	<u>40</u>

Planted area in crops

2 The area of agricultural field in the town is 3.5 km². Find the area of rice field?

Math sentence $3.5 \times 0.17 = 0.595$
Answer 0.595 km²

2 The following strip graph shows the result of the survey what kind of vehicles passed for a day.

The total number of vehicles was 1000, how many bicycles were passed?

Math sentence $1000 \times 0.25 = 250$
Answer 250

3 The following table shows land use in a town. Complete the circle graph and strip graph.

Land use	Forest	Agriculture	Residence	The others
Ratio (%)	52	23	8	17

4 The table below shows the proportion of garbage in a city in 2000, 2010, and 2020.

	2000 (%)	2010 (%)	2020 (%)
Burnable garbage	64	62	55
Unburnable garbage	12	11	9
Recyclable garbage	24	27	36

1 Complete the strip graph below

Changes in the proportion of garbage

2 How many times the proportion of recyclable garbage is compared to unburnable garbage in 2020.

Math sentence $36 \div 9 = 4$
Answer 4 times

3 Total amount of garbage in 2000 was 1.2 million ton. How much the amount of burnable garbage in 2000?

Math sentence $1.2 \times 0.64 = 0.768$
Answer 0.768 million ton (768 thousand ton)

