

Unit

Ratios and percentages

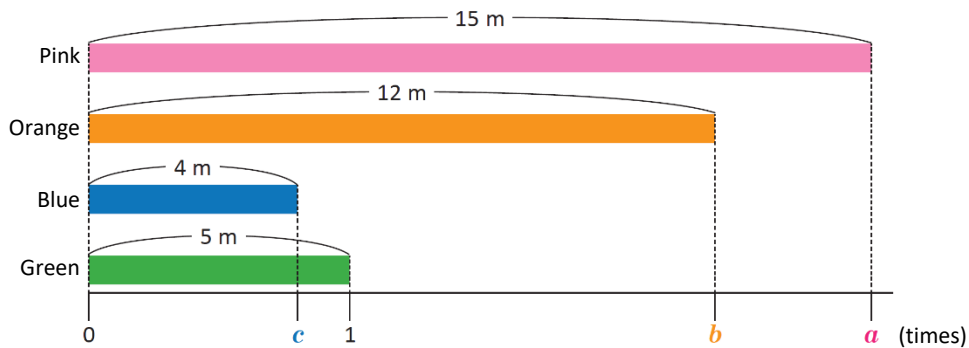
In this unit, you will learn how to

- Determine the ratio between two quantities
- Calculate the ratio value
- Use different notations to express ratios
- Solve problems involving percentage calculations

1.1 Comparison between quantities: number of times

Analyze

Look at the bars and the number line.



- How many times is the length of the pink ribbon relative to the size of the green ribbon?
- How many times is the length of the orange ribbon relative to the length of the green ribbon?
- How many times is the length of the blue ribbon compared to the size of the green ribbon?

Solution

- a. **PS:** $15 \div 5$

$$15 \div 5 = 3$$

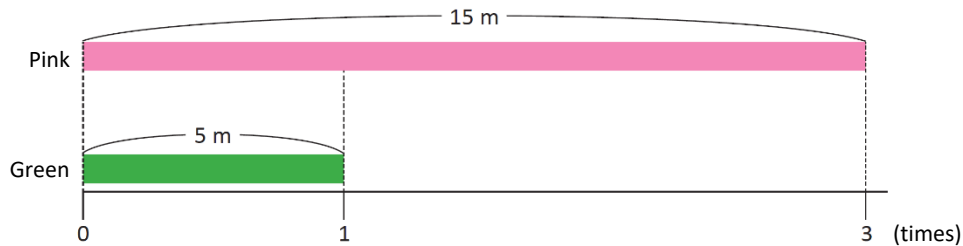
The length of the pink ribbon is three times the length of the green ribbon.

A: Three times.



Julia

In the diagram, the number of times the pink ribbon is relative to the green ribbon is represented by **a**. So, **a** is equal to three.



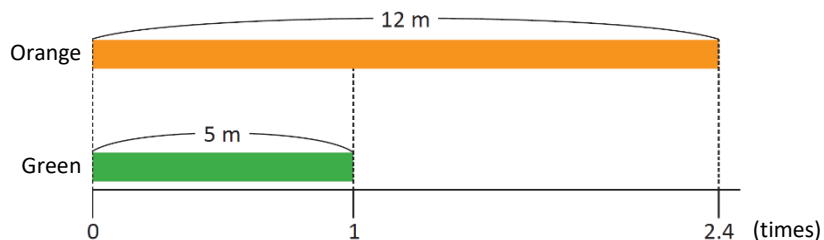
- b. **PS:** $12 \div 5$

$$12 \div 5 = 2.4$$

The length of the orange ribbon is 2.4 times the length of the green ribbon.

A: 2.4 times.

In the diagram, the number of times the orange ribbon is relative to the green ribbon is represented by **b**. So, **b** is equal to 2.4.



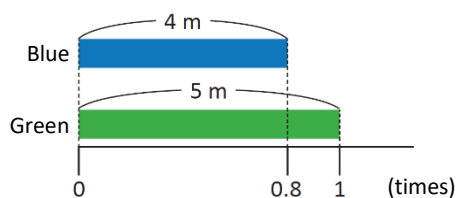
c. PS: $4 \div 5$

$$4 \div 5 = 0.8$$

The length of the blue ribbon is 0.8 times the length of the green ribbon.

A: 0.8 times.

In the chart, c is equal to 0.8.



Understanding

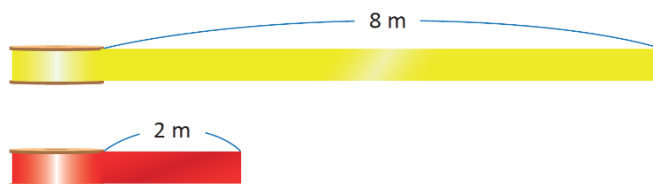
A number of times is also a comparison between quantities through the quotient between them; it can be a natural number, a decimal number, or a fraction.

The number of times one quantity is calculated over another:

$$\text{Number of times} = \text{Quantity to compare} \div \text{Base quantity}$$

Solve

1. Martha has a red ribbon measuring two meters and a yellow one measuring eight meters. Find the number of times the ribbon is yellow relative to the red one.



2. Anthony is 10 years old and his dad is 42 years old. How many times is dad's age relative to Anthony's age?



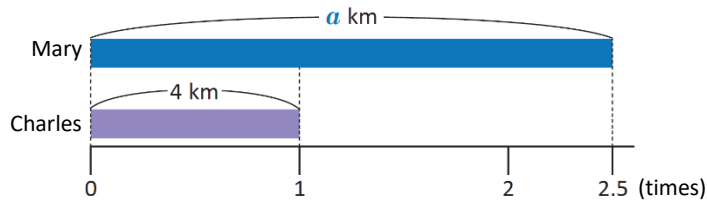
3. In a football tournament, George scored 12 goals and Xavier 9. Find the number of times Xavier's goals are relative to George's goals.



1.2 Calculation of the quantity to compare

Analyze

Charles and Mary went for a run together. Charles completed 4 km, while Mary covered 2.5 times what Charles did. How many kilometers did Mary run?



Remember:

$$\text{Number of times} = \frac{\text{Quantity to compare}}{\text{Base quantity}}$$

How can you calculate the amount to compare if you only know the base amount and the number of times?



Solution



PS: 4×2.5

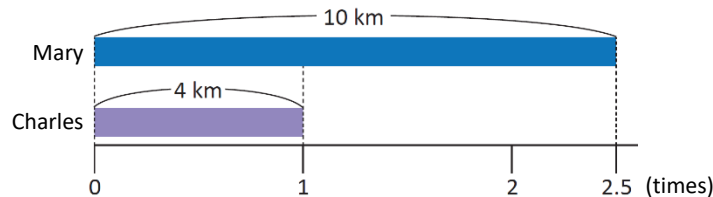
Do the multiplication to find the number of kilometers that Mary traveled:

$$4 \times 2.5 = 10$$

Therefore, Mary traveled 10 km.

A: 10 km

In the figure, the number of kilometers traveled by Mary is represented by a . Thus, $a = 10$:



I can also verify that, by dividing the quantity to be compared (10 km) by the base quantity (4 km), the number of times (2.5) is obtained.

Understanding

When the base quantity and the number of times are known, then the quantity to compare is calculated as follows:

$$\text{Quantity to compare} = \text{Base quantity} \times \text{Number of times}$$

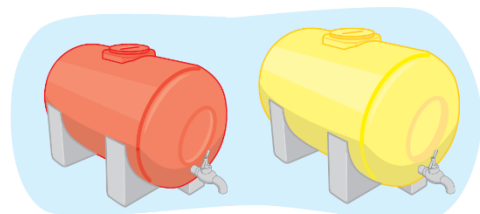
Solve

1. Joseph weighs 45 kg, while Martha weighs 0.8 times what Joseph weighs. How much does Martha weigh?

Remember that the base quantity may be greater than the quantity to be compared.



2. A red tank has a total capacity of 300 liters, while a yellow tank has 1.75 times the capacity of the red tank. What is the capacity of the yellow tank?

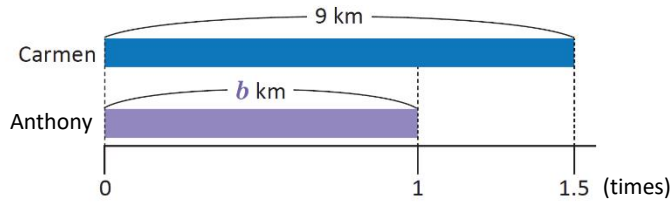


3. Carmen and Beatrice competed in the long jump. Carmen jumped 2m, and Beatrice jumped 0.75 times what Carmen jumped. How many meters did Beatrice jump?

1.3 Calculating the base quantity

Analyze

On one day, Carmen toured 1.5 times what Anthony did. If Carmen traveled 9 km, how many kilometers did Anthony travel?



If:

$$\text{Quantity to compare} = \text{Base quantity} \times \text{Number of times}$$

How can you calculate the base quantity, if you only know the quantity to compare and the number of times?



Solution



Ana

PS: $9 \div 1.5$

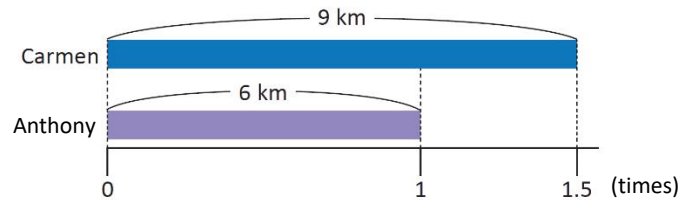
I divide to find the number of kilometers Anthony traveled:

$$9 \div 1.5 = 6$$

Then, Anthony traveled 6 km.

A: 6 km

As per the graph, the number of kilometers traveled by Anthony is represented by b . Thus, $b = 6$:



I can also see that by dividing the quantity to be compared (9 km) by the base amount (6 km), you get the number of times (1.5).

Understanding

When the quantity to be compared and the number of times is known, then the base quantity calculated:

$$\text{Base quantity} = \text{Quantity to compare} \div \text{Number of times}$$

Solve

1. In a swimming class, Martha swam three times what Ana swam. If Martha swam 1.5 km, how many kilometers did Ana swim?
2. In a classroom, the number of boys is 1.4 times the number of girls. If there are 21 boys, how many girls are there in the room?
3. In a rectangle, the length is 3.5 times the width. If the length is 42 cm, how much is the width?
4. At a parents' meeting, the number of men was 0.4 times the number of women. If 32 men attended, how many women attended?

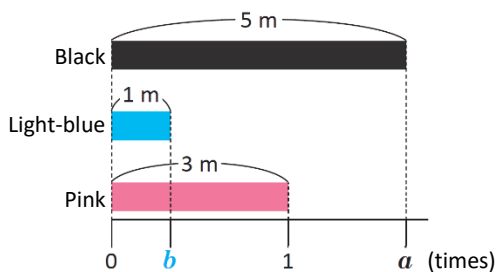
Remember to simplify before performing the calculation.



1.4 Ratio and Ratio value

Analyze

Look at the ribbons on the number line:



- How many times is the black ribbon longer than the pink ribbon?
- How many times is the Light-blue ribbon shorter compared to the pink?

Solution

a. **PS:** $5 \div 3$

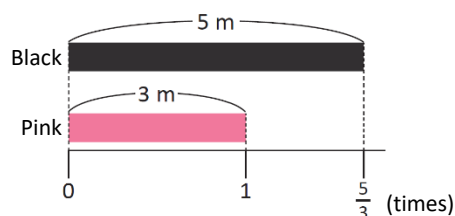


Charles

If I calculate the quotient, I get: $5 \div 3 = 1.66666\dots$
but, the division $5 \div 3$ can also be expressed

$$\text{as } 5 \times \frac{1}{3} = \frac{5}{3}.$$

A: $\frac{5}{3}$ times.

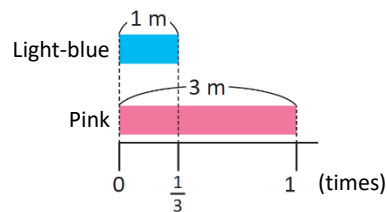


b. **PS:** $1 \div 3$

As per the previous case: $1 \div 3 = 0.33333\dots$ So,

I write the division $1 \div 3$ as $1 \times \frac{1}{3} = \frac{1}{3}$.

A: $\frac{1}{3}$ times.



Understanding

In general, the comparison between two quantities using the quotient between them is called **ratio**. If the values are a and b , **the ratio of a and b** is represented as $a : b$.

The number resulting from calculating the quotient $a \div b$ is called **ratio value**; it can be a natural number, a decimal number, or a fraction (if written as $\frac{a}{b}$).

When quantities being compared have the same unit, the value of the ratio indicates the number of times it is relative to the other.



Solve

- Joseph saved \$ 8 and Julia \$ 3. She writes the ratio of the amount saved by Joseph and the amount saved by herself and calculates the ratio value. What is the interpretation of this result, using the number of times?
- A tank has a total capacity of 2 liters, and a pot, a full capacity of 7 liters. Write the ratio between the tank's capacity and the pot's capacity, then find the ratio value. What is the interpretation of this result, using the number of times?



1.5 Ratio between heterogeneous quantities

Analyze

During a race, Michael covered 33 m in 6 seconds, while John covered 51 m in 10 seconds.

- How many meters did each travel in one second?
- Who was moving faster?

Solution

- To calculate the number of meters Michael traveled in one second, I divide 33 m by 6 seconds:



Carmen

$$33 \div 6 = 5.5$$

Michael ran 5.5 m in one second. Similarly, In John's case I divide, 51 m by 10 seconds:

$$51 \div 10 = 5.1$$

John ran 5.1 m in 1 second.

Notice that you are comparing the distance traveled (in meters), and the time it took to complete it (in seconds); This is also a ratio.



- From the previous statement, I observe that Michael was moving faster because he traveled more meters in one second.

A: Michael ran faster.

Understanding

The quantities compared in a ratio can also be in different units of measure. When the quantity units for a and b are different, the ratio value $a : b$ indicates the number of units available for a and b . It means how many elements there are for each unit of a and b (quantity per unit).

For example, if Michael traveled 33 m in 6 seconds, the ratio between the meters traveled and his time is $33 : 6$. While the ratio value is $33 \div 6 = 5.5$, this indicates that Michael traveled 5.5 meters for every second.

Solve

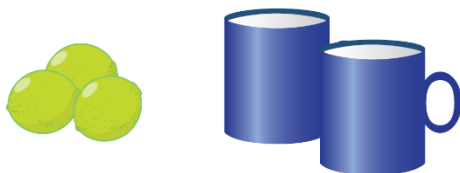
- A car travels 298 km in 4 hours.
 - Write the ratio of the kilometers traveled to the time in hours, and find the value of the ratio .
 - How is this result interpreted?
- There are 20 girls and 10 boys in a classroom..
 - Write the ratio between the number of girls and boys, and find the value of the ratio.
 - How is this result interpreted?



1.6 Antecedent y consequent

Analyze

In an old lemonade recipe, the number of lemons and cups of water are in a ratio of 3 : 2. If 6 cups of water are used, how many lemons should be used?



Solution



Joseph

The ratio values is $\frac{3}{2}$ (or 1.5). Then, for each cup of water you need $\frac{3}{2}$ lemons. And, for 6 cups of water, $6 \times \frac{3}{2}$ lemons will be used.

$$\cancel{6}^3 \times \frac{\cancel{2}^3}{1} = 3 \times 3 = 9$$

A: 9 lemons.

The 3 : 2 ratio indicates that two cups of water are used for every three lemons.



Beatrice

- For six lemons, four cups of water are used (both portions double).
- For nine lemons, six cups of water are used (both portions triple).

A: 9 lemons.

Understanding

In a ratio $a : b$, quantity a is called the antecedent, and quantity b is called the consequent. In addition, it is true that:

$$\text{antecedent} = \text{consequent} \times \text{ratio value}$$

Note that calculating the antecedent is similar to calculating the quantity to compare:

$$\text{Quantity to compare} = \text{Base quantity} \times \text{Number of times}$$

Replace base quantity for consequent and number of times for ratio value.



Solve

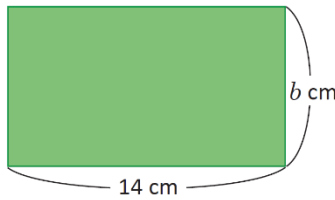
1. Twenty tickets are placed in a bag for a raffle. The number of winning tickets and the total number of tickets in the bag has a ratio of 1 : 4. How many winning papers are there?
2. Anthony practices basketball. One day, he made 15 throws. If the ratio of the shots made to the total number of shots was 4 : 5, how many shots did he make?
3. A restaurant estimated the ratio for the number of people served in one night to the profit they made was 1 : 10. If the restaurant's profit was \$300 that night, how many people did they serve?



1.7 Consequent calculation

Analyze

The length and width of a rectangle are at a ratio of 7 : 4. If the length measures 14 cm, how much does the width measure?



Solution



The ratio value is $\frac{7}{4}$ (or 1.75), so the length is $\frac{7}{4}$ times the width. Then, Mario divides the length by $\frac{7}{4}$ and the result will be the width:

$$14 \div \frac{7}{4} = 14 \times \frac{4}{7} = 2 \times 4 = 8$$

A: 8 cm

The ratio 7 : 4 indicates that, for every 7 cm of the length, there is 4 cm of width. Then:



- For 14 cm in length, there is 8 cm in width (quantities doubled).

A: 8 cm

Understanding

In one ratio it is true that:

$$\text{Consequent} = \text{Antecedent} \div \text{Ratio value}$$

Calculate the consequent is similar to calculating the base quantity:

$$\text{Base quantity} = \text{Quantity to compare} \div \text{Number of times}$$

Remove quantity to compare and write antecedent. Instead of number of times, write ratio value.



Solve

1. In each case, calculate the consequent:

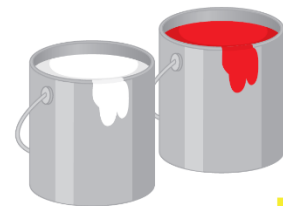
a. Antecedent = 1, ratio value = $\frac{1}{2}$

b. Antecedent = 6, ratio value = $\frac{3}{4}$

c. Antecedent = 10, ratio value = 2

d. Antecedent = 12, ratio value = $\frac{4}{3}$

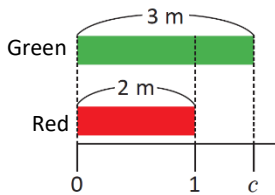
2. Charles prepared pink paint; the ratio of the white to red paint milliliters was 4 : 5. If he used twelve ml of white, how many did he use of red?



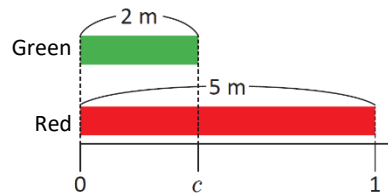
1.8 Practice what you learned

1. Write the length ratio between the green and red ribbons. Then calculate the value of the ratio:

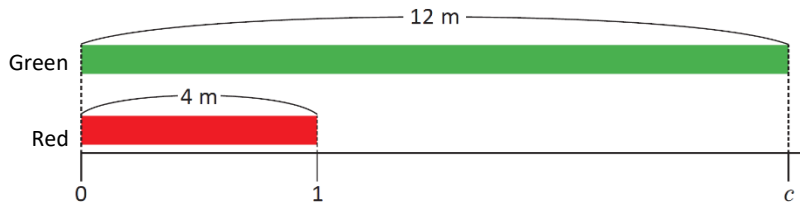
a.



b.



c.



Solve the following problems:

2. In the Salvadoran diet, two tortillas provide 31 g of carbohydrates, 1 g of fat, 3 g of protein, and 150 calories.
 - a. Write the ratios, and calculate the ratio value between the number of carbohydrates, the number of tortillas, the amount of fat, and the number of tortillas
 - b. How do you interpret the above results?
3. Anthony saved \$ 15, and then he spent \$ 5. What is the ratio and ratio value between the money spent and money saved? How do you interpret this result?
4. The ratio of the length to the width of a rectangle is 3 : 2. If the width is 10 cm, how long is the length?
5. On a bus, the ratio between the number of seats occupied and unoccupied is 6 : 5; If 24 seats are occupied, how many seats are vacant?
6. The ratio between the number of calories a person burns and the time (in minutes) they spend running is 10 : 1. If a person burned 150 calories, how many minutes did they spend running?
7. A soccer team determined that the ratio between the total games in a championship and the games they won was 5 : 3. If they won 6 games, how many games were played during the tournament?

2.1 Percent or percentage

Analyze

The following table contains the notes of the number of goals and the number of attempts that John made in his last two football training sessions:

Training	Goals	Attempts
First	5	10
Second	9	12



In which training would you say John was most successful?

Solution

The ratios between the number of goals and the number of attempts for the first and second training sessions are 5 : 10 and 9 : 12, respectively. I calculate the ratio value:

First training session

$$5 \div 10 = 0.5$$

Second training session

$$9 \div 12 = 0.75$$



Anthony

During the first training session, John succeeded in half of the attempts. In the second practice, he succeeded 0.75 times the number of attempts.

A: In the second training session.

Understanding

The **percent** or **percentage** is obtained by multiplying the ratio value by 100, i.e.:

$$\text{Percentage} = \text{Ratio value} \times 100$$

After the last digit of the number indicating percentage, the symbol "%" is written. For example, if the ratio value between the number of goals and the number of attempts (in the first training) is multiplied by 100, you get:

$$\text{percentage} = 0.5 \times 100 = 50$$

It is written "50%" and reads: "fifty percent ."This number indicates that 50 out of every 100 attempts are successful.

Solve

1. The following table contains Michael's results in the last two basketball games.

Game	Baskets	Throws
First	12	16
Second	9	15

- Find the value of the ratio between the number of baskets and total throws attempts.
- What percentage of baskets did you get in each game?, how is this result interpreted?

2. Joseph wrote down the results he got by playing cup and ball on Monday, Tuesday ,and Wednesday:

Day	Successful	Attempts
Monday	8	20
Tuesday	10	25
Wednesday	8	16

- Between Monday and Wednesday, which day did you get the best results? Explain using percentages.
- Between Monday and Tuesday, which day did you get the best results? Explain using percentages.

2.2 Relationship between ratios and percentages

Remember

Perform:

a. 0.01×100

b. 0.2×100

Analyze

In Martha's classroom, there are a total of 20 students, of whom 7 are children. What is the percentage of children in this room?

Solution

The ratio between the number of children and the total number of students is 7 : 20. I calculate the value of the ratio, and then I get the percentage:

$$\text{Ratio value: } 7 \div 20 = 0.35$$

$$\text{Percentage: } 0.35 \times 100 = 35$$



The ratio value, 0.35, is equivalent to 35%.

A: 35% of the students in the classroom are children.

Understanding

En general:

- Multiplying the ratio value by 100 yields the percentage:

$$\text{Percentage} = \text{Ratio value} \times 100$$

- Dividing the percentage by 100 yields the ratio value:

$$\text{Ratio value} = \text{Percentage} \div 100$$

Solve

1. Find the percentage that the following ratio values represent:

a. 0.01 b. 0.07

c. 0.75 d. 1

2. Find the ratio value that corresponds to each of the following percentages:

a. 5 % b. 9 %

c. 12 % d. 54 %

3. The total area of a school is 1,200 m², and the court area is 252 m².

a. What is the ratio value between the court area and the total school area?

b. What percentage of the land occupies the court?

Did you know...?

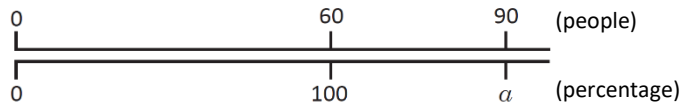
It is widespread to use percentages when the quantities being compared are very large. For example, according to the Projections of the General Directorate of Statistics and Censuses, it is expected that in 2020 the Salvadoran population will be 6,601,409 inhabitants, of which 3,520,577 will be women.

When calculating the ratio value between the number of women and the total population, the result is approximately 0.53. At the same time, the corresponding percentage is 53%. Therefore, it is expected that the estimated population for 2020, 53%, will be women; 53 out of every 100 Salvadorans in 2020 will be women.

2.3 Percentages greater than 100 %

Analyze

A restaurant can accommodate 60 people. According to the restaurant's capacity, if they served 90 people on Saturday, what percentage of people attended?



In this case, the antecedent is greater than the consequent. Therefore, the percentage is greater than 100%.



Solution

I calculate the ratio value for the number of people served and the capacity of the restaurant, and its respective percentage:



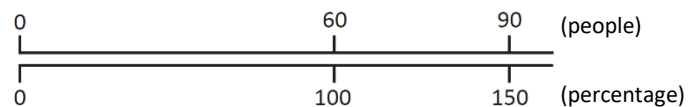
Charles

$$\begin{aligned} \text{Ratio value} &= 90 \div 60 = 1.5 \\ \text{Percentage} &= 1.5 \times 100 = 150 \end{aligned}$$

Then, the percentage of people served in the restaurant was 150%.

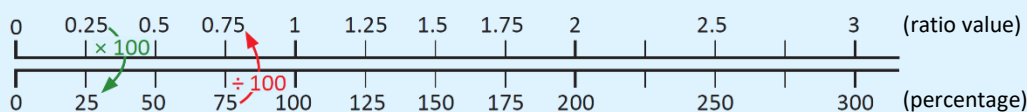
A: 150 %

In the graph, the percentage has been represented as a ; then, $a = 150$.



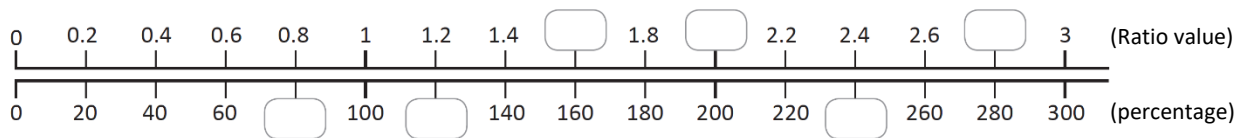
Understanding

When the antecedent is greater than the consequent, the percentage obtained is greater than 100%. This is because the ratio value is greater than 1. The following chart shows some relationships between the ratio value and the corresponding percentage:



Solve

1. Fill in the missing ratio or percentage boxes in the chart:



2. It is recommended that an adult drink 2 liters of water daily. If Mary consumes 2.5 liters, what percentage of water does she consume from the suggested amount?

3. The World Health Organization (WHO) recommends children consume a maximum of 4 g of salt daily; if a child consumes 6 g daily, he could get sick. What percentage of salt relative to the recommended amount can make a child sick?



World Health Organization

2.4 Calculating the antecedent using percentages less than 100 %

Remember

1. How is antecedent calculated using the consequent and ratio value?
2. Find the ratio value for:
 - a. 35 %
 - b. 100 %

Analyze

Mary prepares 200 ml of an orange drink. If 35% of the soft drink content is orange juice, how many milliliters of juice is equivalent? Represent the number of milliliters of juice as a .

The total amount of soft drink (200 ml) corresponds to 100 %. The unknown amount of orange juice (a ml) corresponds to 35 % of the total amount of soft drink.



Solution



I Calculate the ratio value, which is equal to dividing the percentage by 100:

$$\text{Ratio value} = 35 \div 100 = 0.35$$

This value corresponds to the ratio value
 $a : 200$; And as:

$$\text{Antecedent} = \text{Consequent} \times \text{Ratio value}$$

then,

$$a = 200 \times 0.35 = 70$$

A: 70 ml

35% of orange juice means that for every 100 ml of soft drink, then 35 ml would be orange juice. By increasing the beverage to double (200 ml), the amount of orange juice also increases to double, 70 ml.



I check by calculating how much is (in percentage) 70 ml of 200 ml:

$$\text{Ratio value} = 70 \div 200 = 0.35$$

$$\text{Percentage} = 0.35 \times 100 = 35$$

A: 70 ml

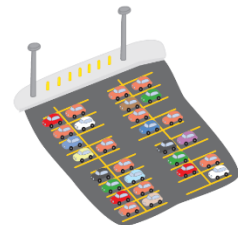
Understanding

In general:

- Calculating the value corresponding to the percentage of a quantity is equivalent to calculating the antecedent of the ratio.
- When the consequent and percentage are known and wants to find the antecedent, proceed with the following steps:
 - ① Find the ratio value from the percentage: ratio value = percentage \div 100.
 - ② Find the antecedent: Antecedent = Consequent \times Ratio value.

Solve

1. Calculate:
 - a. 20 % of 80 liters.
 - b. 90 % of 120 liters.
2. Out of a classroom of 30 students, 80% of students passed the subject of Mathematics. How many students passed?
3. In a parking lot, there are 80 vehicles of which, 5% are green. How many green cars are in the parking lot?



2.5 Calculating the antecedent using percentages greater than 100%

Analyze

Martha's parents must pay \$250 a monthly payment for their home mortgage. Also, they have to pay a 4% fixed interest. How much should you pay each month?

Solution

A 100% of payment is \$250; "4% interest rate per month" indicates that 4% of \$250 is added. So, I have to calculate each monthly payment, including the interest.

- ① The total percentage is: $100\% + 4\% = 104\%$

I use the previous lesson information:

- ② Calculate the ratio value (percentage \div 100): $104 \div 100 = 1.04$
③ Calculate 104 % of 250 (consequent \times ratio value): $250 \times 1.04 = 260$

Martha's parents must pay \$260 each month, which corresponds to the monthly mortgage payment, plus 4% fixed interest.

A: \$260 monthly



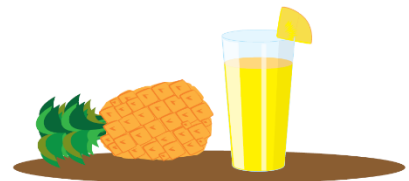
Understanding

In situations involving increases to the percentage, and you want to find the ratio antecedent, do the following.

- ① Find the total percentage: $100\% + \text{percentage increase}$.
② Calculate the ratio value: $\text{percentage} \div 100$.
③ Calculate the antecedent: $\text{antecedent} = \text{consequent} \times \text{ratio value}$.

Solve

1. A pineapple juice typically containing 800 ml is on offer, with 20% more than usual. How many milliliters of juice does it have when on offer?



2. A small printing press wants to purchase a batch of paper that costs \$720. As you want to import it from another country, you must pay a 5% tax on import duties over the original price. How many dollars must the printing press pay for the batch of paper, including taxes?

3. In a restaurant, 9% of the consumption is paid as a tip. If someone consumes \$30, how much will they have to pay, including the tip?



2.6 Calculating prices with VAT

Analyze

Julia's dad will buy a dining set that costs \$160. The seller told him that this price does not include VAT, which is 13% of the original price. How much will the dining set cost you with VAT included?

Note that:

- The price of the dining set without VAT corresponds to 100%
- The dining set price including VAT corresponds to 113%.



Solution



Anthony

In this case there is an increase of 13% to the dining room set price. I apply the steps learned in the previous class:

- ① Total percentage = $100\% + 13\% = 113\%$
- ② Ratio value = $113 \div 100 = 1.13$
- ③ Antecedent = $160 \times 1.13 = 180.8$

A: \$180.80

I find the amount of money you will pay for VAT and add it to \$160 (original dining room price):



Carmen

- ① Amount of money corresponding to 13 %:
Ratio value = $13 \div 100 = 0.13$
Antecedent = $160 \times 0.13 = 20.8$
- ② I add the VAT amount (\$20.80) to the original price:

$$160 + 20.8 = 180.8$$

A: \$180.80

Understanding

Value Added Tax (VAT) is a tax paid at the time of making a purchase. In El Salvador, VAT corresponds to 13% of the original price and can be calculated in two ways:

First option:

- ① Calculate the ratio value corresponding to 113 % (The percentage was obtained by adding 100 % to 13 % VAT).
- ② Calculate the new price by multiplying the actual cost by the ratio value).

Second option:

- ① Calculate 13% of the original price.
- ② Add, to the original price, the amount found in step ①.

In the first option, the value of the ratio corresponding to 113 % is 1.13; then, you can perform a single step by multiplying the original price by 1.13.



Solve

Calculates the price of the following items, including VAT, using the two options shown.

- Desktop computer for \$525.
- A fan for \$30.
- A television for \$449.



2.7 Calculating prices and discounts

Analyze

Mary bought a backpack for 25% off. If the regular price was \$8, how much did Mary pay for the gear?

The price, applying the discount, is equal to 75% of the original price.



Solution



Mario

- ① As the backpack had a 25% discount, Mary only canceled 100% - 25% of the original price, 75%.

- ② 75 % corresponds to a ratio value of 0.75 ($75 \div 100$).

- ③ Price to pay: $8 \times 0.75 = 6$

A: \$6

- ① I calculate 25% of \$8, multiplying by 0.25 (ratio value corresponding to 25%):

$$8 \times 0.25 = 2$$

- ② Subtract from the original amount the value corresponding to the discount:

$$8 - 2 = 6$$

A: \$6



Ana

Understanding

To find the price after applying discounts, you can perform two procedures:

First option:

- ① Calculate the percentage of the discount price:
 $100\% - \text{discount percentage}$
- ② Calculate the ratio value corresponding to the percentage found in ①.
- ③ Find the discounted price, multiply the ratio value by the original price.

Second option:

- ① Calculate the ratio value corresponding to the discount percentage.
- ② Calculate the amount corresponding to the discount.
- ③ Subtract the amount found in ② from the original price.

Solve

At "THE BARGAIN" clothing store, clothes are on sale. Find the price of the following garments by applying the indicated discount:

- a. Girl's dress
Regular price: \$20
30 % off



- b. Men's sweater
Regular price: \$15
20 % off



- c. Boy's T-shirt
Regular price: \$5
5% off



2.8 Calculating consequent using percentages

Remember

Julia read 200 pages of a book on vacation. This amount is 5 times the number of pages Joseph read. How many pages did Joseph read?

Analyze

A one-month-old giraffe measures 260 cm; this height corresponds to 130% of its size just at birth. What was the giraffe's height immediately after birth? Represent this quantity as b cm.

Note that:

- The height of the giraffe at birth corresponds to 100 % (consequent, b cm).
- The height of the giraffe after one month, which is 260 cm, corresponds to 130 % (antecedent).



Solution



Charles

Calculate the ratio value, which is equal to dividing the percentage by 100:

$$\text{Ratio value} = 130 \div 100 = 1.3$$

This number corresponds to the ratio value $260 : b$; and as:

$$\text{Consequent} = \text{Antecedent} \div \text{Ratio value}$$

then,

$$b = 260 \div 1.3 = 200$$

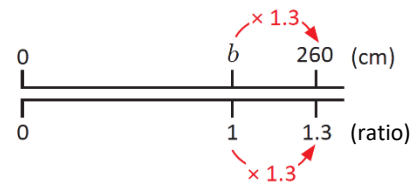
A: 200 cm

Did you know...?

In the double number line graph, for the ratio to increase from 1 to 1.3, multiply 1×1.3 ; then, for the centimeters to increase from b to 260 must multiply $b \times 1.3$, and:

$$b \times 1.3 = 260$$

1.3 times b equals 260, so $b = 260 \div 1.3 = 200$



Understanding

When you know the quantity whose percentage is greater than 100% (antecedent), and you want to find the original (consequential) quantity, do the following:

- ① Calculate the ratio value: **Ratio value = percentage \div 100**
- ② Calculate the consequent, which is the original quantity: **consequent = antecedent \div ratio value**

Solve

1. A TV costs \$678 with tax included. What is the price of the TV excluding tax?

Note that \$678 correspond to 113 %

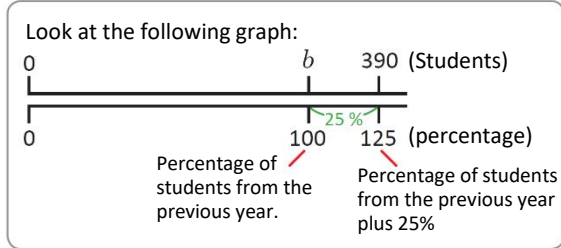


2. Martha weighs 60 kg, and this corresponds to 120% of what she weighed a year ago. How much did Martha weigh a year ago?

2.9 Calculating percentage and consequent

Analyze

At Ana's school this year, there are 390 students. If this number is 25% more students than the previous year, how many students were there last year? It represents the number of students last year as b .



Solution



Julia

"25% more students than last year" indicates that last year's number of students (b students) represents 100%. This year there are $100\% + 25\% = 125\%$ students compared to last year.

This year's 390 students correspond to 125%, and the ratio value $390 : b$ is equal to:

$$125 \div 100 = 1.25$$

I apply previous lesson knowledge, **Consequent = Antecedent \div Ratio value:**

$$b = 390 \div 1.25 = 312$$

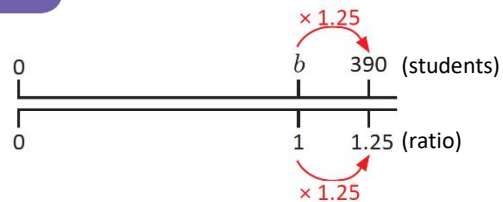
A: 312 students

Did you know...?

For the ratio to increase from 1 to 1.25, multiply 1×1.25 ; then, for the students to increase from b to 390, multiply $b \times 1.25$, and:

$$b \times 1.25 = 390$$

1.25 times b equals 390, so $b = 390 \div 1.25 = 312$



Understanding

In problems where the percentage increases, the amount corresponding to that increase is known as (antecedent); the original (consequent) amount is still unknown. Perform the following:

- ① Find the total percentage corresponding to the increase: $100\% + \text{percentage increase}$.
- ② Calculate the ratio value: $\text{total percentage} \div 100$
- ③ Calculate the original (consequent) quantity: **Consequent = Antecedent \div Ratio value**

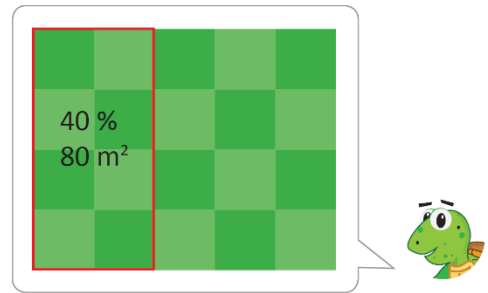
Solve

1. Joseph's height is 156 cm, 20% more than his sister Julia. What is Julia's height in centimeters?
2. After receiving a 10% increase from his previous salary, John's salary is \$440. What was the previous wage?
3. A puppy weighs 168 g a week after birth. This amount is 60% more than the puppy's birth weight. How many grams did it weigh at birth?

2.10 Calculating the consequent using percentages less than 100 %

Analyze

The owner of a piece of land decides to sell it on lots for a more significant profit. So far, it has sold a lot of 80 m^2 , which represents 40% of the total land. What is the total area of the land? Represent the entire area as $b \text{ m}^2$.



Solution



The ratio value $80 : b$ equal a:
 $40 \div 100 = 0.4$

To calculate the quantity b , I Use:

Consequent = Antecedente \div Ratio value

$$b = 80 \div 0.4 = 200$$

A: 200 m^2



Remember, the antecedent may be greater than the consequent.

The total area ($b \text{ m}^2$) represents 100 %. Like $100 \% = 40 \% + 40 \% + 20 \%$, then I can find b by adding the corresponding areas to 40 % and 20 %.

- $40 \% \rightarrow 80 \text{ m}^2$
- $20 \% \rightarrow 40 \text{ m}^2$ (Is half of 40%)

$$b = 80 + 80 + 40 = 200$$

A: 200 m^2



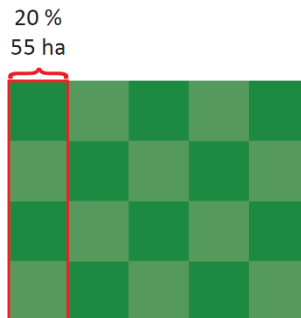
Understanding

Even if the percentage is less than 100%, the consequent is always calculated with the formula:

Consequent = Antecedent \div Ratio value

Solve

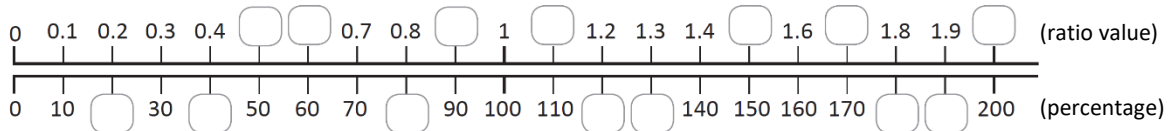
1. A farmer plants 55 ha of corn representing 20% of his land. How many hectares is the land?



2. A worker saves \$56, which is 10% of her monthly salary. How much is her monthly salary?

2.11 Practice what you learned

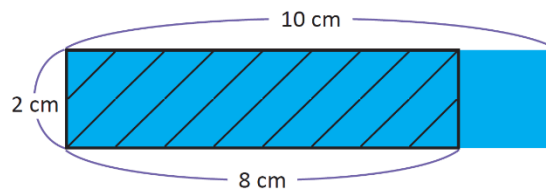
- In the Math exam, Martha got eight correct questions out of a total of ten. What is the percentage of correct answers?
- In a cinema room, 42 seats out of the 120 available are occupied. What is the percentage of occupied seats?
- Fill in the missing ratio and percentage values:



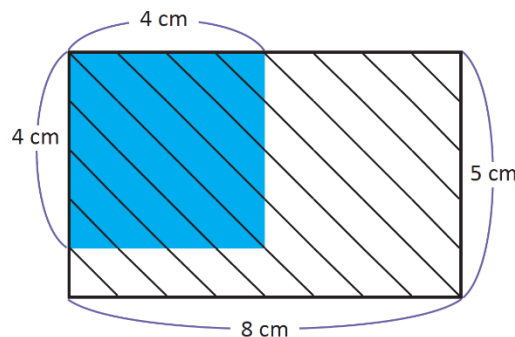
- A spa served 250 people on August 5th and 300 people on August 6th.
 - Calculates the ratio value between the number of people who attended on August 6th and those who attended on August 5th.
 - What percentage of people who attended on the 6th compared to those who attended on the 5th?
- At John's nursery, there are 420 plants of which, 25% are roses. How many roses are in the nursery?
- While waiting for photos to download to his computer; John notices so far, 30% of 50 megabytes have been downloaded. How many megabytes have been downloaded so far?

★ Self-challenge

- Calculate the percentage represented by the shaded area with lines of the rectangle in relation to the area of the blue rectangle.

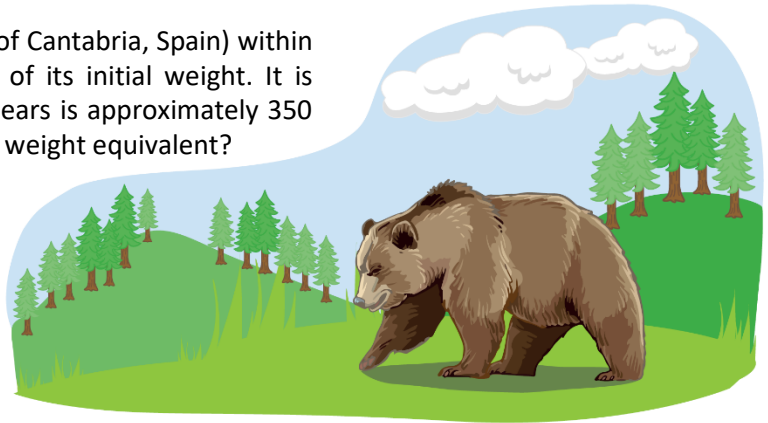


- Calculate the percentage represented by the shaded area with lines of the rectangle in relation to the area of the blue square.



2.12 Practice what you learned

1. A brown bear (living in the mountains of Cantabria, Spain) within a few months of birth reaches 150% of its initial weight. It is known that the birth weight of such bears is approximately 350 grams. How many grams is 150% of his weight equivalent?



2. A shirt that costs \$40 is on sale with 15% off. How many dollars does the shirt cost when you apply the discount.?

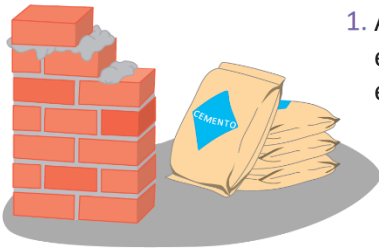
3. At the end of the year, John managed to save \$70, and this represents 140% of what was planned. How many dollars had he planned to save?

4. Ana sold a TV for \$240; this amount is 20% more than the price for which she purchased the TV. How many dollars did Ana pay when she purchased the TV?



5. When a grizzly bear (subspecies of the North American brown bear) hibernates, its heart rate drops to 10 beats per minute, 20% of its standard value. What is the normal heart rate of the grizzly bear?

★ Self-challenge



1. Anthony is building a wall for which he needs eight bags of cement. If each bag costs \$5 without tax, how much will you have to pay for the eight bags after adding 13% tax?

2. A train has covered 65% of its route. If you still have 70 km left to travel, how many kilometers is the total route?

