Unit 7. Inscribed and central angles

Unit competencies

To determine the measurement of angles inscribed and semi-inscribed on a circumference, using theorems and relationships on chords and arcs on a circumference, to study the characteristics and properties of planar figures.

Relation and development

I and II cycle

- Construction of angles using the protractor
- Classification and construction of triangles
- Classification and construction of quadrilaterals
- Classification of geometric bodies
- Symmetrical figures
- Perimeter, area of triangles, and quadrilaterals
- Rectangular and triangular cube and prism patterns
- Length of the circumference and area of the circle
- Length and area of notable circular sectors
- Prism volume
- Translations, turns, and rotational symmetry

Seventh grade

Unit 8: Plane figures and construction of geometric bodies

- Movement of figures in the plane
- Circles, segments, and angles
- Planes, geometric figures, and total area of the prism, pyramid, and cylinder

Eight grade

Unit 4: Parallelism and angles of a polygon

- Adding internal and external angles of a polygon
- Parallel lines and angles

Unit 5: Triangle congruence criteria

- Congruence of triangles

Unit 6: Characteristics of triangles and quadrilaterals

- Triangles
- Parallelograms

Unit 7: Area and volume of geometric solids

- Volume characteristics of geometric solids
- Calculation of the volume of geometric solids
- Volume applications
- Areas of geometric solids
- Area applications

Ninth grade

Unit 5: Similar figures

- Similarity
- Similarity of triangles
- Similarity and parallelism
- Application of similarity and similar triangles

Unit 6: Pythagorean Theorem

- Pythagorean theorem
- Theorem application

Unit 7: Inscribed and central angle

- Central and inscribed angle
- Central and inscribed angle application

Unit Curriculum

Lesson	Hours	Classes
1. Central and inscribed angle	1	1. Elements of the circumference
	1	2. Definition and measurement of inscribed angles
	1	3. Inscribed angle, part 1
	1	4. Inscribed angle, part 2
	1	5. Inscribed angle theorem
	1	6. Practice what you learned
	1	7. Congruent arcs
	1	8. Practice what you learned
2. Application of the central and inscribed angle	1	1. Construction of tangents to a circumference
	1	2. Chords and arcs of the circumference
	1	3. Similar triangle applications
	1	4. Parallelism
	1	5. Four points on a circumference
	1	6. Semi-inscribe angle
	2	7. Practice what you learned
	1	Unit 7, test

16 class hours + Unit 7 test

Key points of each lesson

Lesson 1: Central and inscribed angle
In class 1.2, the central angle theorem is determined intuitively, using geometric instruments, so that in the classes after this lesson, the formal proof of it is carried out.

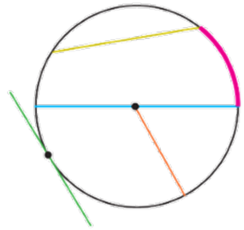
Lesson 2: Application of central and inscribed angle
Having proved the angle measure theorem previously inscribed, this lesson uses this result as the main tool for the deduction of some properties.

Lesson 1 Central and inscribed angle

1.1 Elements of the circumference

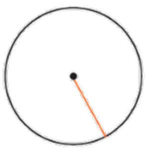
P

Write the name given to the drawn elements on the following circumference:

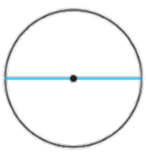


S

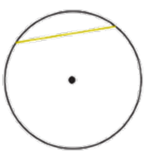
Segments.



The segment that goes from the center to a point in the circumference is called **radius.**

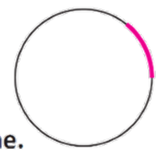


The segment that goes from one point of the circumference to another and passes through the center is called **diameter.**



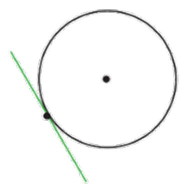
The segment that goes from one point of the circumference to another is called **chord.**

Arc.



Any portion of the circumference of a circle is called an **arc.**

Line.



The line that touches the circumference at a point is called **tangent.**

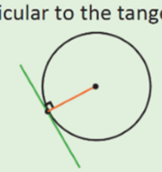
The point where the tangent line touches the circumference is called: **point of tangency**.

C

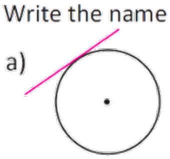
The elements of the circumference are:

- The segments: radius, diameter and chord
- The lines: tangent
- The arc of the circumference

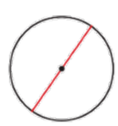
The radius to the point of tagency is perpendicular to the tangent point.

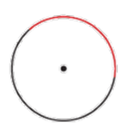


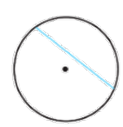
1. Write the name of the elements given fore each circimference:

a)  Tangent line

b)  Radius

c)  Diameter

d)  Arc

e)  Chord

2. Respond the following questions:

a) What is the element that is $\frac{1}{2}$ in diameter? Radius

b) What is the name of the longest chord on a circumference? Diameter

c) How is the tangent line and radiues to the poingt of tangency of a circumference? Perpendicular

d) By placing two dots on the circumference. How many arcs are formed? Two

Achievement Indicator

1.1 Identify the elements of the circumference / CIRCLE.

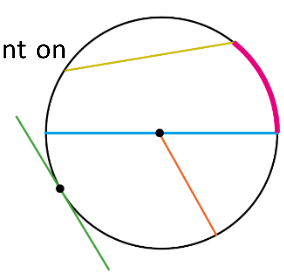
Sequence

From the first to sixth grade, the elements of the circle were taught. In seventh grade, the circle revisited to work with its elements, determine the meaning of the tangent line to the circumference and deduce properties from the characteristics of two intersecting circles. In this class, a reminder is made about the elements of the circle; the difference is they are presented as elements of the circumference; furthermore, it is presented to the tangent line to the circumference as one more element. Students posses a very clear understanding about the relationship between the circle and the circumference, so it is expected that there will be no confusion regarding the class.

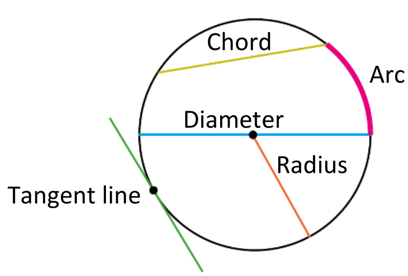
For this case, the first item is considered complete when writing the names of all literals.

Date: **U7 1.1**

Ⓟ Write the name of each element on the circumference.



Ⓢ



Ⓡ 1.
a) Tangent line
b) Radius
c) Diameter
d) Arc
e) Chord

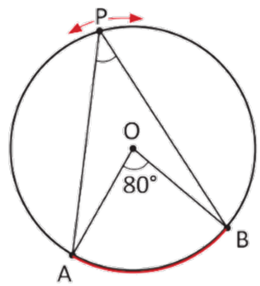
2.
a) Radius
b) Diameter
c) Perpendiculars
d) Two

Homework: Workbook, page 148.

1.2 Definition and measurement of inscribed angles

P

Draw on a piece of paper and measure ∢BPA by moving point P to different places in the circumference. Compare the measument of ∢BPA with ∢BOA.



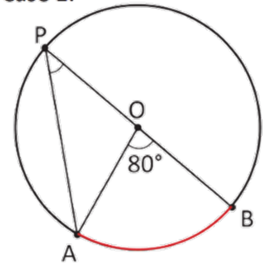
The BOA angle is called **central angle**, because its vertex is the center of the circumference.

Note that ∢BPA and ∢BOA share the same arc $\overset{\frown}{AB}$.

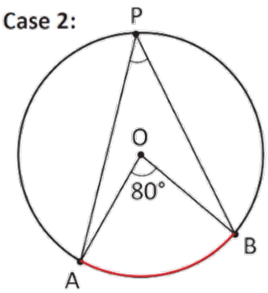
S

Use a ruler and a compass to make the drawing and move point P on the circumference for the following cases:

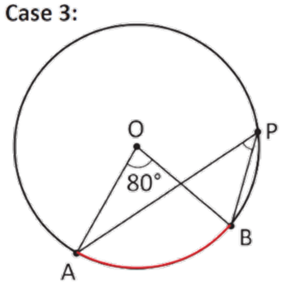
Case 1:



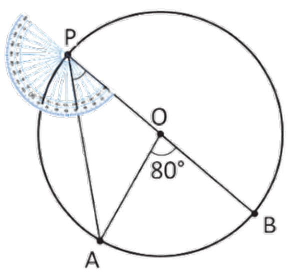
Case 2:

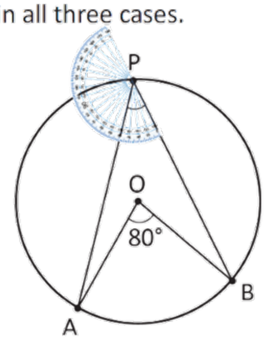


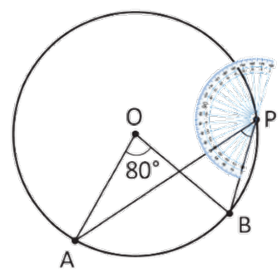
Case 3:



Using the protractor measure ∢BPA in all three cases.







In all three cases the measurement of ∢BPA = 40°.

And ∢BOA = 2∢BPA or ∢BPA = $\frac{1}{2}$ ∢BOA.

C

Subtending the same arc means sharing the same arc.

Angles whose vertex is on the circumference are called:**Inscribed angles.**

In a circumference, the measure of the central angle that subtends the same arc of any inscribed angle is twice the measure of any inscribed angle that subtends the same arc.

Determine the measurement of an angle inscribed to a circumference whose central angle within the same arc measure 160°. Use a scheme as in the initial problem.

Achievement Indicator

1.2 Distinguish the types of inscribed angles on the circumference and their intuitive relationship to the central angle.

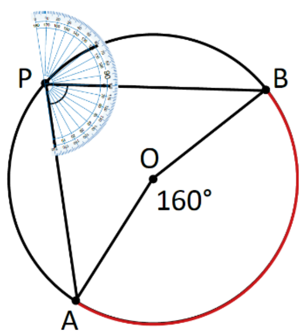
Sequence

The concept of inscribed angle in a circle is introduced in this class, simultaneously; the property related to its measure is presented. The property is raised intuitively from the construction, that is, through the use of geometry instruments. This class is relevant as a basis for the following three. Some elements are retaken, and will be detailed in the purpose section.

Purpose

Ⓟ Provides three possible cases that can occur when moving the point on the circumference. There may be more shapes the students do, although any of the shapes made will resemble one of the cases presented. The first one refers to the case in which the central angle is on one side of the inscribed angle, and the second, to the case in which the central angle is inside the inscribed angle, finally the third, to the case in which the central angle is outside the inscribed angle.

Solution of some items:



Measure of $\measuredangle BPA = 80°$

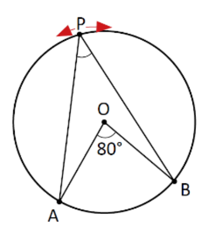
$\measuredangle BOA = 2\measuredangle BPA$

or

$\measuredangle BPA = \frac{1}{2}\measuredangle BOA.$

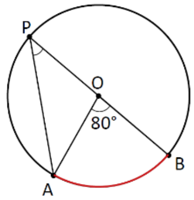
Date: **U7 1.2**

Ⓟ Measure $\measuredangle BPA$ by displacing P at different positions on the circumference. Compare the measure of $\measuredangle BPA$ with that of $\measuredangle BOA$.

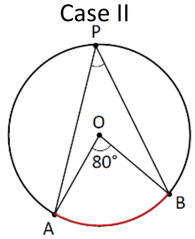


Ⓢ

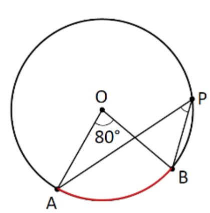
Case I



Case II

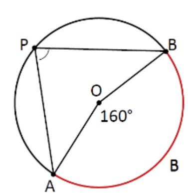


Case III



In all three cases $\measuredangle BPA = 40°$, y $\measuredangle BOA = 2\measuredangle BPA$ or $\measuredangle BPA = \frac{1}{2}\measuredangle BOA$.

Ⓡ



The measure of $\measuredangle BPA = 80°$

$\measuredangle BOA = 2\measuredangle BPA$

or

$\measuredangle BPA = \frac{1}{2}\measuredangle BOA.$

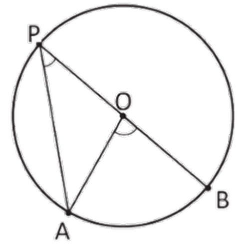
Homework: Workbook, page 149.

Lesson 1

1.3 Inscribed angles, part 1

P

Demonstrate that ∢BOA = 2∢BPA when the center lies somewhere in the ΔBPA.



The diameter is the chord that passes across the center of the circumference.

S

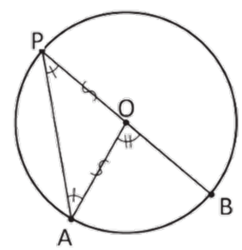
In ΔAOP: OP = OA (are radiuses of the circumference).

So, ∢OPA = ∢PAO (equal sides oposse equal angles).

Else ∢BOA = ∢OPA + ∢PAO (∢BOA is the external angle of ΔAOP).

Therefore, ∢BOA = 2∢OPA.   As ∢OPA = ∢BPA.

Then, ∢BOA = 2∢BPA.



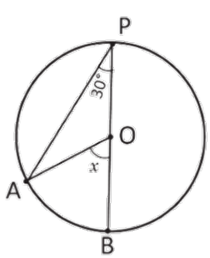
C

In the inscribed angles whose side coincides with the diameter of the circumference it is satisfied that **the measurement of the central angle subtending the same arc is twice the measurement of the inscribed angle.**

E

Determine the value of x for each case.

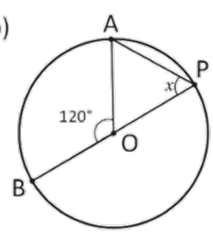
a)



As ∢BOA = 2∢BPA.

Therefore, $x = 2(30°) = 60°$.

b)



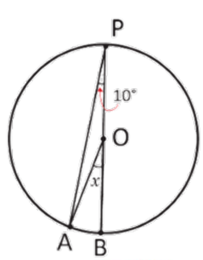
As ∢BOA = 2∢BPA.

Then, $∢BPA = \frac{1}{2}∢BOA$

Therefore, $x = \frac{120°}{2} = 60°$.

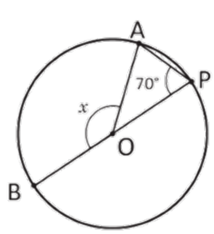
Determine the value of x for each case.

a)



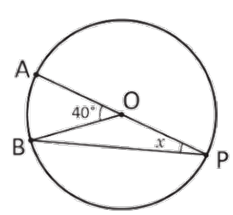
$x = 20°$

b)



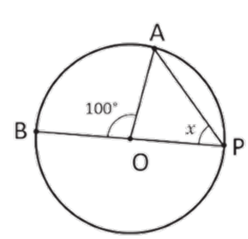
$x = 140°$

c)



$x = 20°$

d)



$x = 50°$

Achievement Indicator

1.3 Determine the measures of inscribed angles whose sides coincide with a diameter of the circumference.

Sequence

The previous class established the property that refers to the measure of an intuitively inscribed angle. During this lesson, it will be formally done using a similar situation as case 1 of the Solution of the class will be viewed.

Purpose

Ⓟ Apply the concept of the radius of a circle, the characteristics of an isosceles triangle, and the property of the measure of an external angle of a triangle to solve the initial problem. The first step in the solution strategy is to determine that the ΔAOP is isosceles, since its sides coincide with two radii of the circumference. Then it is applied that the measure of the external angle BOA is the sum of the two internal angles not adjacent to it, which in this case are equal since ΔAOP is isosceles.

Ⓢ Directly apply the property of the inscribed angle to determine the value of an unknown, at angles that are in a different position than the initial problem.

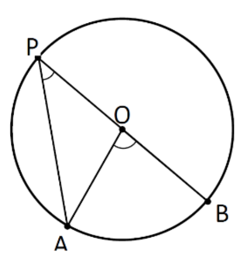
Solution of some items:

a) As $\measuredangle BOA = 2\measuredangle BPA$.
therefore, $x = 2(10°) = 20°$.

c) As $\measuredangle BOA = 2\measuredangle BPA$.
then $\measuredangle BPA = \frac{1}{2}\measuredangle BOA$.
therefore, $x = \frac{40°}{2} = 20°$.

Date: **U7 1.3**

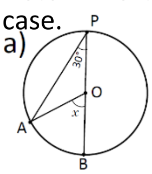
Ⓟ Show that $\measuredangle BOA = 2\measuredangle BPA$.



Ⓢ In the ΔAOP: OP = OA (are radii of the circumference) **(1)**
$\measuredangle OPA = \measuredangle PAO$ (equal sides are opposed to equal angles)
$\measuredangle BOA = \measuredangle OPA + \measuredangle PAO$ ($\measuredangle BOA$ is the outer angle of ΔAOP) **(2)**
$\measuredangle BOA = 2\measuredangle OPA$ (per (1) and (2))
Then, $\measuredangle BOA = 2\measuredangle BPA$.

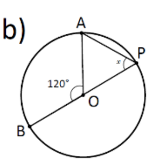
Ⓔ Determine the value of x in each case.

a)



$\measuredangle BOA = 2\measuredangle BPA$
Therefore
$x = 2(30°) = 60°$.

b)



$\measuredangle BOA = 2\measuredangle BPA$
Then,
$\measuredangle BPA = \frac{1}{2}\measuredangle BOA$
Therefore, $x = \frac{120}{2} = 60°$.

Ⓡ a) $x = 20°$ b) $x = 140°$
c) $x = 20°$ d) $x = 50°$

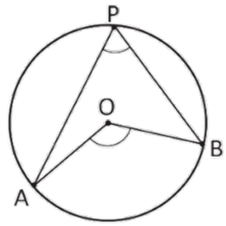
Homework: Workbook, page 150.

Lesson 1

1.4 Inscribed angles, part 2

P

Show that ∢BOA = 2∢BPA when the center is within ∢BPA.



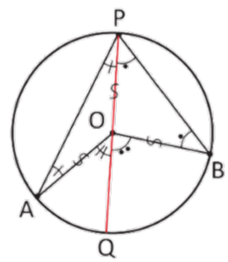
S

Draw the diameter for QP.

∢QOA = 2∢QPA and ∢BOQ = 2∢BPO (as seen in class 3).

Adding both equalities
∢QOA + ∢BOQ = 2∢QPA + 2∢BPQ = 2(∢QPA + ∢BPQ).

Therefore, ∢BOA = 2∢BPA.



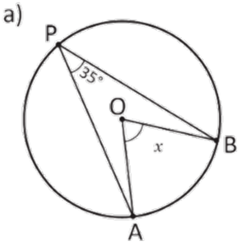
C

In the inscribe angles within the central angle, which subtends the same arc, comply that **the central angle measure is twice the measurement of the inscribed angle.**

E

Determine the value of x for each case.

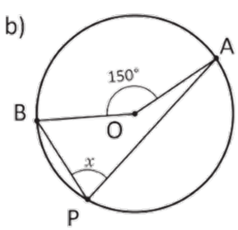
a)



As ∢BOA = 2∢BPA.

Therefore, x = 2(35°) = 70°.

b)



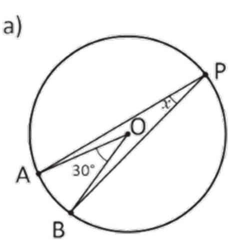
As ∢BOA = 2∢BPA.

Then, ∢BPA = $\frac{1}{2}$∢BOA.

Therefore, $x = \frac{150^\circ}{2} = 75^\circ$.

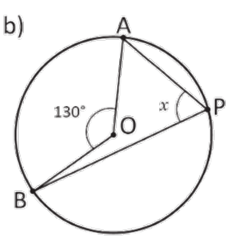
Determine the value of x for each case.

a)



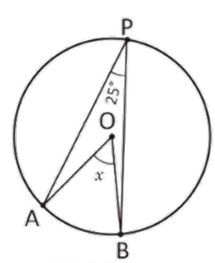
$x = 15^\circ$

b)



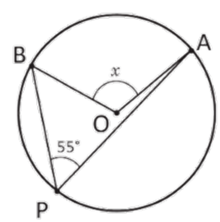
$x = 65^\circ$

c)



$x = 50^\circ$

d)



$x = 110^\circ$

Unit 7

Achievement Indicator

1.4 Determine the measures of inscribed angles whose central angle is inside the inscribed angle.

Sequence

For this class, a situation similar to case 2 of the Solution section of class 1.2 to prove the property. As a strategy for its realization, the demonstration made in the previous class is used.

Purpose

Ⓟ The first step in the solution strategy is to make the auxiliary construction of the diameter QP to reach a situation similar to that of the initial Problem of the previous class and to be able to use the result obtained as a tool to carry out the demonstration.

Ⓢ Directly apply the property of the inscribed angle to determine the value of an unknown at angles that are in a different position than the initial problem.

Solution of some items:

a) As $\sphericalangle BOA = 2\sphericalangle BPA$.

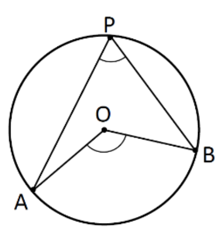
Then, $\sphericalangle BPA = \frac{1}{2}\sphericalangle BOA$.

Therefore, $x = \frac{30°}{2} = 15°$.

c) As $\sphericalangle BOA = 2\sphericalangle BPA$.
Therefore, $x = 2(25°) = 50°$.

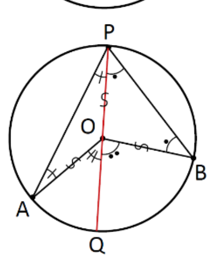
Date: **U7 1.4**

Ⓟ Show that:
$\sphericalangle BOA = 2\sphericalangle BPA$
When the center is inside the $\sphericalangle BPA$.



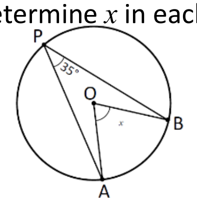
Ⓢ

Draw the diameter for QP.
$\sphericalangle QOA = 2\sphericalangle QPA$ y $\sphericalangle BOQ = 2\sphericalangle BPO$
(as seen in class 3)
Adding both equalities
$\sphericalangle QOA + \sphericalangle BOQ = 2\sphericalangle QPA + 2\sphericalangle BPQ$
$= 2(\sphericalangle QPA + \sphericalangle BPQ)$
Therefore, $\sphericalangle BOA = 2\sphericalangle BPA$.



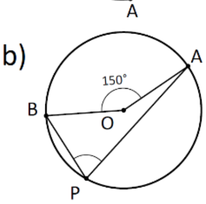
Ⓔ Determine x in each case.

a)



As $\sphericalangle BOA = 2\sphericalangle BPA$.
Therefore
$x = 2(35°) = 70°$.

b)



As $\sphericalangle BOA = 2\sphericalangle BPA$
$\sphericalangle BPA = \frac{1}{2}\sphericalangle BOA$.
Therefore
$x = \frac{150°}{2} = 75°$.

Ⓡ a) $x = 15°$ b) $x = 65°$
c) $x = 50°$ d) $x = 110°$

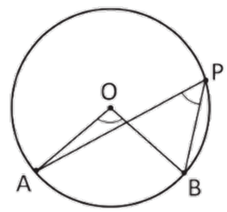
Homework: Workbook, page 151.

Lesson 1

1.5 Inscribed angle theorem

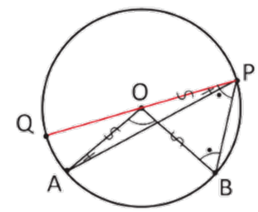
P

Demonstrate that $\measuredangle BOA = 2\measuredangle BPA$ when the center is outside of $\measuredangle BPA$.



S

Draw the diameter for QP.



$\measuredangle AOQ = 2\measuredangle APQ$ and $\measuredangle BOQ = 2\measuredangle BPQ$ (as seen in class 3).

As $\measuredangle BOA = \measuredangle BOQ - \measuredangle AOQ$.

Then, $\measuredangle BOA = 2\measuredangle BPQ - 2\measuredangle APQ = 2(\measuredangle BPQ - \measuredangle APQ) = 2\measuredangle BPA$.

Therefore, $\measuredangle BOA = 2\measuredangle BPA$.

C

In a circumference, for any inscribed angle, it is true to state that **the central angle measure is twice the measure of the inscribed angle that subtends the same arc.**

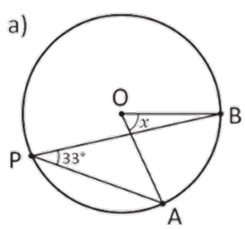
Also the inscribed angles that subtend the same arc have the equal measure.

The result is known as the **Inscribed angle theorem.**

E

Determine the value of x for each case.

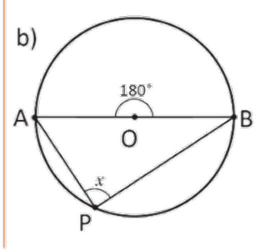
a)



As $\measuredangle BOA = 2\measuredangle BPA$.

Therefore, $x = 2(33°) = 66°$.

b)



As $\measuredangle BOA = 2\measuredangle BPA$.

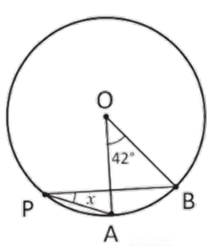
Then, $\measuredangle BPA = \frac{1}{2}\measuredangle BOA$.

Therefore, $x = \frac{180°}{2} = 90°$.

The inscribed angle to the semi-circumference measures 90°.

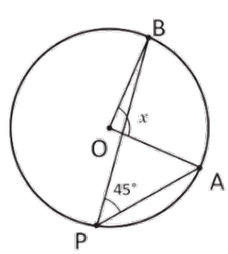
Determine the value of x, y and z for each case.

a)



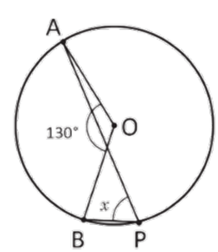
$x = 21°$

b)



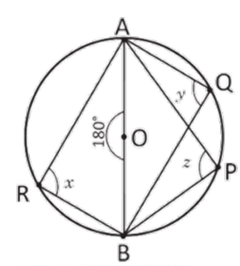
$x = 90°$

c)



$x = 65°$

d)



$x = 90°$ $y = 90°$
$z = 90°$

Achievement Indicator

1.5 It uses the inscribed angle theorem to determine the measurement of angles in the circumference.

Sequence

We take a situation similar to case 3 seen in the Solution section of class 1.2 to carry out the proof of the property. As a strategy for its realization, the demonstration made in class 1.3 is used.

Purpose

Ⓟ The first step in the solution strategy is to carry out the auxiliary construction of the diameter QP to reach a situation similar to that of case 1, as is the initial problem of class 1.3, and to be able to use the result obtained as another tool, for demonstration.

Ⓢ In addition to addressing the Conclusion, it is important to note in the Additional Information Box that the name given to the relationship between the measures of the inscribed and central angle measures is **Inscribed Angle Theorem**. Ⓒ Directly apply the property of the inscribed angle to determine the value of an unknown in angles that are in a different position than the initial problem.

Solution of some items:

a) As $\sphericalangle BOA = 2\sphericalangle BPA$.

Then, $\sphericalangle BPA = \frac{1}{2}\sphericalangle BOA$.

Therefore, $x = \frac{42°}{2} = 21°$.

b) As $\sphericalangle BOA = 2\sphericalangle BPA$.
Therefore, $x = 2(45°) = 90°$.

d) As $\sphericalangle BOA = 2\sphericalangle BPA$.

Then, $\sphericalangle BPA = \frac{1}{2}\sphericalangle BOA$.

Therefore, $z = \frac{180°}{2} = 90°$.

As $\sphericalangle BOA = 2\sphericalangle BQA$.

Then, $\sphericalangle BQA = \frac{1}{2}\sphericalangle BOA$.

Therefore, $y = \frac{180°}{2} = 90°$.

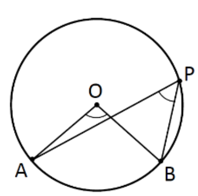
As $\sphericalangle BOA = 2\sphericalangle BRA$.

Then, $\sphericalangle BRA = \frac{1}{2}\sphericalangle BOA$.

Therefore, $x = \frac{180°}{2} = 90°$.

Date: **U7 1.5**

Ⓟ Show that
$\sphericalangle BOA = 2\sphericalangle BPA$.
When the center is outside the $\sphericalangle BPA$.



Ⓢ Draw the diameter for QP.
$\sphericalangle AOQ = 2\sphericalangle APQ$ y $\sphericalangle BOQ = 2\sphericalangle BPQ$
(as seen in class 3)

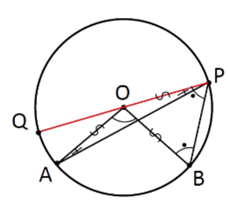
As $\sphericalangle BOA = \sphericalangle BOQ - \sphericalangle AOQ$

$\sphericalangle BOA = 2\sphericalangle BPQ - 2\sphericalangle APQ$

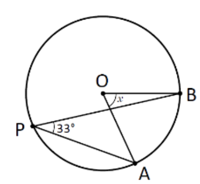
$= 2(\sphericalangle BPQ - \sphericalangle APQ)$

$= 2\sphericalangle BPA$.

Therefore, $\sphericalangle BOA = 2\sphericalangle BPA$.



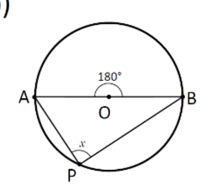
Ⓔ a)



As
$\sphericalangle BOA = 2\sphericalangle BPA$

Therefore,
$x = 2(33°) = 66°$.

b)



As
$\sphericalangle BOA = 2\sphericalangle BPA$
$\sphericalangle BPA = \frac{1}{2}\sphericalangle BOA$.
Therefore,
$x = \frac{180}{2} = 90°$.

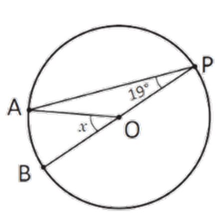
Ⓡ a) $x = 21°$ b) $x = 90°$
c) $x = 65°$ d) $x = 90°$
$y = 90°$
$z = 90°$

Homework: Workbook, page 152.

1.6 Practice what you learned

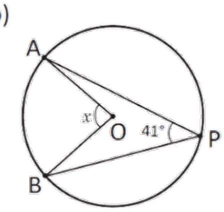
1. Determine the value of x for each case

a)



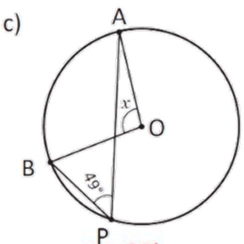
$x = 38°$

b)



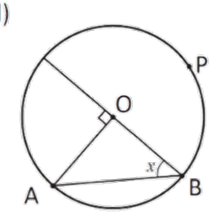
$x = 82°$

c)



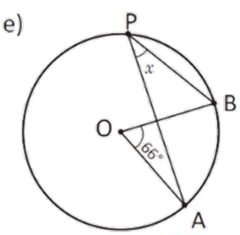
$x = 98°$

d)



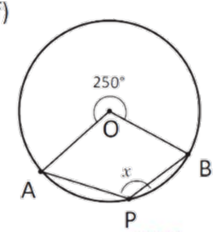
$x = 45°$

e)



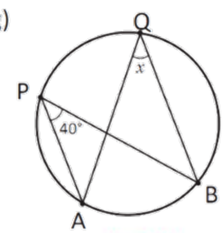
$x = 33°$

f)



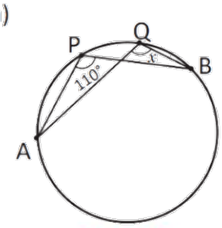
$x = 125°$

g)



$x = 40°$

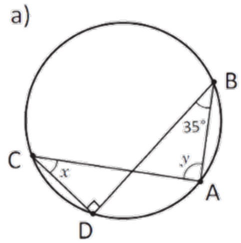
h)



$x = 110°$

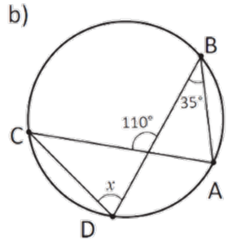
2. Determine the value of x and y according to each case.

a)



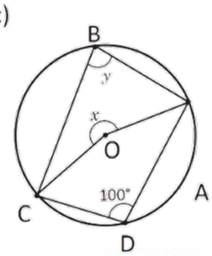
$x = 35°$
$y = 90°$

b)



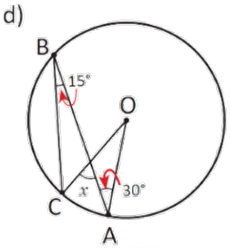
$x = 75°$

c)



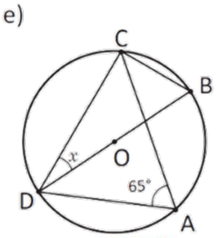
$x = 200°$
$y = 80°$

d)



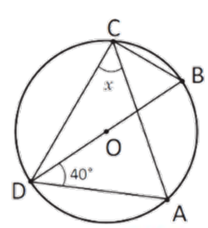
$x = 60°$

e)



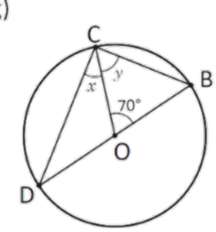
$x = 25°$

f)



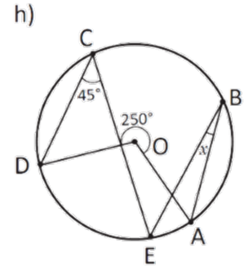
$x = 50°$

g)



$x = 35°$
$y = 55°$

h)



$x = 10°$

Unit 7

Achievement Indicator

1.6 Solve problems corresponding to the central and inscribed angle.

Solution of some items:

1.

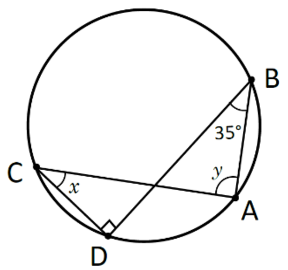
a) As ∢BOA = 2∢BPA
Therefore, x = 2(19°) = 38°.

e) As ∢BOA = 2∢BPA
Then, ∢BPA = $\frac{1}{2}$ ∢BOA.
Therefore, $x = \frac{66°}{2} = 33°$

h) x =∢BQA = ∢BPA = 110°, because both inscribed angles subtend $\overset{\frown}{AB}$.

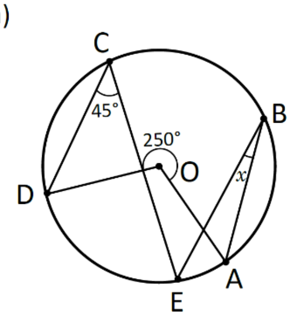
2.

a)



As ∢CED = 90° - 35° = 55°,
Then ∢BEA = ∢CED = 55°.
Therefore, y = 180° - 35° - 55° = 90°.

h)



First draw $\overline{OE}$.
∢AOD = 360° – 250° = 110°
∢EOD = 2(45°) = 90°
∢AOD = ∢AOE + ∢EOD
110° = ∢AOE + 90°
∢AOE = 20°

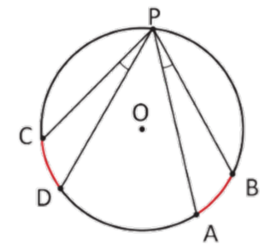
Therefore,
x = ∢ABE = $\frac{1}{2}$ ∢AOE = $\frac{20°}{2}$ = 10°.
x = 10°

Homework: Workbook, page 153.

1.7 Congruent arcs

P

Compare the measurement of ∢BPA with ∢DPC in the following figure, if $\overset{\frown}{CD} = \overset{\frown}{AB}$.



The notation $\overset{\frown}{AB}$, means the portion of the arc between point A and point B.

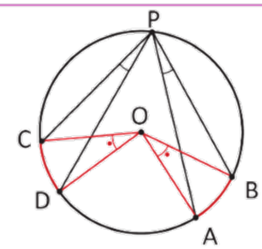
S

Construct the angles ∢BOA and ∢DOC.

∢BOA = ∢DOC ($\overset{\frown}{CD} = \overset{\frown}{AB}$ per hypothesis).

∢BPA = $\frac{1}{2}$ ∢BOA and ∢DPC = $\frac{1}{2}$ ∢DOC (as per inscribed angle).

Therefore, ∢BPA = ∢DPC.



C

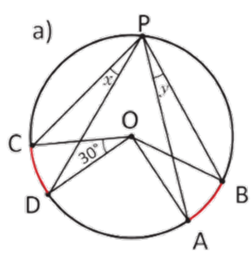
In a circumference, the inscribed angles, which subtend arcs of equal measure, have equal measure.

It is also true that if two inscribed angles are of equal measure, then the arcs that they subtend are also of equal measure.

E

Determine the value of x and y for each case where $\overset{\frown}{CD} = \overset{\frown}{AB}$.

a)

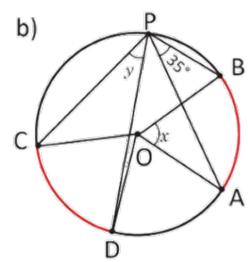


As ∢BOA = ∢DOC.

Therefore,

$x = y = \frac{30°}{2} = 15°$.

b)



As ∢BOA = 2∢BPA.

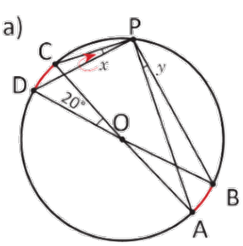
Therefore, $x = 2(35°) = 70°$.

So, ∢BOA = ∢DOC.

Then, y = ∢DPC = ∢BPA = 35°.

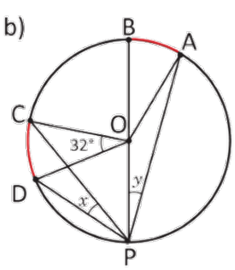
Determine the value of x and y for each case. Consider $\overset{\frown}{AB} = \overset{\frown}{CD}$.

a)



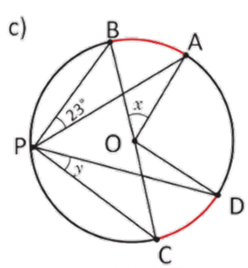
$x = y = 10°$

b)



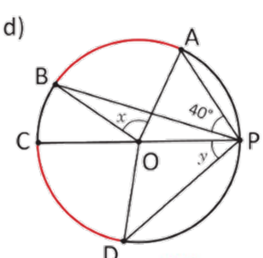
$x = y = 16°$

c)



$x = 46°$
$y = 23°$

d)



$x = 80°$
$y = 40°$

Achievement Indicator

1.7 Determine the measure of inscribed angles that subtend arcs of equal measure.

Sequence

For this class, It is established that the property of inscribed angles that subtend arcs of equal measure have an equal measure, and reciprocally if two inscribed angles are of equal measure, then the arcs they subtend are also of equal measure. To demonstrate this, the auxiliary construction of the respective central angles is made. This strategy is used because, in seventh grade, the arc length of circular segments was worked whose angle was considered the central angle in a circle, so you already know that if two arcs are equal, then the central angles must be equal. They subtend.

Purpose

Ⓟ As a first step to making the comparison, the central angles ∢BOA and ∢DOC, are drawn, then determined that these central angles are of equal measure because $\overset{\frown}{CD} = \overset{\frown}{AB}$ ((the arc length of a circular sector, was worked on in the seventh grade).

Ⓢ Directly apply the property of the inscribed angle to determine the value of an incognito at angles in a different position than the initial problem.

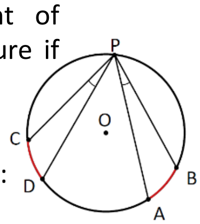
Solution of some items:

a) As ∢BOA = ∢COD.
Therefore, $y = x = \frac{20°}{2} = 10°$.

c) As ∢BOA = 2∢BPA.
Therefore, $x = 2(23°) = 46°$.
Therefore, ∢BOA = ∢DOC
Then, y = ∢DPC = ∢BPA = 23°.

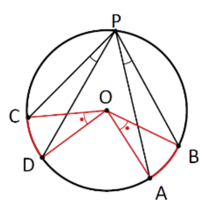
Date: **U7 1.7**

Ⓟ Compare the measurement of ∢BPA with ∢DPC in the figure if $\overset{\frown}{CD} = \overset{\frown}{AB}$



Ⓢ The angles were constructed: ∢BOA and ∢DOC

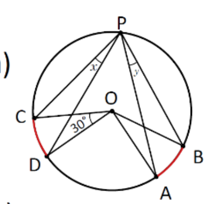
∢BOA = ∢DOC
($\overset{\frown}{CD} = \overset{\frown}{AB}$ per Hypothesis)



∢BPA = $\frac{1}{2}$ ∢BOA and
∢DPC = $\frac{1}{2}$ ∢DOC
(per inscribed angle)

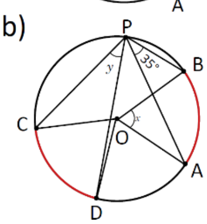
Therefore, ∢BPA = ∢DPC.

Ⓔ a)



As
∢BOA = ∢DOC.
Therefore,
$y = x = \frac{30}{2} = 15°$.

b)



As
∢BOA = 2∢BPA.
Therefore,
$x = 2(35°) = 70°$.
Moreover
∢BOA = ∢DOC.
Then,
y = ∢DPC = ∢BPA = 35°.

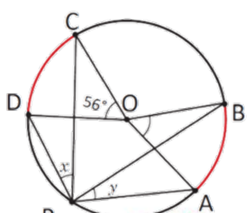
Ⓡ a) $x = y = 10°$ b) $x = y = 16°$
c) $x = 46°$, $y = 23°$ d) $x = 80°$, $y = 40°$

Homework: Workbook, page 154.

1.8 Practice what you learned

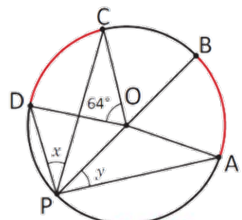
1. Determine the value of x and y for each case. Consider $\widehat{AB} = \widehat{CD}$.

a) $\widehat{AB} = \widehat{CD}$



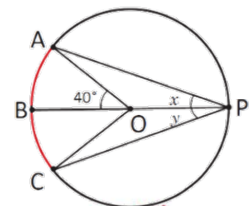
$x = 28°$
$y = 28°$

b) $\widehat{AB} = \widehat{CD}$



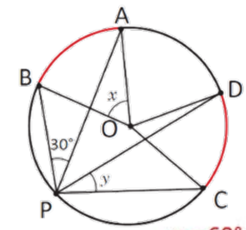
$x = 32°$
$y = 32°$

c) $\widehat{AB} = \widehat{BC}$



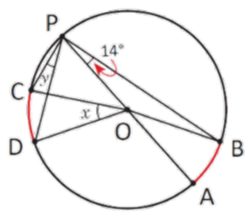
$x = 20°$
$y = 20°$

d) $\widehat{AB} = \widehat{CD}$



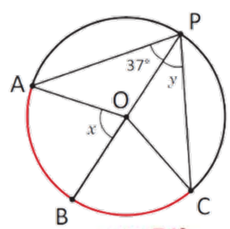
$x = 60°$
$y = 30°$

e) $\widehat{AB} = \widehat{CD}$



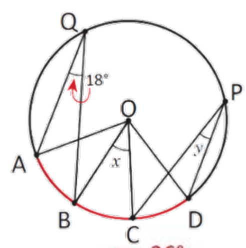
$x = 28°$
$y = 14°$

f) $\widehat{AB} = \widehat{BC}$



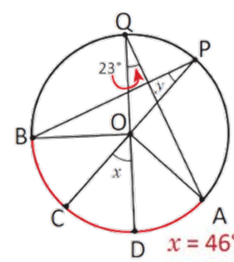
$x = 74°$
$y = 37°$

g) $\widehat{AB} = \widehat{BC} = \widehat{CD}$



$x = 36°$
$y = 18°$

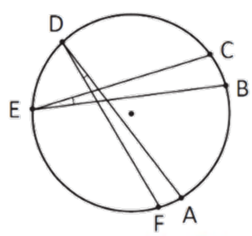
h) $\widehat{BC} = \widehat{CD} = \widehat{DA}$



$x = 46°$
$y = 23°$

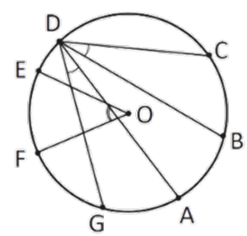
2. On the following circumferences, determine the arcs that are of equal measure.

a) ∢ADF = ∢CEB



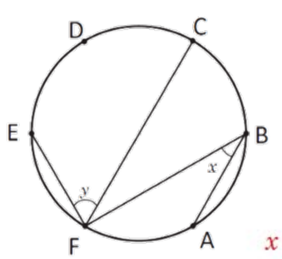
$\widehat{AF} = \widehat{CB}$

b) ∢FOE = 2∢CDB y ∢BDC = ∢ADG



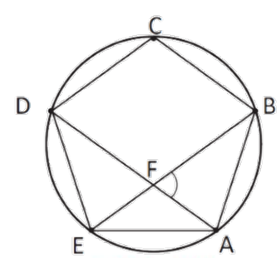
$\widehat{CB} = \widehat{EF} = \widehat{AG}$

3. Use the figure and determine the value of x and y if the points A, B, C, D, E, and F divide the circumference into six equal arcs.



$x = 30°$
$y = 60°$

4. In the figure A, B, C, D and E is a regular pentagon, draw the diagonals AD and BE. Determine the measure of ∢BFA.



∢BFA = 72°

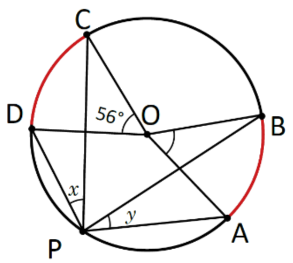
Achievement Indicator

1.8 Solves problems corresponding to the central and inscribed angle.

Solution of some items:

1.

a)



$\measuredangle BOA = \measuredangle DOC$ because $\overset{\frown}{BA} = \overset{\frown}{CD}$,

$\measuredangle BOA = \measuredangle DOC = 56°$ they are opposite angles by the vertex.

Therefore,

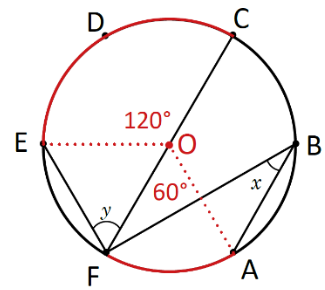
$x = y = \frac{56}{2} = 28°$.

3.

Since there are six equal arcs, the 360° of the circumference must also be divided into six equal angles.

360 ÷ 6 = 60°.

That is, for each arc corresponds to a central angle of 60°.



As $\measuredangle COE = 2\measuredangle CFE$.

Then, $\measuredangle CFE = \frac{1}{2}\measuredangle COE$.

Therefore, $y = \frac{120°}{2} = 60°$.

As $\measuredangle AOF = 2\measuredangle ABF$.

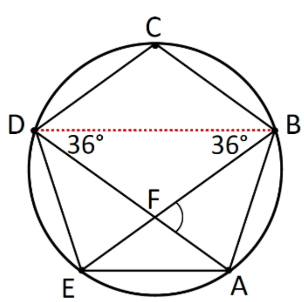
Then, $\measuredangle ABF = \frac{1}{2}\measuredangle AOF$.

Therefore, $x = \frac{60°}{2} = 30°$.

4.

Since you have a regular pentagon, each arc delimited by its vertices has the same measure. Therefore, each arc corresponds to a central angle of $\frac{360°}{5} = 72°$. Then, $\measuredangle FBD = \measuredangle FDB = \frac{72°}{2} = 36°$.

In ΔBFD, $\measuredangle BFA = \measuredangle FBD + \measuredangle FDB = 72°$.

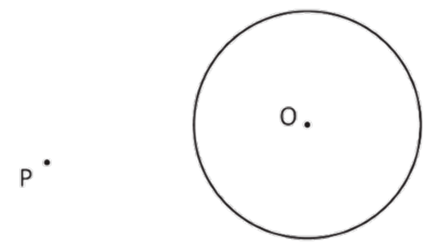


Homework: Workbook, page 155.

2.1 Construction of tangents to a circumference

P

Given the following circumference and the point marked as P, construct with a ruler and compass the lines that pass through point P and are tangent to the circumference.



S

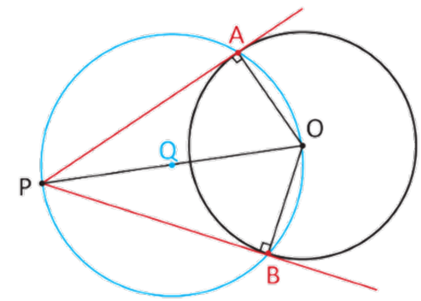
Taking the midpoint of the segment PO, denoted by Q.

Draw the circumference with the center Q and radius QO.

Draw dots for A and B where the circumference intersects.

Then, ∢OAP = ∢PBO = 90° (both subtend a 180° arc).

Therefore, the lines PA and PB are tangents to the circumference of center O.



The line perpendicular to the radius at a point in the circumference is the tangent to the circumference.

C

Using the inscribed angle results, one can construct the lines passing through a point P and tangent to a given circumference following the steps of the solution.

1. Draw a new circumference and P point outside the circumference, and construct the tangents to the circumference passing through the point P.

2. Based on the exercises in class, respond:

   a) Are PA and PB segments, the same?

   b) Why?

You can apply triangle congruence to justify your answer.

Achievement Indicator

2.1 Construct the tangents to a circle from a point outside the circle.

Sequence

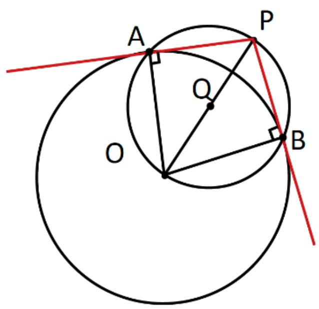
In seventh grade, the concept of a tangent line to a circle was introduced, so students already know these types of lines. For this class, two tangent lines are constructed to pass through a point external to the circumference. In addition, using the property of inscribed angles, it is concluded that a line perpendicular to the radius at a point on the circumference is the tangent line at that point.

Purpose

Ⓟ After carrying out the construction of the tangent lines, the information contained in the remember section, must be indicated. It is established that a perpendicular line to a radius about a point on the circumference is a tangent line.

Solution of some items:

1.



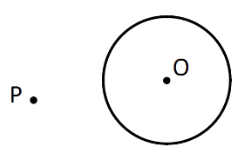
2.

a) Yes

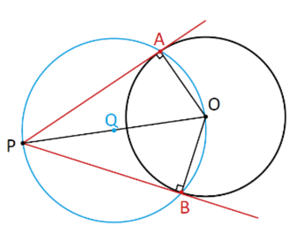
b) Because the ΔOAP and ΔOBP are right-angle triangles and their hypotenuses and one of their legs that correspond to the radii are of equal measure (right-angle triangle congruence criterion). Therefore, PA = PB.

Date: **U7 2.1**

Ⓟ Construct the lines that pass through P and are tangent to the circumference.



Ⓢ

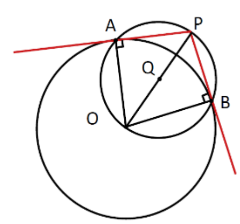


1. Taking the midpoint of $\overline{PO}$, denoted by Q, the circumference with center Q and radius $\overline{QO}$ is drawn.

3. Points A and B where the circles intersect are marked.

4. So, ∢OAP = ∢PBO = 90° (both subtend an arc of 180°).
   Therefore, the lines PA and PB are tangents to the circle with the center O.

Ⓡ 1.

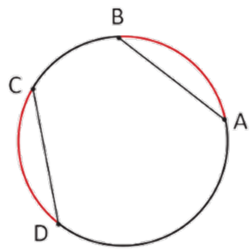


Homework: Workbook, page 156.

2.2 Chords and arcs of the circumference

P

In the following figure $\overset{\frown}{AB} = \overset{\frown}{CD}$. Compare the length of chords AB and CD.



S

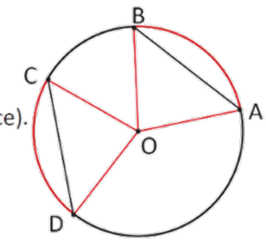
Draw the radiuses for OA, OB, OC and OD.

$\measuredangle BOA = \measuredangle DOC$ (because $\overset{\frown}{AB} = \overset{\frown}{CD}$).

OA = OB = OC = OD (are radiuses of the circumference).

Then, $\Delta BOA \cong \Delta DOC$ (as per SAS criterion).

Therefore, AB = CD (per congruence).



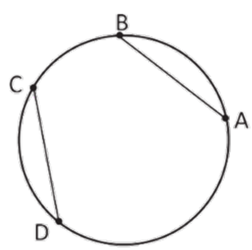
To apply the SAS congruence criterion, two sides and the angle between them must be congruent.

C

In a circumference if the measure of the two arcs is equal, then the measure of the chord that subtends those arcs is equal.

E

In the following figure AB = CD. Compare the length of $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ arcs.

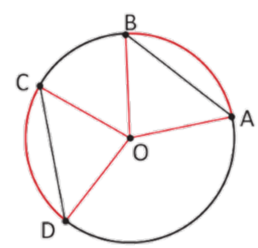


Draw the radiuses of OA, OB, OC and OD.

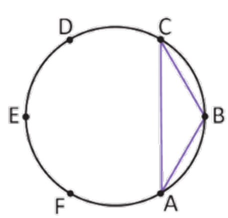
Then, $\Delta BOA \cong \Delta DOC$ (per LLL criterion).

Then , $\measuredangle BOA = \measuredangle DOC$ (per congruence).

Therefore, $\overset{\frown}{AB} = \overset{\frown}{CD}$ (the central angle is equal).



Points A, B, C, D, E, and F divide the circumference into six equal arcs. Clasify the figures formed by connecting the points indicated in each statement. Look at the example:



a) ABC   BA = BC (because $\overset{\frown}{BA} = \overset{\frown}{BC}$).
R. ABC is an isosceles triangle.

b) ABDE   c) ACE   d) ACD

e) ABCDEF   f) DEF   g) ABCD

Achievement Indicator

2.2 Use the chords and congruent arcs to classify figures with equal sides.

Sequence

First, the ΔBOA and ΔDOC are constructed, which are isosceles because each side has the same measure since they are radii of the circumference. Then by the SAS criterion, it is determined that the triangles are congruent (the red sides are of equal measure and the angle between them, since AB = CD).

Purpose

Ⓟ First, the ΔBOA and ΔDOC are constructed, which are isosceles because each side has the same measure since they are radii of the circumference. Then by the SAS criterion, it is determined that the triangles are congruent (the red sides are of equal measure and the angle between them, since AB = CD).

Determine that AB = CD, with similar construction to ΔBOA and ΔDOC, with the difference that the LLL criterion is applied to determine that the triangles are congruent since it is established as a hypothesis that AB = CD. Then from the established congruence, it is concluded that the arcs are equal since subtended by angles of equal measure.

Solution of some items:

b) ∢ABD = 90° (Because $\overline{AD}$ is a diameter).
In the same way: ∢BDE = ∢DEA = ∢EAB = 90°.
R. ABDE is a rectangle.

c) AC = CE = EA (because AC = CE = EA) ACE is an equilateral triangle.

d) ∢ACD = 90° (because $\overline{AD}$ is a diameter)
R. ACD is a right-angle triangle.

e) $\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD} = \overset{\frown}{DE} = \overset{\frown}{EF} = \overset{\frown}{FA}$
∢ABC = ∢BCD = ∢CDE = ∢DEF = ∢EFA
(because AB = BC = CD = DE = EF = FA)
R. ABCDEF is a regular hexagon

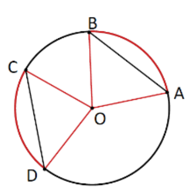
f) DE = EF (because DE = EF)
R. DEF is an isosceles triangle.

g) AB = CD (because $\overset{\frown}{AB} = \overset{\frown}{CD}$)
$\overline{BC} \parallel \overline{AD}$ (because ∢ACB = ∢DBC as $\overset{\frown}{AB} = \overset{\frown}{CD}$)
R. ABCD it is an isosceles trapezoid.

Date: **U7 2.2**

Ⓟ In the figure $\overset{\frown}{AB} = \overset{\frown}{CD}$. Compare the length of AB and CD chords.

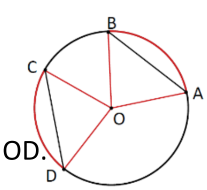
Ⓢ



Draw the radiuses OA, $\overset{\frown}{OB}$, $\overset{\frown}{OC}$ and OD.
∢BOA = ∢DOC (because AB = CD)
OA = OB = OC = OD (are radiuses of the circumference).

Then, ΔBOA ≅ ΔDOC (per criterion SAS).
Therefore, AB = CD (per congruence).

Ⓔ If AB = CD then:



By drawing OA, OB, OC and OD.
ΔBOA ≅ ΔDOC. (Per LLL)
∢BOA = ∢DOC (per congruence)
Therefore, $\overset{\frown}{AB} = \overset{\frown}{CD}$.
(The central angle is equal)

Ⓡ b) AB = DE and AE = BD
(because $\overset{\frown}{AB} = \overset{\frown}{DE}$ and $\overset{\frown}{AE} = \overset{\frown}{BD}$)
R. ABDE is a rectangle

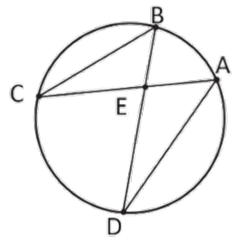
Homework: Workbook, page 157.

Lesson 2

2.3 Similar triangle application

P

Determine if the following figure satisfies $\Delta AED \sim \Delta BEC$.



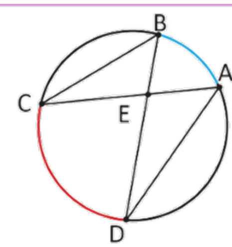
S

In the figure $\measuredangle AED = \measuredangle BEC$ (are opposite by the vertex).

$\measuredangle DBC = \measuredangle DAC$ (subtend the same arc).

But $\measuredangle EBC = \measuredangle DBC$ and $\measuredangle DAE = \measuredangle DAC$.

Therefore, $\Delta AED \sim \Delta BEC$ (per criterion AA).



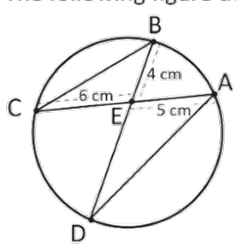
To apply the AA criterion it is only necessary two to be congruent.

C

It is necessary to observe the inscribed angles that subtend the same arc to determine the similarity between triangles.

E

The following figure determines the measure of the ED segment.



AS $\Delta AED \sim \Delta BEC$.

Then, $\dfrac{ED}{EC} = \dfrac{AE}{BE}$.

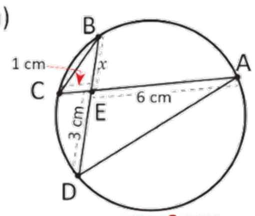
Therefore, $ED = EC \times \dfrac{AE}{BE} = 6 \times \dfrac{5}{4} = 7.5$.

ED = 7.5 cm

When two triangles are alike, the ratio between their homologous sides remains constant.

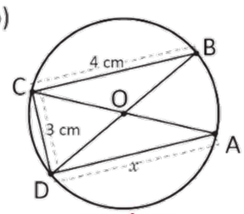
1. Determine x in the following figures:

a)



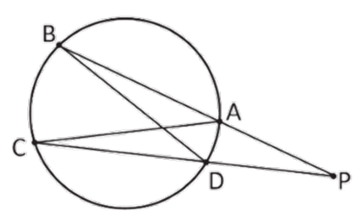
$x = 2$ cm

b)



$x = 4$ cm

2. In the following figures determine what conditions are required for $\Delta ACP \sim \Delta DPB$.



Is something else necessary?

Achievement Indicator

2.3 Solve problems with similar triangles using the inscribed angle theorem.

Sequence

Previously, the opposite angles theorem has been worked, and it was determined if two triangles are similar. Similarly, in class 1.7 of this unit, the students learned that two inscribed angles have the same measure if they subtend arcs of equal measure. So in this class, these facts are used to show, and to determine the similarity between triangles like those in the Initial Problem; it is necessary to observe the inscribed angles that subtend the same arc.

Purpose

Ⓟ After making the similarity of the triangles, have the students read the information contained in the box on the clue.

Ⓢ After making the similarity of the triangles, have the students read the information contained in the box on the clue.

Solution of some items:

1.

a) As $\Delta AED \sim \Delta BEC$ (per AA similarity criterion).

Then, $\frac{ED}{EC} = \frac{AE}{BE}$.

Therefore,

$BE = x = AE \times \frac{EC}{ED} = 6 \times \frac{1}{3} = 2$

$x = 2$ cm

b) In the triangles ΔADC and ΔBCD, $\sphericalangle ADC = \sphericalangle BCD = 90°$, CA = DB and $\overline{CD}$ is common.

Therefore, $\Delta ADC \cong \Delta BCD$.

Then $x = BC = 4$

$x = 4$ cm

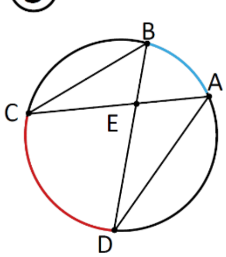
2. In ΔACP and ΔDBP, $\sphericalangle ACP = \sphericalangle DBP$ (since they are inscribed angles subtend by $\overset{\frown}{AD}$), $\sphericalangle P$ is common.

Hence, $\Delta ACP \sim \Delta DBP$ (Per AA similarity criterion).
No other conditions are necessary.

Date: **U7 2.3**

Ⓟ The figure determines whether $\Delta AED \sim \Delta BEC$.

Ⓢ

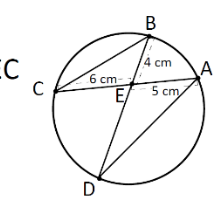


In the figure $\sphericalangle AED = \sphericalangle BEC$.
(They are opposite by the vertex)
$\sphericalangle DBC = \sphericalangle DAC$.
(They subtend the same arc)

But $\sphericalangle EBC = \sphericalangle DBC$ and $\sphericalangle DAE = \sphericalangle DAC$

Therefore, $\Delta AED \sim \Delta BEC$.
(Per criterion AA).

Ⓔ In the figure $\Delta AED \backsim \Delta BEC$



Then, $\frac{ED}{EC} = \frac{AE}{BE}$.

Therefore, $ED = EC \times \frac{AE}{BE}$
$= 6 \times \frac{5}{4}$
$= 7.5$

Ⓡ 1.
a) $x = 2$ cm
b) $x = 4$ cm

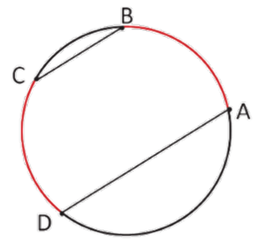
Homework: Workbook, page 158.

Lesson 2

2.4 Parallelism

P

In the figure $\overset{\frown}{AB} = \overset{\frown}{CD}$. Determine if the segments AD and BC are parallels or secants.

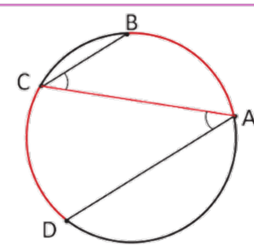


S

Draw the chord AC.

Then, $\measuredangle BCA = \measuredangle DAC$ (since $\overset{\frown}{AB} = \overset{\frown}{CD}$).

Therefore, BC ∥ AD (alternate interior angles are equal).

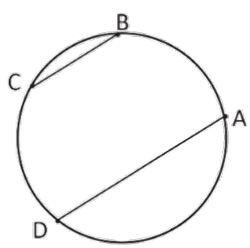


C

If having two arcs of equal measure on a circumference, then the chords determined by the beginning of one arc and the end of the other are parallel.

E

Compare the arcs $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ from the circumference, if BC ∥ AD.

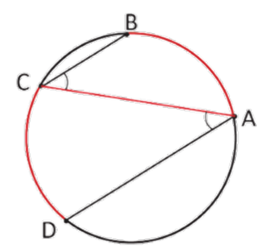


Draw the chord AC.

$\measuredangle BCA = \measuredangle DAC$ (alternate interior angles).

Therefore, $\overset{\frown}{AB} = \overset{\frown}{CD}$ (incribed angle theorem).

This result is reciprocal to the initial exercise.



Determine which of the following statements, are sufficient conditions to four consecutive points A, B, C, and D; on a circumference. Once connected, there is at least a pair of parallel chords.

a) $\overset{\frown}{AC} = \overset{\frown}{AD}$	b) $\measuredangle DBC = \measuredangle BDA$	c) CB = DA	d) $\overset{\frown}{CB} = \overset{\frown}{AD}$
e) AB = BC	f) $\measuredangle ACD = \measuredangle ADB$	g) AC = BD	h) $\Delta ABC \cong \Delta DCB$

Achievement Indicator

2.4 Use congruent arcs to determine parallelism between chords.

Sequence

Acknowledging that if there are two arcs of equal measure in a circumference, then the chords determined by the end of one arc and the beginning of the other are parallel.

In eighth grade, was reviewed the conditions of parallelism between two lines. In the problem, the construction of $\overline{AC}$ is made, and since ∢ACB = ∢CAD (by subtending equal arcs), it is determined that $\overline{BC}$ and $\overline{AD}$ are parallel (∢ACB and ∢CAD are internal alternates). Furthermore, in-class 1.7, it was determined that if 2 arcs have the same measure, then the inscribed angles that subtend them have the same measure.

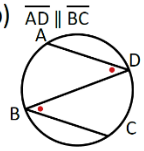
Purpose

Ⓟ In the Example, we work on the reciprocal of the property in the Conclusion, that is, from the fact that $\overline{BC} \parallel \overline{AD}$ determine that AB = CD.

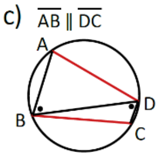
Solution of some items:

Sufficient conditions (b, c, d, g and h):

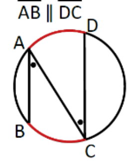
b) $\overline{AD} \parallel \overline{BC}$



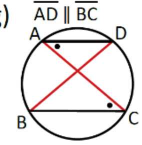
c) $\overline{AB} \parallel \overline{DC}$



d) $\overline{AB} \parallel \overline{DC}$

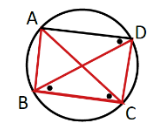


g) $\overline{AD} \parallel \overline{BC}$



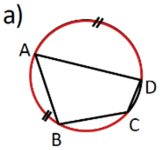
If AC = BD then, $\overset{\frown}{AC} = \overset{\frown}{BD}$.
Therefore, $\overset{\frown}{AB} = \overset{\frown}{CD}$.

h) $\overline{AD} \parallel \overline{BC}$



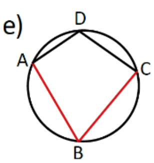
Non sufficient conditions (a, e and f):

a)



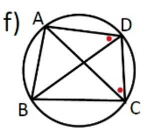
Point B could be moved along $\overset{\frown}{AC}$

e)



Point D could be moved along $\overset{\frown}{AC}$.

f)

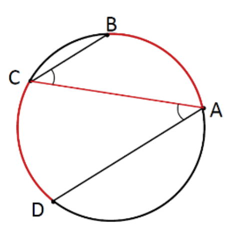


Point C could be moved along $\overset{\frown}{BD}$.

Date: **U7 2.4**

Ⓟ In the figure $\overset{\frown}{AB} = \overset{\frown}{CD}$. Determine whether the AD and BC segments are parallel or secant.

Ⓢ

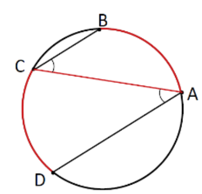


Draw the chord AC.

Then, ∢BCA = ∢DAC. (Since $\overset{\frown}{AB} = \overset{\frown}{CD}$)

Therefore, BC ∥ AD.
(Internal alternating angles are equal).

Ⓔ If BC ∥ AD:



Drawing the chord AC.

∢BCA = ∢DAC
(Alternate interior angles).

Therefore $\overset{\frown}{AB} = \overset{\frown}{CD}$
(Inscribed angle theorem).

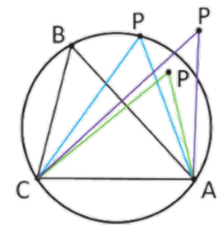
Ⓡ b), c), d), g) and h)

Homework: Workbook, page 159.

2.5 Four points on a circumference of a circle

P

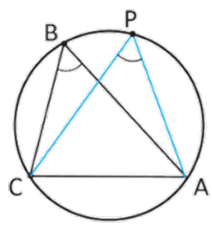
Considering ∢ABC = ∢APC and that both angles share the AC segment.
It shows that A, B, C and P are on the same circumference.



S

Point P has three options; on, in or out of the circumference.

Option 1

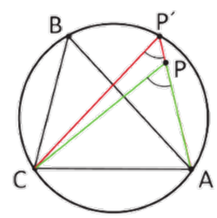


In this case:

∢ABC = ∢APC.

Therefore, A, B, C and P should stay in the same circumference.

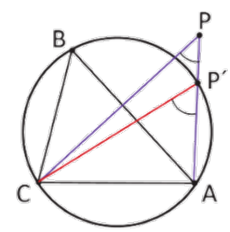
Option 2



Drawing ∢AP'C, then
∢ABC = ∢AP'C < ∢APC
Since ∢APC = ∢AP'C + ∢P'CP

Therefore, **∢ABC < ∢APC.**

Option 3



Drawing ∢AP'C, then
∢ABC = ∢AP'C > ∢APC.
Since ∢AP´C = ∢APC + ∢PCP'.

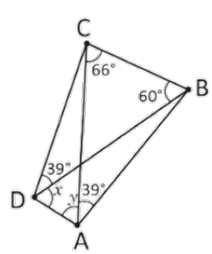
Therefore, **∢ABC > ∢APC.**

C

If two equal angles also share a segment at their openings, then the four points are on the same circumference.

E

Determine the value x and y.

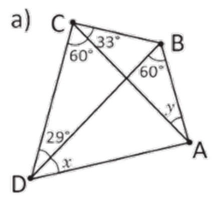


Since ∢CAB = ∢CDB and both share the CB, then A, B, C, D are on the same circumference.

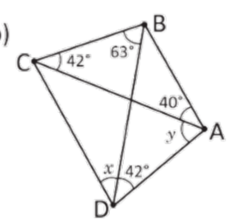
It must satisfied that **∢BCA = ∢BDA**, then x = 66°.
Moreover, it must meet that **∢CBD = ∢CAD**, then y = 60°.

Determine the value of x and y.

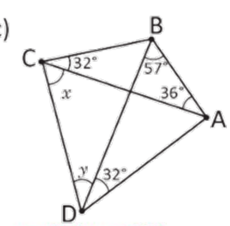
a)



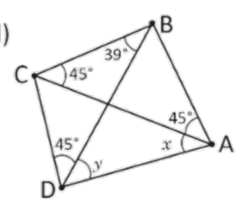
b)



c)



d)



x = 33° y y = 29°

x = 40° y y = 63°

x = 57° y y = 36°

x = 39° y y = 45°

Achievement Indicator

2.5 Determine the conditions for four points to be on a circle.

Sequence

For this class, it is determined that if two angles are equal and share a segment at their openings, then the four points are on the same circumference. The results obtained, from the position that a point P occupies on the circumference (inside, on and outside) are analyzed.

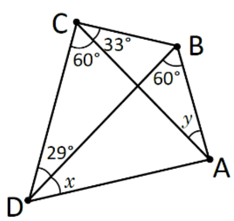
Purpose

Ⓟ The wording of the initial problem must be: Let A, B, and C is fixed points on the circumference and P another point that can be inside, on, or outside the circumference. If ∢ABC = ∢APC holds and both angles share segment AC, show that point P is on the same circumference.

Ⓢ The solution addresses the three possible cases that could occur; to determine the angles have different measures when the point is not on the circumference.

Solution of some items:

a)



Since ∢ACD = ∢ABD and both share the DA segment, then A, B, C, D are on the same circumference.

As ∢ADB and ∢ACB subtend the same arc then:

x = ∢ADB = ∢ACB = 33

Then, as ∢BAC and ∢BDC subtend the same arc then:

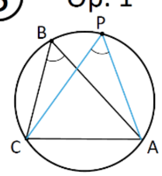
y = ∢BAC = ∢BDC = 29

x = 33° and y = 29°

Date: **U7 2.5**

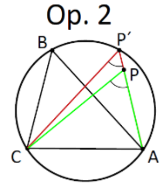
Ⓟ If A, B and C are fixed on the circumference and P can be in, on or out of it. If ∢ABC = ∢APC and share $\overline{AC}$. Show that P is on the circumference.

Ⓢ Op. 1



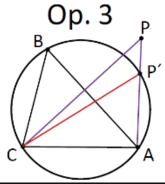
In this case:
∢ABC = ∢APC.

Op. 2



∢ABC = ∢AP'C.
∢APC = ∢AP'C + ∢P'CP > ∢AP'C.
Therefore, ∢ABC = ∢AP'C < ∢APC.

Op. 3



∢ABC = ∢AP'C.
∢AP'C = ∢APC + ∢PCP' > ∢APC.
Therefore, ∢ABC = ∢AP'C > ∢APC.

Ⓔ Determining x and y.
∢ABC = ∢APC and
share $\overline{CB}$
A, B, C and D are on the same circumference.
∢BCA = ∢BDA, x = 66°
∢CBD = ∢CAD, y = 60°

Ⓡ a) x = 33° and y = 29°
b) x = 40° and y = 63°
c) x = 57° and y = 36°
d) x = 39° and y = 45°

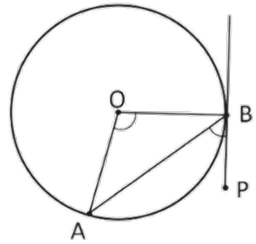
Homework: Workbook, page 160.

Lesson 2

2.6 Semi-Inscribed angle

P

Compare the measurement of ∢ABP with ∢BOA in the following figure:



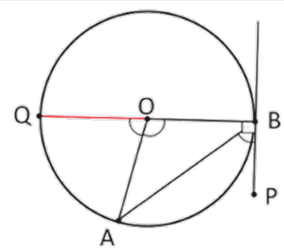
S

Draw the diameter for QB.

Then, ∢AOQ = 2∢ABO (Incribed Angle Theorem).

So, ∢AOQ = 180° - ∢BOA (Supplementary angle).

Then 2 ∢ABO = 180° - ∢BOA, therefore, ∢ABO = 90° - $\frac{\sphericalangle BOA}{2}$.



Therefore, ∢PBA = $\frac{\sphericalangle BOA}{2}$, or ∢BOA = 2∢PBA (per complementary angle, since PB ⊥ BO).

C

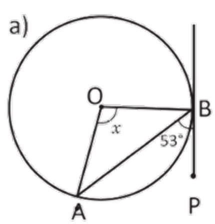
The angle formed by a tangent and a chord of the circumference is called: **semi-inscribed angle**.

In a circumference, **the measurement of a semi-inscribed angle is equal to half the measurement of the central angle, which subtends the same arc as the chord.**

E

Determine the value of x for each case:

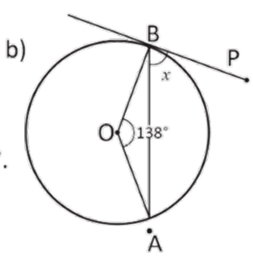
a)



As ∢BOA = 2∢PBA.

Therefore, x = 2(53°) = 106°.

b)

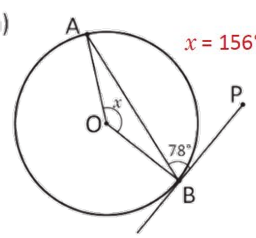


As ∢PBA = $\frac{1}{2}$ ∢BOA.

Therefore, $x = \frac{138°}{2} = 69°$.

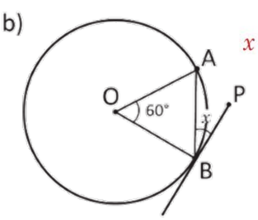
Determine the value of x for each case:

a)



x = 156°

b)



x = 30°

Unit 7

Achievement Indicator

2.6 Determine the measurements of semi-inscribed angles using center angle measure.

Sequence

The semi-inscribed angle term is introduced as well as the property referring to its measure. A similar situation, was presented in the initial Problem of class 1.3 (that is, that the central angle is on one side of the inscribed angle) is constructed as a first step in the strategy to make the formal deduction of the property.

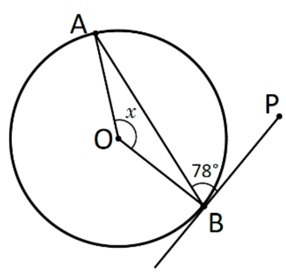
Purpose

Ⓟ Using the inscribed angle theorem and the supplementary angle condition, perform the angle comparison. Initially, the auxiliary construction of the diameter QB is performed to construct an inscribed angle similar to case 1 of the Solution of class 1.2.

Ⓢ Point out the importance of auxiliary constructions (in this case, the diameter) to perform some geometrical demonstrations.

Solution of some items:

a)

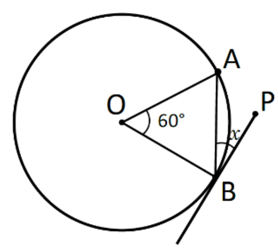


As ∢BOA = 2∢PBA.

Therefore, $x = 2(78°) = 156°$

$x = 156°$.

b)



As ∢PBA = $\frac{1}{2}$ ∢BOA.

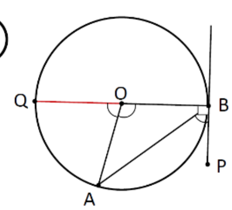
Therefore, $x = \frac{60°}{2} = 30°$

$x = 30°$.

Date: **U7 2.6**

Ⓟ Compare the measure of ∢ABP with ∢BOA in the figure.

Ⓢ



Draw the diameter of QB.
∢AOQ = 2∢ABO.
(Inscribed angle theorem)

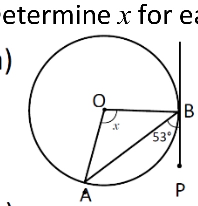
∢AOQ = 180° – ∢BOA.
(Supplementary angle)

2∢ABO = 180° – ∢BOA, that is, ∢ABO = 90° – $\frac{∢BOA}{2}$

Therefore, ∢PBA = 90° – ∢ABO = $\frac{∢BOA}{2}$, or ∢BOA = 2∢PBA. (Per complementary angle, since PB ⊥ BO).

Ⓔ Determine x for each case.

a)

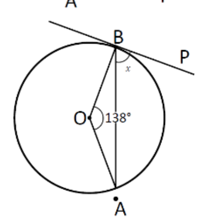


∢BOA = 2∢PBA

Therefore,

$x = 2(53°) = 106°$.

b)



∢PBA = $\frac{1}{2}$ ∢BOA

Therefore,

$x = \frac{138°}{2} = 69°$.

Ⓡ a) $x = 156°$

b) $x = 30°$

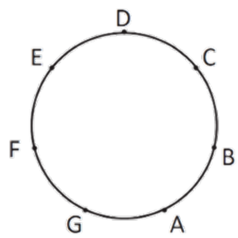
Homework: Workbook, page 161.

Lesson 2

2.7 Practice what you learned

1. Draw a circumference and a dot on the outside of it. Use a ruler and a compass to draw the tangents across P.

2. Dots A, B, C, D, E, F, G divide the circumference into seven equal arcs. Classify the figures formed by connecting the dots indicated in each statement.



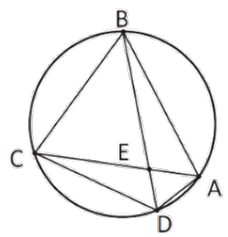
a) ABC
Isosceles triangle
AB = BC

b) ACDF
Isosceles triangle
CD ∥ AF and AC = FD

c) ADG
Isosceles triangle
AD = DG

d) ABCDEFG
Regular heptagon. The sides and angles are congruent respectively.

3. The following figures A, B, C, D are in the circumference. Respond:



a) What are the angles ∢EAB and ∢EDC?

b) What are the angles ∢ABE and ∢ACD? Why?

c) What are the angles ΔABE and ΔDCE? Why?

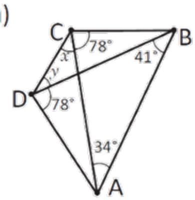
4. Determine which of the following statements are sufficient conditions to four consecutive points A, B, C, D on a circumference. Once connected, there is at least a pair of parallel chords.

a) $\overset{\frown}{AC} = \overset{\frown}{BD}$ b) ∢CAB = ∢CDB c) AC = AD d) ΔABC ~ ΔCDA

2.8 Practice what you learned

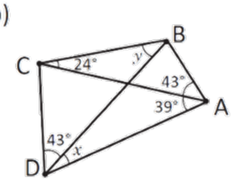
Determine the value of x or y, accordingly:

a)



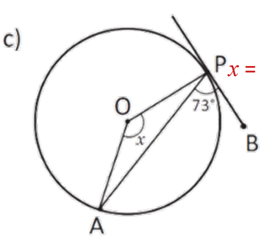
$x = 41°$
$y = 34°$

b)



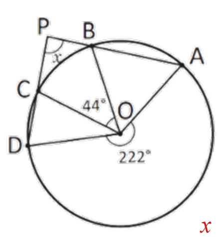
$x = 24°$
$y = 39°$

c)



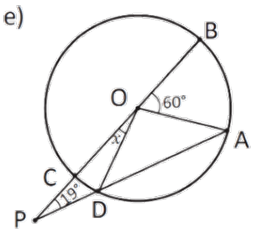
$x = 146°$

d)



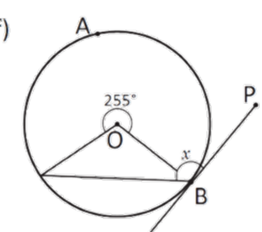
$x = 89°$

e)



$x = 22°$

f)



$x = 127.5°$

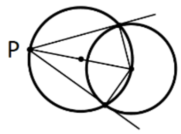
Achievement Indicator

2.7 and 2.8 Solve problems corresponding to the central and inscribed angle application.

Solution of some items:

Class 2.7

1. An example of a solution might be:



3.

a) ∢EAB = ∢EDC because they both subtend $\overset{\frown}{BC}$.

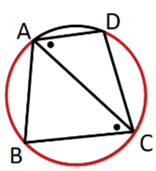
b) ∢ABE = ∢ACD because they both coincide with inscribed angles that subtend $\overset{\frown}{AD}$.

c) ΔABE is similar to ΔDCE because 2 of the angles are equal (AA).

4.

Sufficient conditions:

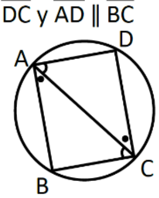
a) $\overline{AD} \parallel \overline{BC}$



$\overset{\frown}{AC} = \overset{\frown}{BD}$ then $\overset{\frown}{AB} = \overset{\frown}{CD}$

Therefore, ∢ACB = ∢CAD.

d) $\overline{AB} \parallel \overline{DC}$ y $\overline{AD} \parallel \overline{BC}$

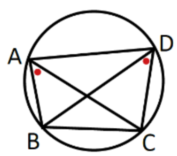


As ΔABC ~ ΔCDA then,

∢CAB = ∢ACD, ∢BCA = ∢DAC.

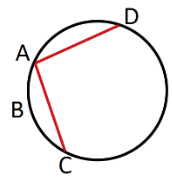
Not sufficient conditions:

b)



Point D could move along $\overset{\frown}{AC}$.

c)



Point B could move along $\overset{\frown}{AC}$.

Class 2.8

a) Since ∢ADB = ∢ACB and both share the segment AB, then A, B, C, D are on the same circumference.

As ∢ACD and ∢ABD subtend the same arc, then:

x = ∢ACD = ∢ABD = 41°

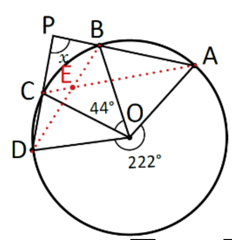
Then, since ∢BAC and ∢BDC subtend the same arc, then:

y = ∢BDC = ∢BAC = 34°

x = 41° and y = 34°

Homework: Workbook, page 162.

d)



First draw $\overline{BD}$ and $\overline{AC}$.

Since ∢AOD = 222° is central then the inscribed angles: ∢ABD = ∢ACD = 111° because both subtend $\overset{\frown}{AD}$. Therefore ∢BOC = 44° is central, so the inscribed angle ∢CAB = 22° because both subtend $\overset{\frown}{BC}$.

Also:

∢ACP = 180° – ∢ACD

= 180° – 111°

= 69°

Because the angles are on $\overline{DP}$.

Finally:

$22 + 69 + x = 180$

$x = 89°$