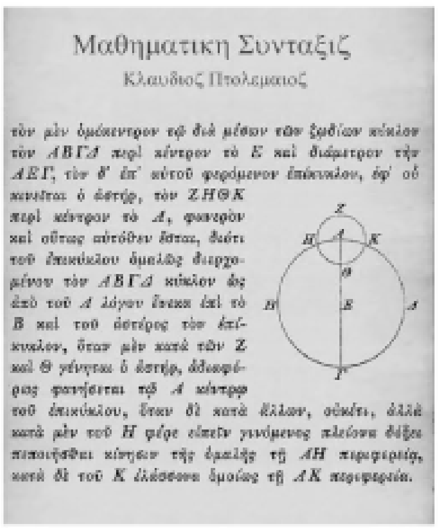
Unit 7

Inscribed and central angles

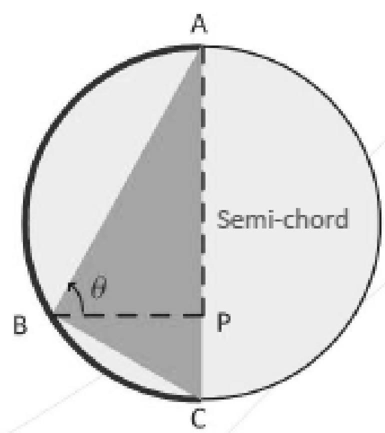

Μαθηματικη Συνταξις
Κλαυδιος Πτολεμαιος

A sheet from the astronomical treatise Almagest.

Ancient civilizations used astronomy to predict abundant hunting, planting, or the arrival of winter.

In the astronomical treatise *Almagest*, the Greco-Egyptian mathematician Claudius Ptolemy (second century) made a mathematical description of the geocentric system (the planets revolve around the Earth). One of his contributions to mathematics was a theorem on cyclic quadrilaterals, in which essential properties of inscribed angles are used.

The Trigonometry, which studies the relationship between the sides and angles of a triangle, was developed by astronomical studies. During the V and VI centuries, the Indian mathematicians Varahamihira and Brahmagupta formulated numerous trigonometric properties using the semi-chord (a triangle inscribed in the circle with one side as the circle's diameter). Furthermore, the cyclic quadrilaterals are based on the study of the inscribed angles



The angle inscribed ABC is straight This construction allowed the collection of important relationships.

The contents will be developed by addressing the definition of the theorem of the inscribed angle, which establishes a relationship with the central angle. Also, study the construction of tangent lines on the circumference, the definition of semi-inscribed angles, and the relationship between chords and arcs.

Elements of the circumference

The elements of the circumference are:

Radius: The segment that goes from the center to a point in the circumference.

Diameter: The segment that goes from one point of the circumference to another and passes through the center.

Chord: The segment that goes from one point of the circumference to another.

Arc: Any portion of the circumference of a circle.

Tangent: The line that touches the circumference at a point.

Please choose the appropriate letter for each element of the circumference, place it inside the parentheses with the corresponding definition. Some items can repeat.

Chord	An element whose size is half the size of the diameter.
Tangent	Segment drawn between two different points on the circumference.
Radius	Element perpendicular to a radius at a point, on the circumference.
Arc	The segment that goes from the center to a point on the circumference.
Diameter	An element of the circumference determined by the opening of a central angle.
	Length of an element twice, the size of the radius.
	Longest rope in a circumference.
	Part of the circumference delimited by two points.

Use the circumference provided and draw its elements according to the color indicated.

Chord: Red

Arc: Yellow

Tangent: Blue

Diameter: Sky-blue

Radius: Green

How long did it take to solve the problems?

1.1

1.

a) ( )

b) ( )

c) ( )

d) ( )

e) ( )

( )

( )

( )

2.

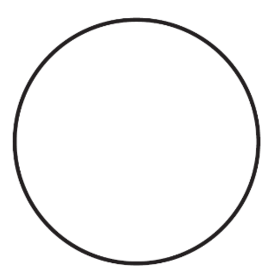
a)

d)

b)

e)

c)



1.2 Definition and measurement of inscribed angles

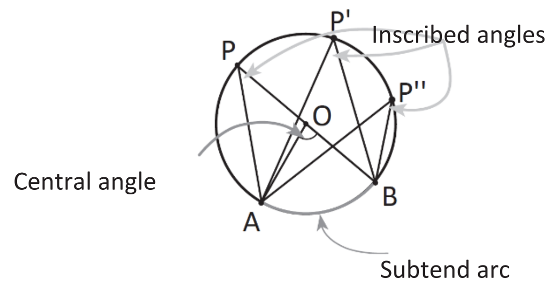
Connect the elements of the circumference to its definition.

1. Diameter	a) Segment drawn between two different points on the circumference
2. Tangent	b) The segment that goes from the center to a point on the circumference.
3. Radius	c) Part of the circumference delimited by two points.
4. Arc	d) Line touching the circumference at a single point.
5. Chord	e) Segment drawn between two points on the circumference and passes through the center.

Angles whose vertex is on the circumference are called **inscribed angles.**

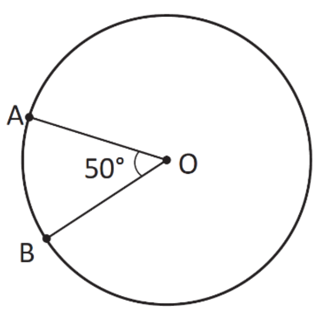
In a circle, the measure of the central angle that subtends the same arc as any inscribed angle is twice the measure of any inscribed angle that subtends the same arc.

Remember that subtend means to share the same arc.

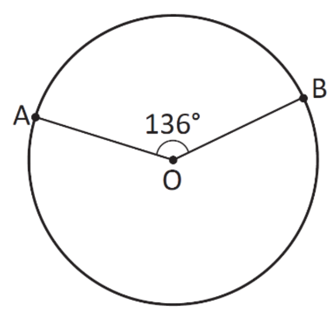


Draw three different inscribed angles on the following circles and determine their measurement.

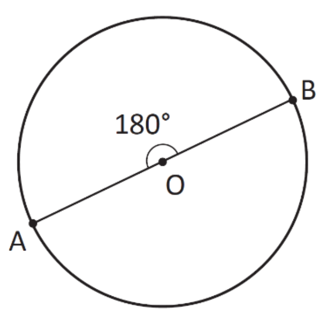
a)



b)



c)



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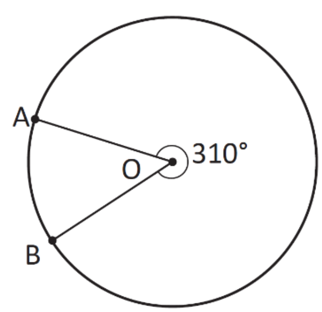


Unit 7

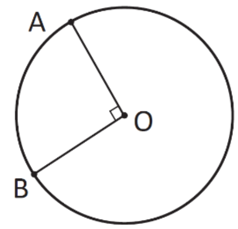
How long did it take to solve the problems?

1.3 Inscribed angles, part 1

R 1. Write the definition of the elements of the circumference.

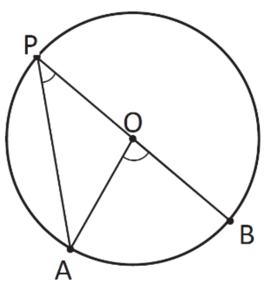
a) Diameter ____
____
b) Tangent ____
____
c) Radius ____
____
d) Arc ____
____
e) Chord ____
____

2. Draw three different angles inscribed on the circumference and determine their measurement.

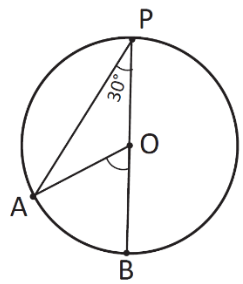


C In a circumference, for any inscribed angle, it is proper to state that **the central angle measure is twice the measure of the inscribed angle that subtends the same arc.**

For Example:



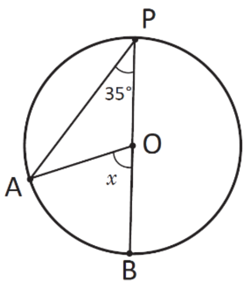
$\measuredangle BOA = 2\measuredangle BPA$

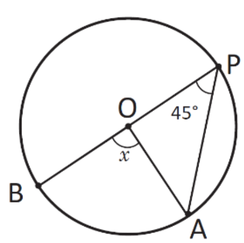


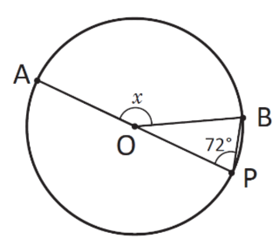
As $\measuredangle BOA = 2\measuredangle BPA$

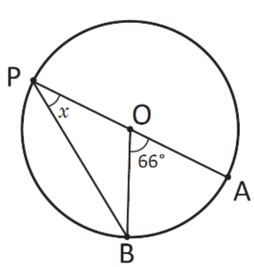
$\measuredangle BOA = 2(30) = 60°$

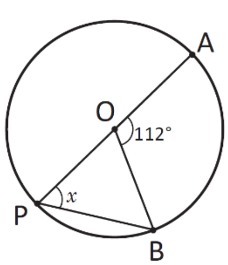
Determine the value of x for each case.

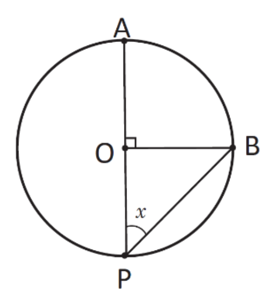
a)


b)


c)


d)


e)


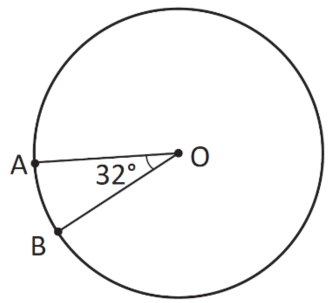
f)


How long did it take to solve the problems?

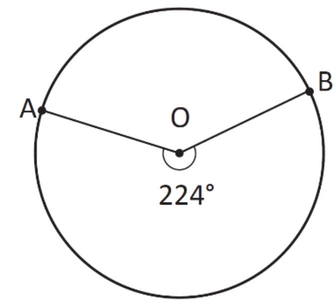
1.4 Inscribed angles, part 2

R 1. Draw three different inscribed angles on the following circumferences and determine their measurement:

a)

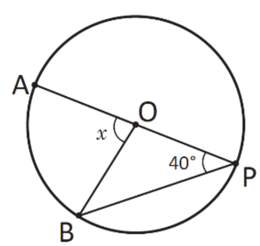


b)

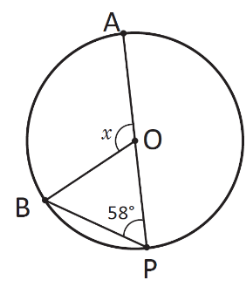


2. Determine the value of x for each case:

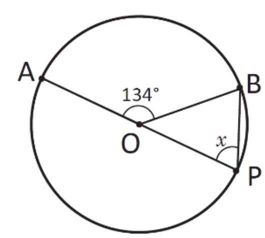
a)



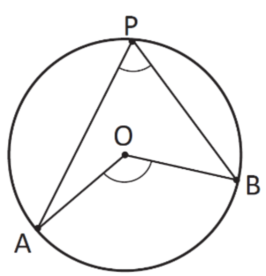
b)



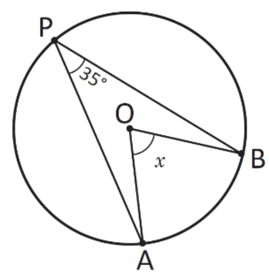
c)



C In the inscribe angles within the central angle, which subtends the same arc, comply that **the central angle measure is twice the measurement of the inscribed angle.** For example:



$\sphericalangle BOA = 2\sphericalangle BPA$

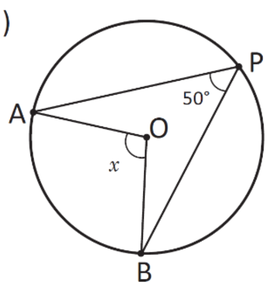


As $\sphericalangle BOA = 2\sphericalangle BPA$

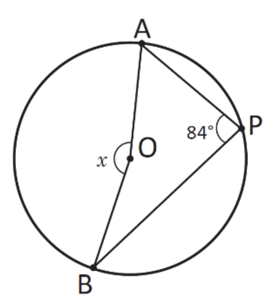
$\sphericalangle BOA = 2(35) = 70°$

Determine the value of x for each case:

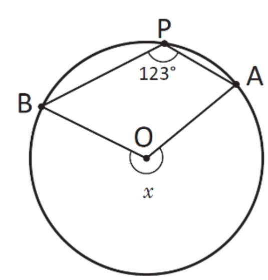
a)



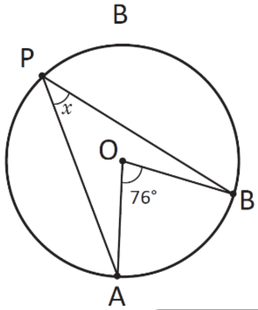
b)



c)



d)



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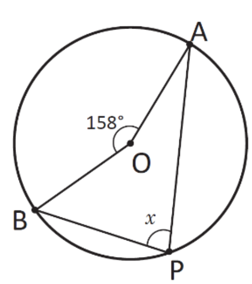
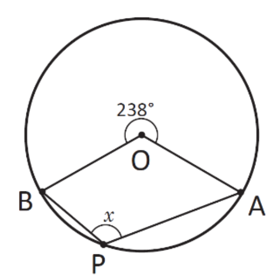


f)



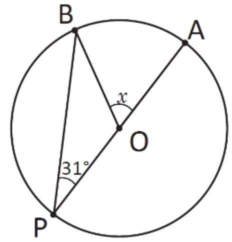
How long did it take to solve the problems?

1.5 Inscribed angle Theorem

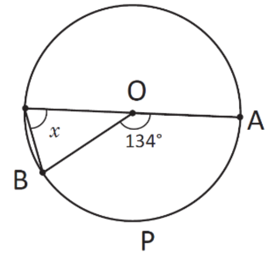
R

1. Determine the value of x for each case:

a)



b)



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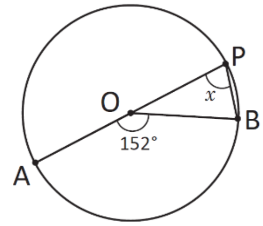
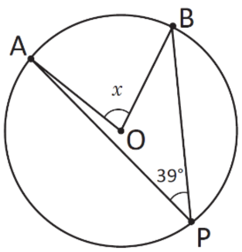
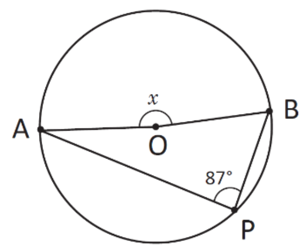


2. Determine the value of x for each case:

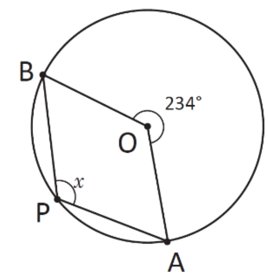
a)



b)



c)



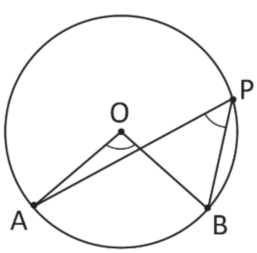
C

In a circumference, for any inscribed angle, it is true to state that **the central angle measure is twice the measure of the inscribed angle that subtends the same arc.**

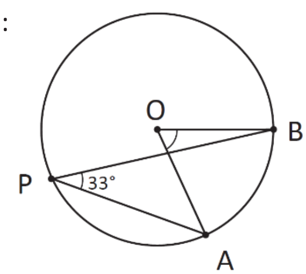
This result is known as the inscribed angle theorem.

Also, the inscribed angles that subtend the same arc have an equal measure.

For example:



∢BOA = 2∢BPA

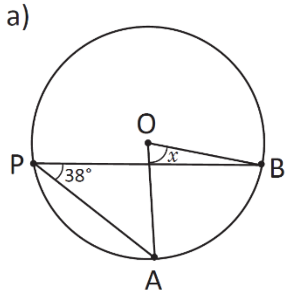


Como ∢BOA = 2∢BPA.

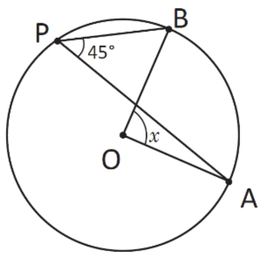
∢BOA = 2(33) = 66°.

Determine the value of x, y and z for each case:

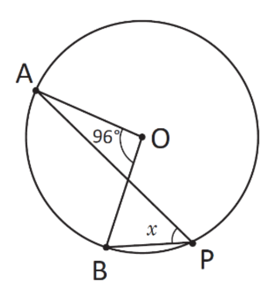
a)



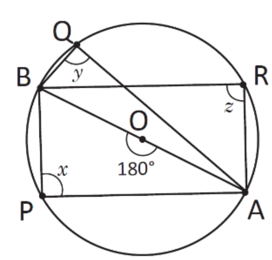
b)



c)



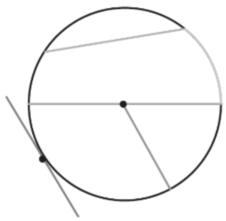
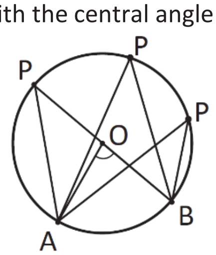
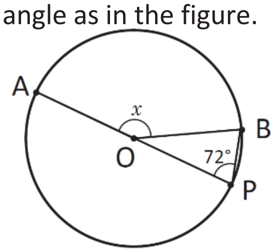
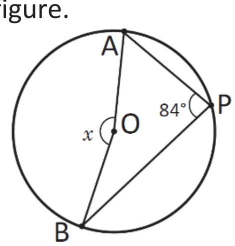
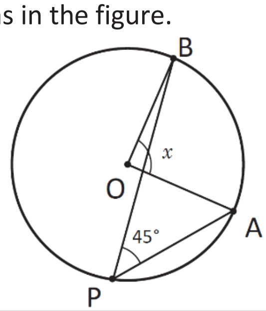
d)



How long did it take to solve the problems?

1.6 Self-Assessment

Solve and mark with an "x" the box you consider appropriate according to what you learned. Be mindful of your response.

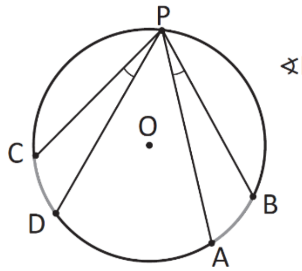
Item	Yes	Could improve	No	Notes
1. I identify the elements of the circumference in the figure below. 				
2. I understand the definition of an inscribed angle and identify its possible relationship with the central angle of the same arc. 				
3. I apply the inscribed angle theorem when the center is somewhere in the angle as in the figure. 				
4. I use the inscribed angle theorem when the center is inside the angle, as in the figure. 				
5. I apply the inscribed angle theorem when the center is outside the angle as in the figure. 				

How long did it take to solve the problems?

1.7 Congruent arcs

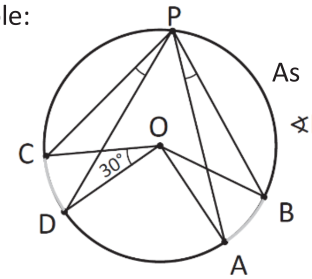
In a circumference, the inscribed angles, which subtend arcs of equal measure, have equal measure.

It is also true that if two inscribed angles are of equal measure, then the arcs that they subtend are also of equal measure.



∢BPA = ∢DPC

For example:

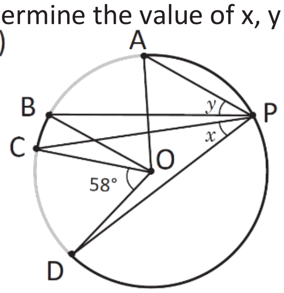


As ∢BOA = ∢DOC.

∢BPA = ∢DPC = $\frac{30}{2}$ = 15°.

1. Determine the value of x, y and z for each case. Consider CD = AB.

a)



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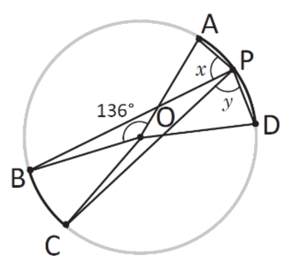
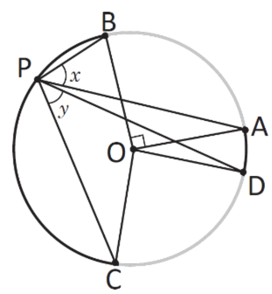
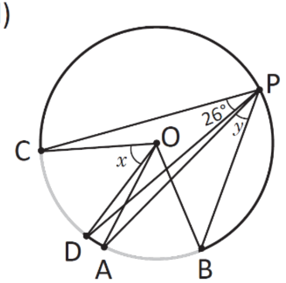


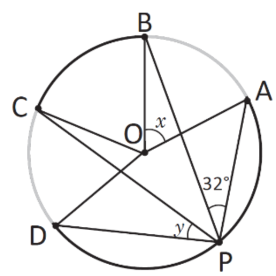
c)



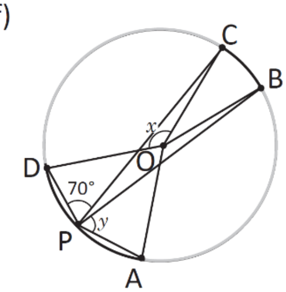
d)



e)

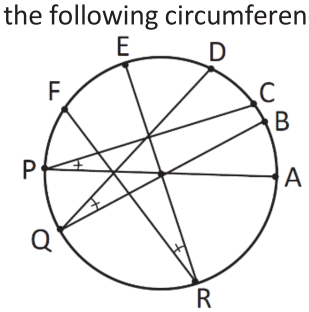


f)

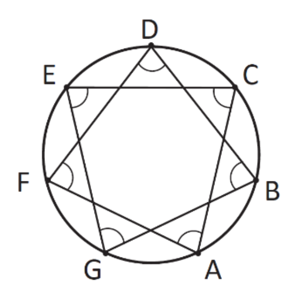


2. On the following circumferences, determine the arcs that are of equal measure.

a)



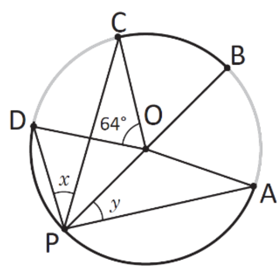
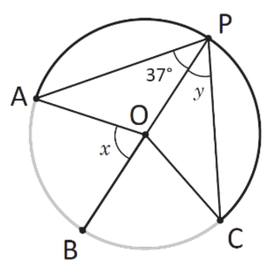
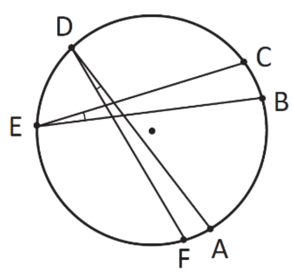
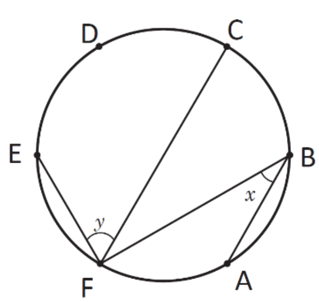
b)



How long did it take to solve the problems?

1.8 Self-Assessment

Solve and mark with an "x" the box you consider appropriate according to what you learned. Be mindful of your response.

Item	Yes	Could improve	No	Notes
1. I apply the properties of the arcs with identical measurements to determine the angle sizes, as in the figures.  				
2. I use the properties of inscribed angles of equal measure to establish which arcs have a similar measure, as in the figure below. 				
3. I correctly apply the results of the inscribed angle theorem and its reciprocal to solve problems such as the following: Determines the value of x and y if in the following figure the points A, B, C, D, E, F divide the circumference into six equal arcs. 				

How long did it take to solve the problems?

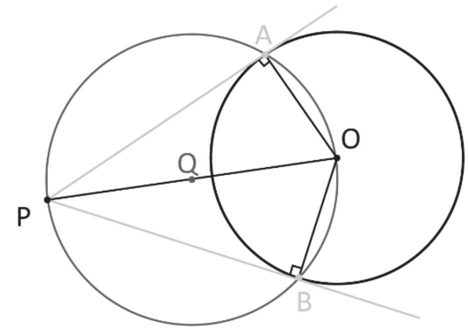
2.1 Constrution of tangents to a circumference

Using the inscribed angle results, construct the lines that pass through a point P and tangent to a given circumference, following the steps:

1. Calculate the midpoint segment between PO.

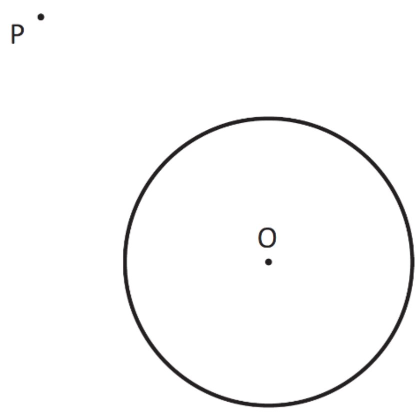
2. Draw the circumference for the diameter of PO.

3. Draw points for A and B where the circumference intersects.

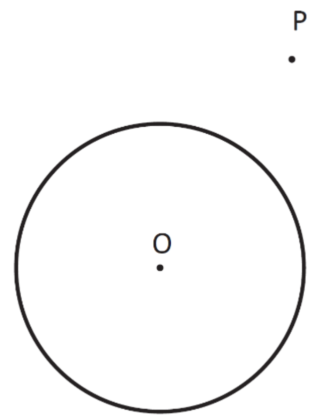


1. Construct the tangents for each circumference passing through the point P.

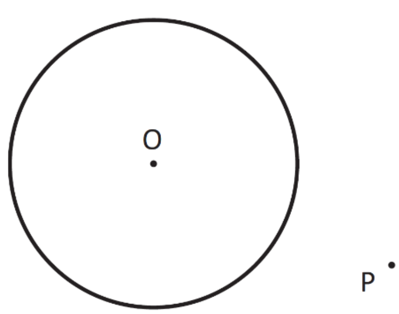
a)



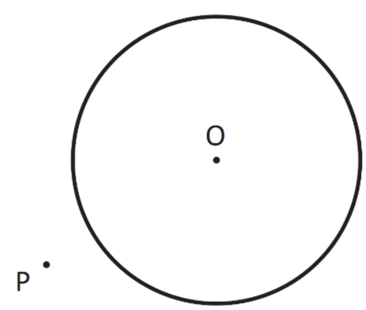
b)



c)



d)



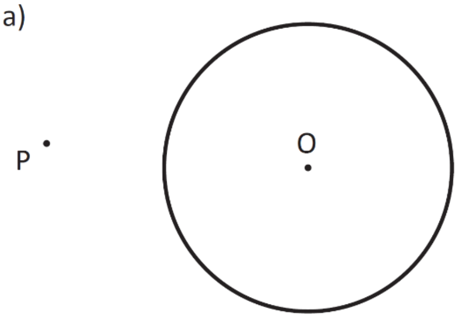
2. Why are the segments of the line tangent to the point of tangency equal?

How long did it take to solve the problems?

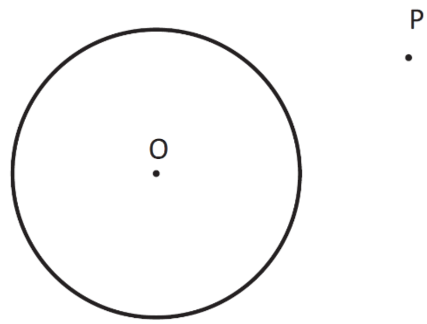
2.2 Chords and arcs of the circumference

Construct the tangents for each circumference passing across the P point.

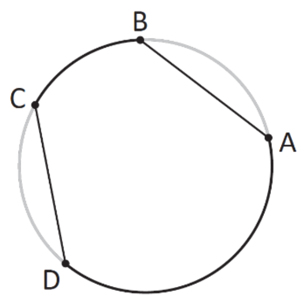
a)



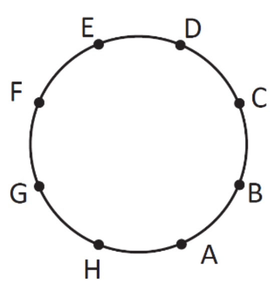
b)



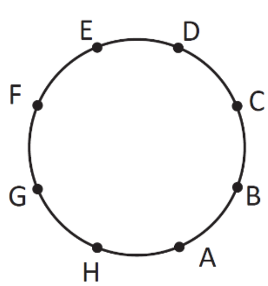
In a circumference if the measure of the two arcs is equal, then the measure of the chord that subtends those arcs is equal.



Points A, B, C, D, E, F, G, H divide the circumference into eight equal arcs. Classify the figures represented by each statement.



a) ACEG b) CEG c) CDGH d) BFGA



e) EGA f) BEH g) BCF h) ABCDEFGH

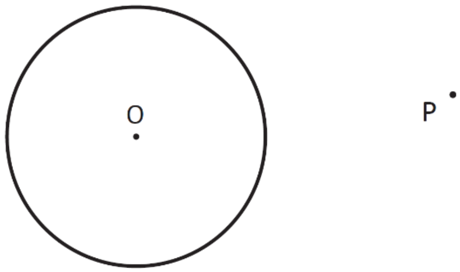
Unit 7

2.3 Similar triangle applications

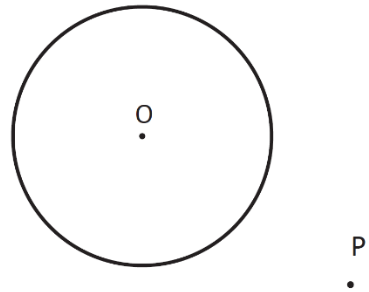
R

1. Construct the tangents for each circumference passing across the P point

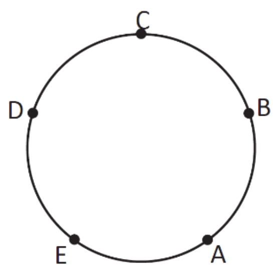
a)



b)



2. Points A, B, C, D, E, and F divide the circumference into six equal arcs. Classify the figures representing each statement. Look at the example:

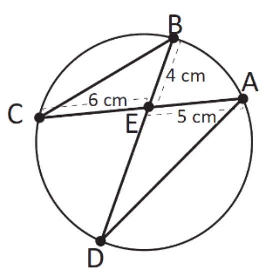


a) ABD b) CDE c) ABDE d) ABCDE

C

It is necessary to observe the inscribed angles that subtend the same arc to determine the similarity between triangles. And can be used to determine the length of some segments.

For example:



ΔAED ~ ΔBEC. Since there are two angles opposite by the vertex and also <DBC = <DAC. By the AA criterion, it is assumed that ΔAED ~ ΔBEC.

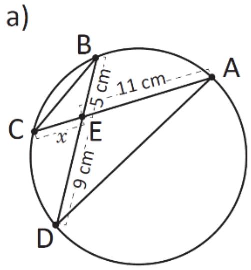
As $\Delta AED \sim \Delta BEC$, then $\frac{ED}{CE} = \frac{AE}{BE}$.

Therefore $ED = CE \times \frac{AE}{BE} = 6 \times \frac{5}{4} = 7.5$

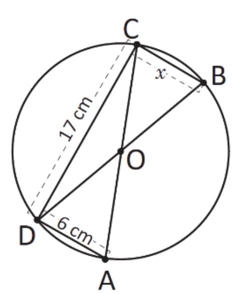
ED = 7.5 cm

Determine x in the following figures:

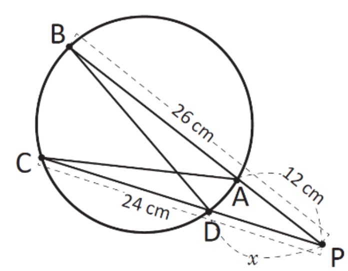
a)



b) Si $\widehat{CB} = \widehat{DA}$



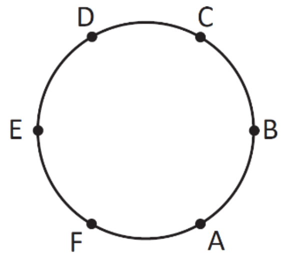
c)



How long did it take to solve the problems?

2.4 Parallelism

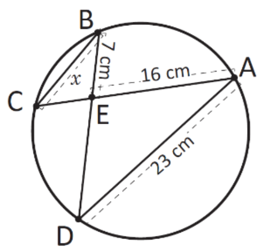
1. Points A, B, C, D, E, and F divide the circumference into six equal arcs. Clasify the figures formed by connecting the points indicated in each statement. Look at the example:



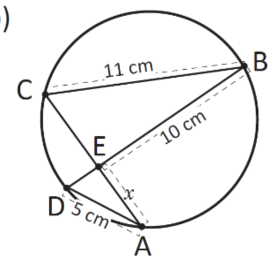
a) BDF b) ABDE c) CDEF d) ABCDEEF

2. Determine x in the following figures:

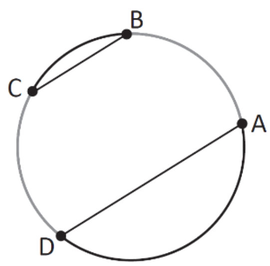
a)



b)



If having two arcs of equal measure on a circumference, then the chords determined by the beginning of one arc and the end of the other are parallel.



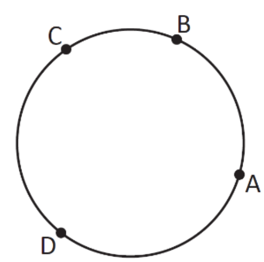
If AB = CD, then AD || BC.

One condition A is sufficient for another condition B if the proposition "If A then B" is met.

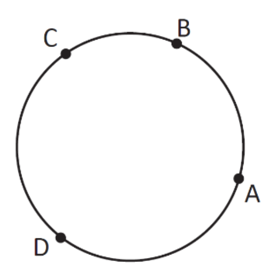
Determine which of the following statements, are sufficient conditions to four consecutive points A, B, C, and D; on a circumference. Once connected, there is at least a pair of parallel chords.

$\overset{\frown}{ABC}$ denotes the arc held from A to C, passing through B.

a) $\overset{\frown}{ABC} = \overset{\frown}{BCD}$ b) ∢DAC = ∢BDA c) CD = BA



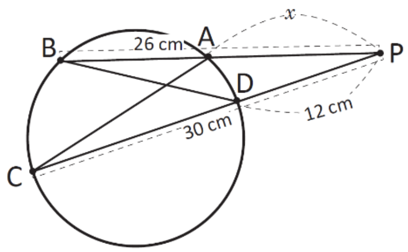
d) AC = BD e) CB = BA f) ΔABC ≅ ΔDCB



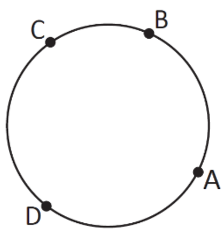
How long did it take to solve the problems?

2.5 Four points on a circumference of a circle

1. Determine x in the following figures:



2. Determine which of the following statements, are sufficient conditions to four consecutive points A, B, C, and D; on a circumference. Once connected, there is at least a pair of parallel chords.



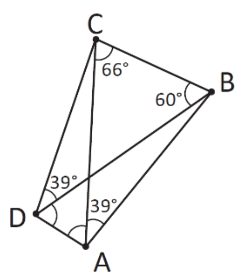
a) $\widehat{BA} = \widehat{DC}$

b) CB = BA

c) ΔABC ≅ ΔDCB

If two equal angles also share a segment at their openings, then the four points are on the same circumference.

For example:

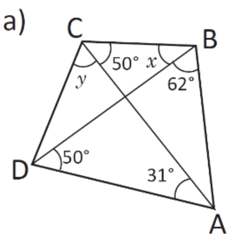


Since <CAB = <CDB and both share CB segment, then A, B, C, D are on the same circumference.

It must satisfied that <BDA = <BCA = 66°.
Moreover, it must meet that <CAD = <CBD = 60°.

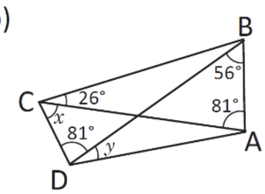
Determine the value of x and y.

a)

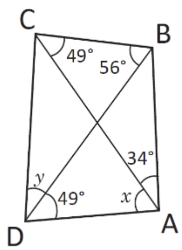


Si

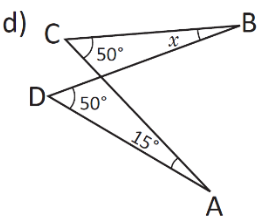
b)



c)



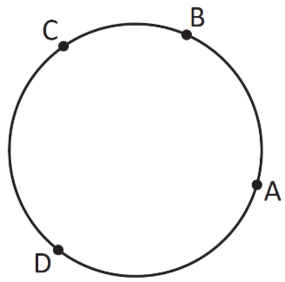
d)



How long did it take to solve the problems?

2.6 Semi-Inscribed angle

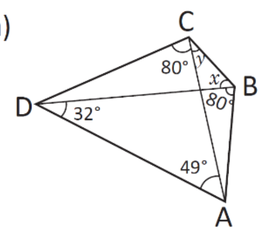
1. Determine which of the following statements, are sufficient conditions to four consecutive points A, B, C, and D; on a circumference. Once connected, there is at least a pair of parallel chords.



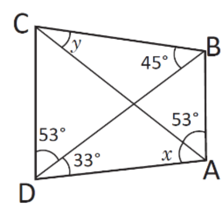
a) $\widehat{ABC} = \widehat{BCD}$ b) $\sphericalangle BDA = \sphericalangle DBC$ c) $\Delta BCD \sim \Delta BCA$

2. Determine the value of x and y.

a)

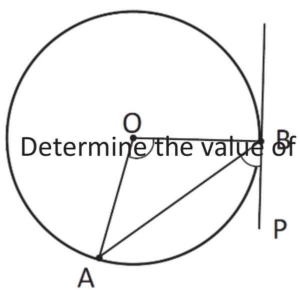


b)



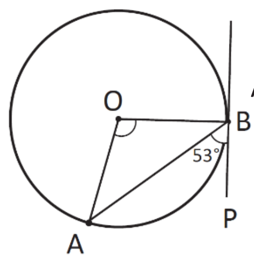
The angle formed by a tangent and a chord of the circumference is called: **semi-inscribed angle. In a** circumference**, the measurement of a semi-inscribed angle is equal to half the measurement of the central angle, which subtends the arc as the chord.**

For example:



$\sphericalangle BOA = 2\sphericalangle PBA$

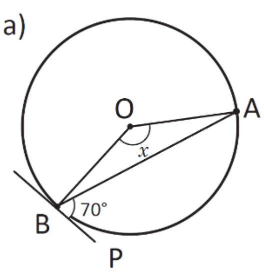
Determine the value of x for each case:



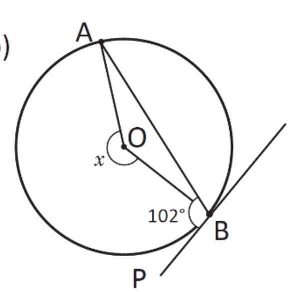
As $\sphericalangle BOA = 2\sphericalangle PBA$

$\sphericalangle BOA = 2(53) = 106°$

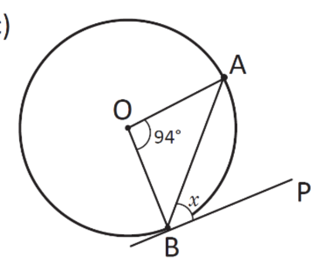
a)



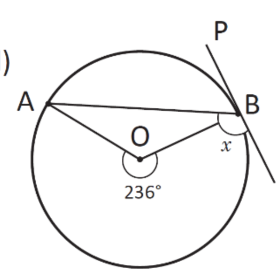
b)



c)



d)



Unit 7

2.7 Self-Assessment

Solve and mark with an "x" the box you consider appropriate according to what you learned. Be mindful of your response.

Item	Yes	Could improve	No	Notes
1 I correctly construct the tangents to a circumference passing through a point P.				
2 I correctly apply that when the measurement of two arcs is equal, the same happens with the chords; to determine what type of figure is formed in a circumference divided into equal arcs.				
3 I use the inscribed angle to find similar triangles and determine measurements of sides.				
4 I can determine sufficient and necessary conditions to have two parallel chords of four points on a circumference.				

2.8 Self-Assessment

Solve and mark with an "x" the box you consider appropriate according to what you learned. Be mindful of your response.

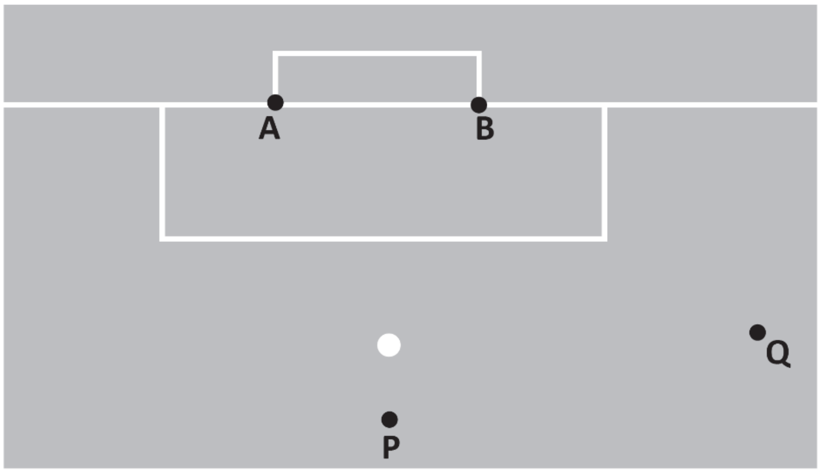
Item	Yes	Could improve	No	Notes
1 I determine correctly when four points are on a circumference and use the result to find the measurements of other angles.				
2 I determine the relationship between a semi-inscribed angle and the central angle that subtends the same arc.				
3 I apply the inscribed angle theorem to solve problems with angles within the circumference.				
4 I apply the inscribed angle theorem to solve problems with angles outside the circumference.				

How long did it take to solve the problems?

Application problems

1. **Shooting angle.** In a free throw game, one player is located at point P and another at point Q. Please, measures the angles <APB, <AQB; and respond:

a) According to the shooting angle, which of them has the best chance of scoring?
b) Mark another point P' that has the same shooting angle as P.



Next, draw a circumference that passes through A, B, and P and considers AB and inscribed angles with the same measure as the <APB.

2. **Map.** A tourist has the scale map shown in the image and needs to know some missing data. Help the tourist by following these steps:

a) Using a protractor measure of the angles <VPA and <VQA is 45°.
b) Find the distance between the tallest tree and the volcano.
c) Justify that points P, Q, A, and V are on a circumference on the map.
d) What is the distance between the Q community and the volcano?