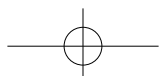# Practice Book for
# Mathematics
## Answer Book

**Grade 6**

## JICA

Japan International
Cooperation Agency

## 1 - 1 Letters and Math Sentences
### Math Sentences that Use Variables (1)

**Example**

In a math sentence, $x$ is a variable. It is a letter that represent any number.

The formula to calculate the area of a rectangle is width multiplied by length. A 5 cm wide tape is cut into several pieces. Write a math sentence for the area of this rectangular tape when the length is $x$ cm long. Calculate the area when $x = 30$.

5 cm — $x$ cm

If the length is 10 cm, the area is $5 \times 10 = 50$ (cm$^2$)
If the length is 15 cm, the area is $5 \times 15 = 75$ (cm$^2$)
If the length is 20 cm, the area is $5 \times 20 = 100$ (cm$^2$)
If the length is 25 cm, the area is $5 \times 25 = 125$ (cm$^2$)
If the length is $x$ cm, the area is $5 \times x$ (cm$^2$)

Answer $5 \times x$ (cm$^2$)  When $x$ is 30 cm, the area is $150$ cm$^2$

**1** The letter $x$ is used in math to mean a value is not yet known. It is called a "variable". Let's practice writing $x$.

$x$ $x$ $x$ $x$ $x$ $x$ $x$

**2** The formula to calculate the area of a parallelogram is the base multiplied by the height. The parallelogram has a base of 4 cm and a height of $x$ cm. Write a math sentence for the area. Calculate the area when $x = 6$ cm.

Answer $4 \times x$ (cm$^2$)  When $x = 6$, the area is $24$ cm$^2$

$x$ cm
4 cm
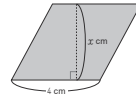
**3** 1.8 L of juice is shared equally among $x$ amount of people. How much juice will each person get? Write a math sentence for the word problem. Calculate the amount of juice when $x = 9$.

Answer $1.8 \div x$ (L)  When $x$ is 9, the amount of juice is $0.2$ L

1

## 1 - 2 Letters and Math Sentences
### Math Sentences that Use Variables (2)

**Example**

A parallelogram has a base of $x$ cm, a height of 6 cm and an area of $y$ cm$^2$. Write the math sentence for the word problem. Calculate $y$ if $x = 5$ cm. Since the formula for a parallelogram is base multiplied by the height, the math sentence is:

If the base is 1 cm, the area is $1 \times 6 = 6$ (cm$^2$)
If the base is 2 cm, the area is $2 \times 6 = 12$ (cm$^2$)
If the base is 3 cm, the area is $3 \times 6 = 18$ (cm$^2$)
If the base is 4 cm, the area is $4 \times 6 = 24$ (cm$^2$)
If the base is $x$ cm, the area is $x \times 6 = y$ (cm$^2$)
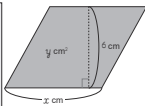
Answer $x \times 6 = y$  When $x = 5$, $y$ is $30$

$y$ cm$^2$  6 cm
$x$ cm

**1** The letter $y$ is also used in math to mean a value is not yet known. It is also called a variable. Let's practice writing $y$.

$y$ $y$ $y$ $y$ $y$ $y$

**2** A rectangle has a length of $x$ cm and a width of 8 cm. It has an area of $y$ cm$^2$. Write the math sentence for the word problem. Calculate the area $y$ when $x = 4$ cm.

Answer $x \times 8 = y$  When $x = 4$, $y$ is $32$

**3** The formula for the circumference of a circle is the diameter multiplied by 3.14. A circle has a diameter of $x$ cm and circumference of $y$ cm. Write the math sentence for the circumference. Calculate the circumference $y$ when $x = 10$ cm.

Answer $x \times 3.14 = y$  When $x = 10$, $y$ is $31.4$

$x$ cm

**4** There was 2 L of juice. I drank $x$ L. There is $y$ L of juice left. Calculate the amount of juice left, $y$ when $x = 0.6$ L.

Answer $2 - x = y$  When $x = 0.6$, $y$ is $1.4$

2

## 1 - 3 Letters and Math Sentences
### How to Read Math Sentences

**Example** Write a word problem that fits the following math sentence.

$$20 + x = y$$

Answer (Example) I have 20 pencils. My mother bought $x$ more pencils for me. Now I have a total of $y$ pencils.

**1** Match the word problem with the math sentence. Write the letter in the ☐.

① $80 + x = y$  (b)  ② $80 - x = y$  (c)
③ $80 \times x = y$  (d)  ④ $80 \div x = y$  (a)

(a) There is a rectangle with an area of 80 cm$^2$ and the length of $x$ cm. The width is $y$ cm.
(b) My father weighs 80 kg and I weigh $x$ kg. Both of us together weigh $y$ kg.
(c) I had 80 pieces of paper. Because I used $x$ pieces, I have only $y$ pieces left.
(d) There are $x$ marbles. Each of marble weighs 80 g. The total weight of all the marbles is $y$ g.

**2** Write a word problem that matches the math sentence.

Examples:

① $x + 30 = y$  I had $x$ sheets of paper. I received 30 sheets of paper from my friend. Now I have a total of $y$ sheets of paper.

② $x - 30 = y$  I had $x$ pieces of candy. I gave my brother 30 pieces. Now I have $y$ pieces of candy left.

③ $x \times 30 = y$  There are 30 balls. Each of ball weighs $x$ g. The total weight of all balls 30 balls is $y$ g.

④ $x \div 30 = y$  There is a rice paddy with an area of $x$ m$^2$. When the length is 30 m, the width is $y$ m.

3

## 1 - 4　Letters and Math Sentences
### Review

**1**　Write a math sentences for the following word problems.

**1**　$2.4$ L of milk is divided into $x$ glasses equally.　How much milk does each glass contain?

Answer　$2.4 \div x$　(L)

**2**　My older sister is $x$ years old.　My younger sister is $7$ years old.　What is the difference between their ages?

Answer　$x - 7$　(years old)

**3**　A triangle has a base of $3$ cm and a height of $x$ cm.　What is the area of this triangle?

The formula for calculating the area of a triangle is $\frac{1}{2}$ multiplied by the base multiplied by the height.

Answer　$3 \times x \div 2$　(cm$^2$)

**2**　Calculate the following problems.

**1**　One orange weighs $0.2$ kg.　The box weighs $0.6$ kg.　How much does the box weigh when $x$ oranges are put into the box?　Write the math sentence to calculate the box's total weight.

Answer　$0.2 \times x + 0.6$　(kg)

**2**　If there are $10$ oranges into a box, what is the total weight?

$0.2 \times 10 + 0.6 = 2 + 0.6 = 2.6$　Answer　2.6 kg

**3**　If there are $20$ oranges into a box, what is the total weight?

$0.2 \times 20 + 0.6 = 4 + 0.6 = 4.6$　Answer　4.6 kg

**3**　Write the math sentence for the word problem.

**1**　There were $x$ pencils to be shared equally among $4$ students.　Each student received $y$ pencils.

Answer　$x \div 4 = y$

**2**　There are $20$ questions on a math practice sheet.　I already solved $x$ questions.　I have $y$ questions left.

Answer　$20 - x = y$

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**3**　There are $x$ cans of juice.　Each can contains $240$ mL.　The total amount of juice is $y$ mL.

Answer　$240 \times x = y$

**4**　$x$ kg of sand is put into a container that weighs $0.3$ kg.　The total weight is $y$ kg.

Answer　$x + 0.3 = y$

**4**　There are $6$ marbles.　One marble weighs $x$ g.　Six marbles weight $y$ g.

**1**　Write a math sentence showing the relationship between $x$ and $y$.

Answer　$x \times 6 = y$

**2**　When $x = 6.5$, what is $y$?

$6.5 \times 6 = 39$　Answer　39　(g)

**3**　When $y = 26.4$, what is $x$?

$x \times 6 = 26.4$
$26.4 \div 6 = 4.4$　Answer　4.4　(g)

**5**　Match the word problem with the math sentence.　Write the letter in the ☐.

**1**　$50 + x = y$　(d)
**2**　$50 - x = y$　(a)
**3**　$50 \times x = y$　(b)
**4**　$50 \div x = y$　(c)

(a)　There are $50$ pieces of paper.　$x$ pieces have been used.　There are $y$ pieces of paper left.
(b)　My school rented school buses for a trip.　Each bus has $50$ seats.　The school rented $x$ buses. $y$ students can attend the trip.
(c)　There is a rectangle flowerbed with an area of $50$ m$^2$ and the length of $x$ m.　The width of this flowerbed is $y$ m.
(d)　There are $50$ male students and $x$ female students.　$y$ is the total number of students.

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## 2 - 1　Multiplication and Division of Fractions
### Multiplying a Fraction by a Whole Number (1)

**Example**　One dL of paint can cover a $\frac{2}{7}$ m$^2$ area.　How much area can $3$ dL of paint cover?

There are six $(2 \times 3)$ $\frac{1}{7}$ s.

Math sentence

$\frac{2}{7} \times 3 = \frac{2 \times 3}{7} = \frac{6}{7}$

Answer　$\frac{6}{7}$ m$^2$

To multiply a fraction by a whole number, multiply the numerator by the whole number.

$\frac{\bullet}{\blacksquare} \times \blacktriangle = \frac{\bullet \times \blacktriangle}{\blacksquare}$

**1**　$\frac{2}{9}$ kg of flour is needed to make one cake.　How much flour is needed to make $4$ cakes?

Complete the number line diagram and table.

How many $\frac{1}{9}$ s are there?

Math sentence

$\frac{2}{9} \times 4 = \frac{2 \times 4}{9} = \frac{8}{9}$

Answer　$\frac{8}{9}$ kg

**2**　Calculate the following multiplication problems. Leave the answer as improper fractions.

**1**　$\frac{3}{5} \times 2 = \frac{3 \times 2}{5} = \frac{6}{5}$

**2**　$\frac{3}{10} \times 3 = \frac{3 \times 3}{10} = \frac{9}{10}$

**3**　$\frac{4}{7} \times 5 = \frac{4 \times 5}{7} = \frac{20}{7}$

**4**　$\frac{3}{5} \times 6 = \frac{3 \times 6}{5} = \frac{18}{5}$

**5**　$\frac{1}{3} \times 7 = \frac{1 \times 7}{3} = \frac{7}{3}$

**6**　$\frac{5}{8} \times 9 = \frac{5 \times 9}{8} = \frac{45}{8}$

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## 2 - 2　Multiplication and Division of Fractions
### Multiplying a Fraction by a Whole Number (2)

**Example 1**　Calculate $\frac{7}{12} \times 4$

Student A
$\frac{7}{12} \times 4 = \frac{7 \times 4}{12} = \frac{\overset{7}{\cancel{28}}}{\underset{3}{\cancel{12}}} = \frac{7}{3}$
He simplified the answer at the end.

Student B
$\frac{7}{12} \times 4 = \frac{7 \times \overset{1}{\cancel{4}}}{\underset{3}{\cancel{12}}} = \frac{7}{3}$
She simplified it in the middle of calculation.

Student A simplified the product after calculating it (calculated twice), but Student B simplified in the middle of calculation (calculated once).　The way of Student B is easier to calculate.　This is the ingenuity of calculation.

**1**　Calculate the following multiplication problems. Simplify when possible. Leave the answer as improper fractions.

**1**　$\frac{5}{6} \times 3 = \frac{5 \times \overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}}} = \frac{5}{2}$

**2**　$\frac{5}{8} \times 4 = \frac{5 \times \overset{1}{\cancel{4}}}{\underset{2}{\cancel{8}}} = \frac{5}{2}$

**3**　$\frac{4}{15} \times 3 = \frac{4 \times \overset{1}{\cancel{3}}}{\underset{5}{\cancel{15}}} = \frac{4}{5}$

**4**　$\frac{11}{12} \times 18 = \frac{11 \times \overset{3}{\cancel{18}}}{\underset{2}{\cancel{12}}} = \frac{33}{2}$

**5**　$\frac{3}{8} \times 20 = \frac{3 \times \overset{5}{\cancel{20}}}{\underset{2}{\cancel{8}}} = \frac{15}{2}$

**6**　$\frac{13}{6} \times 12 = \frac{13 \times \overset{2}{\cancel{12}}}{\underset{1}{\cancel{6}}} = \frac{26}{1} = 26$

**Example 2**　Calculate $1\frac{2}{3} \times 4$

$1\frac{2}{3} \times 4 = \frac{5}{3} \times 4 = \frac{5 \times 4}{3}$
$= \frac{20}{3}$ or $6\frac{2}{3}$

We can change a mixed number to an improper fraction, and then calculate it by using the formula shown in the previous page.

Mixed numbers make it easy to imagine the size of numbers, but we convert mixed numbers to improper fractions when calculating.

**2**　Calculate the following multiplication problems.　Simplify when possible.

**1**　$1\frac{5}{6} \times 9 = \frac{11}{6} \times 9 = \frac{11 \times \overset{3}{\cancel{9}}}{\underset{2}{\cancel{6}}} = \frac{33}{2}$ or $16\frac{1}{2}$

**2**　$1\frac{5}{8} \times 6 = \frac{13}{8} \times 6 = \frac{13 \times \overset{3}{\cancel{6}}}{\underset{4}{\cancel{8}}} = \frac{39}{4}$ or $9\frac{3}{4}$

**3**　$3\frac{3}{4} \times 2 = \frac{15}{4} \times 2 = \frac{15 \times \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}}} = \frac{15}{2} = 7\frac{1}{2}$

**4**　$2\frac{2}{9} \times 6 = \frac{20}{9} \times 6 = \frac{20 \times \overset{2}{\cancel{6}}}{\underset{3}{\cancel{9}}} = \frac{40}{3} = 13\frac{1}{3}$

## 2 - 3 Multiplication and Division of Fractions
### Dividing a Fraction by a Whole Number (1)

**Example** Two dL of paint can cover $\frac{4}{7}$ m². How much area can 1 dL of paint cover?

Area we can paint with 2 dL. Area we can paint with 1 dL.

**Math sentence**

$$\frac{4}{7} \div 2 = \frac{4 \div 2}{7} = \frac{2}{7}$$

Answer $\frac{2}{7}$ m²

**1** $\frac{8}{9}$ L of oil can run a machine for 4 hours. How many L of oil is needed to run the machine for 1 hour?

Complete the number line diagram and table.

**Math sentence**

$$\frac{8}{9} \div 4 = \frac{8 \div 4}{9} = \frac{2}{9}$$

Answer $\frac{2}{9}$ L

**2** Calculate the following division problems.

**1** $\frac{9}{4} \div 3 = \frac{9 \div 3}{4} = \frac{3}{4}$

**2** $\frac{4}{5} \div 2 = \frac{4 \div 2}{5} = \frac{2}{5}$

**3** $\frac{6}{7} \div 3 = \frac{6 \div 3}{7} = \frac{2}{7}$

**4** $\frac{12}{5} \div 6 = \frac{12 \div 6}{5} = \frac{2}{5}$

**5** $\frac{15}{8} \div 3 = \frac{15 \div 3}{8} = \frac{5}{8}$

**6** $\frac{20}{3} \div 5 = \frac{20 \div 5}{3} = \frac{4}{3}$ or $1\frac{1}{3}$

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## 2 - 4 Multiplication and Division of Fractions
### Dividing a Fraction by a Whole Number (2)

**Example 1** Calculate $\frac{3}{5} \div 2$

Three (3) is not divisible by 2 as the whole number. Therefore, we cannot do $\frac{3 \div 2}{5}$. How can we calculate this?

$$\frac{3}{5} \div 2 = \frac{3}{5 \times 2} = \frac{3}{10}$$

To divide a fraction by a whole number, multiply the fraction by the whole number's reciprocal. Remember, a whole number can be written as a fraction, $\blacktriangle = \frac{\blacktriangle}{1}$. It's reciprocal is $\frac{1}{\blacktriangle}$.

$$\frac{\bullet}{\blacksquare} \div \blacktriangle = \frac{\bullet}{\blacksquare \times \blacktriangle}$$

**1** Calculate the following division problems. Simplify when possible.

**1** $\frac{4}{5} \div 3 = \frac{4}{5 \times 3} = \frac{4}{15}$

**2** $\frac{5}{6} \div 2 = \frac{5}{6 \times 2} = \frac{5}{12}$

**3** $\frac{3}{8} \div 5 = \frac{3}{8 \times 5} = \frac{3}{40}$

**4** $\frac{4}{7} \div 3 = \frac{4}{7 \times 3} = \frac{4}{21}$

**5** $\frac{10}{3} \div 4 = \frac{\overset{5}{\cancel{10}}}{3 \times \cancel{4}} = \frac{5}{6}$

**6** $\frac{12}{5} \div 15 = \frac{\overset{4}{\cancel{12}}}{5 \times \cancel{15}} = \frac{4}{25}$

**Example 2** Calculate $1\frac{3}{7} \div 5$

$$1\frac{3}{7} \div 5 = \frac{10}{7} \div 5 = \frac{\overset{2}{\cancel{10}}}{7 \times \cancel{5}}$$
$$= \frac{2}{7}$$

When dividing a mixed number by a whole number, convert the mixed number to an improper fraction first. Multiply the improper fraction by the whole number's reciprocal.

**2** Calculate the following division problems. Simplify when possible.

**1** $1\frac{4}{7} \div 6 = \frac{11}{7} \div 6 = \frac{11}{7 \times 6} = \frac{11}{42}$

**2** $2\frac{2}{3} \div 5 = \frac{8}{3} \div 5 = \frac{8}{3 \times 5} = \frac{8}{15}$

**3** $1\frac{3}{4} \div 2 = \frac{7}{4} \div 2 = \frac{7}{4 \times 2} = \frac{7}{8}$

**4** $2\frac{3}{5} \div 6 = \frac{13}{5} \div 6 = \frac{13}{5 \times 6} = \frac{13}{30}$

**5** $3\frac{1}{9} \div 4 = \frac{28}{9} \div 4 = \frac{\overset{7}{\cancel{28}}}{9 \times \cancel{4}} = \frac{7}{9}$

**6** $1\frac{1}{6} \div 7 = \frac{7}{6} \div 7 = \frac{\cancel{7}}{6 \times \cancel{7}} = \frac{1}{6}$

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## 2 - 5 Multiplication and Division of Fractions
### Review

**1** Calculate the following multiplication problems. Simplify when possible.

**1** $\frac{5}{8} \times 3 = \frac{5 \times 3}{8} = \frac{15}{8}$ or $1\frac{7}{8}$

**2** $\frac{3}{4} \times 9 = \frac{3 \times 9}{4} = \frac{27}{4}$ or $6\frac{3}{4}$

**3** $\frac{5}{12} \times 3 = \frac{5 \times \cancel{3}}{\cancel{12}} = \frac{5}{4}$ or $1\frac{1}{4}$

**4** $\frac{1}{6} \times 10 = \frac{1 \times \overset{5}{\cancel{10}}}{\cancel{6}} = \frac{5}{3}$ or $1\frac{2}{3}$

**5** $1\frac{5}{6} \times 4 = \frac{11}{6} \times 4 = \frac{11 \times \overset{2}{\cancel{4}}}{\cancel{6}} = \frac{22}{3}$ or $7\frac{1}{3}$

**6** $3\frac{2}{5} \times 10 = \frac{17}{5} \times 10 = \frac{17 \times \overset{2}{\cancel{10}}}{\cancel{5}} = \frac{34}{1} = 34$

**2** Calculate the following division problems. Simplify when possible.

**1** $\frac{9}{8} \div 3 = \frac{9 \div 3}{8} = \frac{3}{8}$

**2** $\frac{32}{7} \div 8 = \frac{32 \div 8}{7} = \frac{4}{7}$

**3** $\frac{2}{9} \div 6 = \frac{\cancel{2}}{9 \times \cancel{6}} = \frac{1}{27}$

**4** $\frac{6}{7} \div 4 = \frac{\cancel{6}}{7 \times \cancel{4}} = \frac{3}{14}$

**5** $3\frac{5}{9} \div 8 = \frac{32}{9} \div 8 = \frac{\overset{4}{\cancel{32}}}{9 \times \cancel{8}} = \frac{4}{9}$

**6** $2\frac{2}{5} \div 6 = \frac{12}{5} \div 6 = \frac{\overset{2}{\cancel{12}}}{5 \times \cancel{6}} = \frac{2}{5}$

**3** Write the math sentence for the following word problems and calculate. Simplify when possible.

Complete the number line diagrams and tables.

**1** $\frac{7}{12}$ kg of flour is needed to make one loaf of bread. How much flour is needed to make 3 loaves?

Flour that is needed to make one loaf of bread. Flour that is needed to make 3 loaves of bread.

**Math sentence**

$$\frac{7}{12} \times 3 = \frac{7 \times \cancel{3}}{\cancel{12}} = \frac{7}{4}$$ or $1\frac{3}{4}$

Answer $\frac{7}{4}$ kg or $1\frac{3}{4}$ kg

**2** A 1 m wire weighs $\frac{2}{25}$ kg. How much will 5 m of this wire weigh?

**Math sentence**

$$\frac{2}{25} \times 5 = \frac{2 \times \cancel{5}}{\cancel{25}} = \frac{2}{5}$$

Answer $\frac{2}{5}$ kg

**3** A pump can pump $\frac{12}{5}$ L of water in 3 minutes. How much water can it pump in 1 minute?

**Math sentence**

$$\frac{12}{5} \div 3 = \frac{12 \div 3}{5} = \frac{4}{5}$$

Answer $\frac{4}{5}$ L

**4** $2\frac{2}{9}$ L of milk tea is shared equally among 4 people. How many L of tea will each person get?

**Math sentence**

$$2\frac{2}{9} \div 4 = \frac{20}{9} \div 4 = \frac{\overset{5}{\cancel{20}}}{9 \times \cancel{4}} = \frac{5}{9}$$
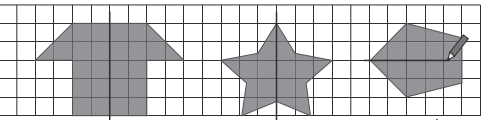
Answer $\frac{5}{9}$ L

10

11

4

**3 - 1**  Symmetric Figures
**Line Symmetry** (1)

**Instruction**  Balanced figures.

Look at the figures below and find out how they can be grouped.

They look balanced. Why do we see so?

**Example 1**  The following figures can be overlapped when folded in two. Where should the figures be folded? Draw the folding line.

A figure has line symmetry when it can be folded along a straight line and both sides overlap exactly. The folding line is called the **axis of symmetry** or **line of symmetry**.

**1**  The following figures can be overlapped when folded in two. Draw the line of symmetry.
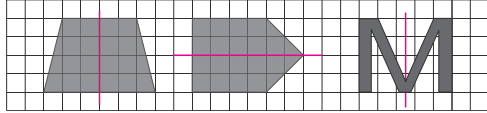
**Example 2**  The following figure is a line symmetric figure. What happens with the length of the overlapping sides and the size of the overlapping angles? Answer the following questions.

**1**  What is the respective overlapping point for point C?

G
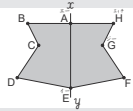
12

**2**  What is the respective overlapping sides for side BC?

HG

**3**  What is the respective overlapping angle for angle D?

F

● When a line symmetric figure is folded on the line of symmetry, the overlapping points, the overlapping sides, the overlapping angles are called **corresponding points**, **corresponding sides**, and **corresponding angles** respectively.
● In line symmetric figures, the length of the corresponding sides and the size of the corresponding angles are respectively equal.

Corresponding points
Corresponding sides
Corresponding angles
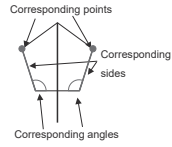
**2**  The following figure is a line symmetric figure. Answer the following questions.

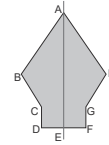**1**  Which are the respective overlapping points for point B?

H

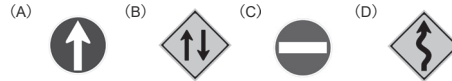**2**  Which are the respective overlapping sides for side CD?

GF

**3**  Which are the respective overlapping angles for angle F?

D

Let's Try!

**1**  Find the symmetric signs below.

(A)     (B)     (C)     (D)

A,C

**2**  Find another symmetric sign in your surroundings.

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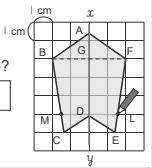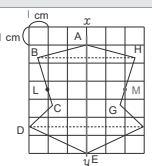**3 - 2**  Symmetric Figures
**Line Symmetry** (2)

**Example 1**  The figure on the right is a line symmetric figure that has straight line $xy$ as the line of symmetry. Answer the following questions.

**1**  Draw the corresponding point L to point M.

**2**  The length of straight line BG is 2 cm. How many cm is the length of straight line FG?

The straight line BF, connecting the corresponding points B and F, and line $xy$ intersect at a right angle.

2 cm

**1**  The figure on the right is a line symmetric figure that has straight line $xy$ as the line of symmetry. Answer the following questions.

**1**  Draw in the figure the corresponding point M to point L.

**2**  Find the length of DF.

6 cm

● For line symmetric figures, the intersection between the straight line that connects two corresponding points and the line of symmetry is perpendicular.
● The length from the line of symmetry to each of the corresponding points is equal.

**Example 2**  The following figure represents half of a line symmetric figure that has the straight line $xy$ as the line of symmetry. Draw the remaining half.
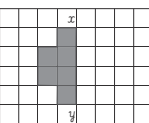
Remember the properties of line symmetry.
● The line connecting two corresponding points and the line of symmetry is perpendicular.
● The line of symmetry is the middle line.

14

| 1. Measure the length between each point and the line of symmetry. | 2. Plot the corresponding points. | 3. Connect all the points, point A, point F, and its corresponding points to draw the remaining half. |
|---|---|---|

**2**  The following figure represents half of a line symmetric figure that has the straight line $xy$ as the line of symmetry. Draw the remaining half.

**1**     **2**

**Example 3**  The following figure represents half of a line symmetric figure that has the straight line $xy$ as the line of symmetry. Draw the remaining half.

Using a compass, you can measure the lengths easily.

**3**  The following figure represents half of a line symmetric figure that has the straight line $xy$ as the line of symmetry. Draw the remaining half.

**1**     **2**

15
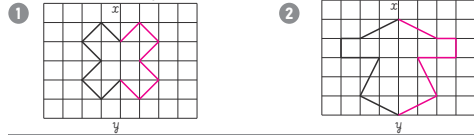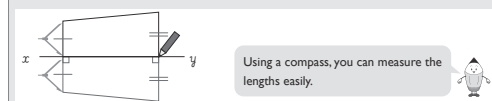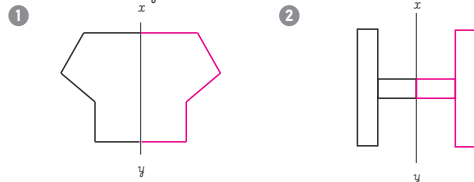
5

## 3 - 3  Symmetric Figures
### Point Symmetry (1)

**Instruction**  Point Symmetry

If the following figure is rotated 180° around the central point "·", it matches the original figure.

0°    45°    90°    180°

Point of symmetry

- A figure has a point of symmetry if the figure matches the original shape when it is rotated 180° with respect to a central point.
- The central point is called the **point of symmetry** or the **centre of symmetry**.

**Example 1**  Find point symmetric figures.

A    B    C    D

A and C

**1**  Find point symmetric figures.

A    B    C    D

A and B

**Instruction**  Properties of Point Symmetry

If the following figure is rotated 180° around the central point "·", it matches the original figure.

- When a point symmetric figure is rotated 180° around the point of symmetry, the matching points, the matching sides, the matching angles are called corresponding points, corresponding sides, and corresponding angles respectively.
- In point symmetric figures, the length of the corresponding sides and the size of the corresponding angles are respectively equal.

**Example 2**  The figure on the right is a point symmetric figure considering point O as the point of symmetry. Answer the following questions.

**1**  What is the respective matching point for point B?    E

**2**  What is the respective matching side for side CD?    FA

**3**  What is the respective matching angle for angle D?    A

**2**  The figure on the right is a point symmetric figure considering point O as the point of symmetry. Answer the following questions.

**1**  What is the name of point O?    Point of symmetry (centre of symmetry)

**2**  Which are the respective matching point for point A?    D

**3**  Which are the respective matching side for side BC?    EF

**4**  Which are the respective matching angle for angle C?    F

**3**  The figure on the right is a point symmetric figure considering point O as the point of symmetry. Answer the following questions.

**1**  What is the respective matching point for point B?    F

**2**  What is the respective matching side for side DE?    HA

## 3 - 4  Symmetric Figures
### Point Symmetry (2)

**Example 1**  The figure on the right is a point symmetric figure considering point O as the point of symmetry. Answer the following questions.

**1**  The straight line AD, BE, and CF are connecting corresponding points. Where do these lines intersect?    Point of symmetry (centre of symmetry)

**2**  Draw in the figure the corresponding point H to point G

- For point symmetric figures, the straight line that connects two corresponding points passes through the point of symmetry.
- The length from the point of symmetry to each of the corresponding points is equal.

For Example 1, the length of line AO is the same as line DO.

**1**  The figure on the right is a point symmetric figure considering point O as the point of symmetry. Answer the following questions.

**1**  The straight line AD, BE, and CF are connecting corresponding points. Where do these lines intersect?    Point of symmetry (centre of symmetry)

**2**  Which line is the same length as line BO?    EO

**3**  Draw in the figure the corresponding point H to point G

**Example 2**  The figure below is a point symmetric figure. Draw the point of symmetry on the figure.

If we want to find the point of symmetry, we draw the straight lines that connects two corresponding points to find the intersection.

**2**  The figure below is a point symmetric figures. Draw the point of symmetry in the figures.

**1**    **2**

**Example 3**  The following figure represents half of a point symmetric figure that has the point O as the point of symmetry. Draw the remaining half.

Remember the properties of line symmetry.
- The straight line that connects two corresponding points passes through the point of symmetry.
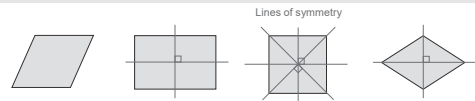- The point of symmetry is the middle of the line.

| 1. Measure the length between each point and the line of the symmetry. | 2. Plot the corresponding points and connect them. | 3. Connect all the points and extend sides to draw the remaining half. |
|---|---|---|

**3**  The following figure represents half of a point symmetric figure that has the point O as the point of symmetry. Draw the remaining half.

**1**    **2**

**3**    **4**

## 3 - 5　Symmetric Figures
### Polygons and Symmetry

**Example 1**　Look at the following quadrilaterals and answer the following questions.

Lines of symmetry

❶　Write ○ or × in the table to separate line symmetric figures and point symmetric figures. In addition, for line symmetric figures how many lines of symmetry does each have?
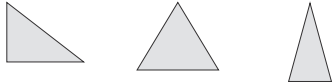
> The point of symmetry is the middle of the line.

|  | Line symmetry | Number of lines of symmetry | Point symmetry |
|---|---|---|---|
| Parallelogram | × | 0 | ○ |
| Rectangle | ○ | 2 | ○ |
| Square | ○ | 4 | ○ |
| Rhombus | ○ | 2 | ○ |

❷　Draw the point of symmetry on each point symmetric figure.

**1**　Look at the following triangles and answer the following questions.

Write ○ and × in the table to separate line symmetric figures and point symmetric figures. In addition, for line symmetric figures how many lines of symmetry does each have?

|  | Line symmetry | Number of lines of symmetry | Point symmetry |
|---|---|---|---|
| Right triangle | × | 0 | × |
| Equilateral triangle | ○ | 3 | × |
| Isosceles triangle | ○ | 1 | × |

**2**　Investigate the following regular polygons and answer the following questions.
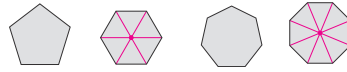
❶　Write ○ and × in the table to separate line symmetric figures and point symmetric figures. In addition, for line symmetric figures, how many lines of symmetry does each have?

> Are there any findings on number of lines of symmetry and sides?

|  | Line symmetry | Number of lines of symmetry | Point symmetry |
|---|---|---|---|
| Pentagon | ○ | 5 | × |
| Hexagon | ○ | 6 | ○ |
| Heptagon | ○ | 7 | × |
| Octagon | ○ | 8 | ○ |

❷　Draw the point of symmetry on each point symmetric figure.

**Example 2**　A hexagon is a line symmetric figure as well as a point symmetric figure. What are the corresponding sides to side AB with the following conditions?
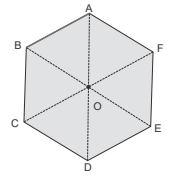
❶　When line AD is the line of symmetry.　AF

❷　When line BE is the line of symmetry.　CB

❸　When point O is the point of symmetry.　DE

**3**　An octagon is a line symmetric figure as well as a point symmetric figure. What are the corresponding sides to side AB with the following conditions?

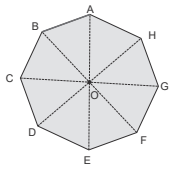❶　When line BF is the line of symmetry.　CB

❷　When line CG is the line of symmetry.　ED

❸　When point O is the point of symmetry.　EF

## 3 - 6　Symmetric Figures
### Review

**1**　Fill in the blanks with words.

❶　A figure has **line symmetry** when it can be folded along a straight line and both sides overlap exactly. The folding line is called the axis of symmetry or **line of symmetry**.

❷　A figure has **point symmetry** if the figure matches the original one when it is rotated $180°$ with respect to a central point. This central point is called the **point of symmetry** or centre of symmetry.

**2**　Find the line symmetric figures and point symmetric figures from the figures below.

A　　B　　C　　D
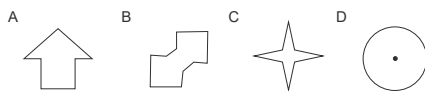
Line symmetry　**A, C, D**　　Point symmetry　**B, C, D**

**3**　The figure on the right is a line symmetric figure. Answer the following questions.

❶　What is the name of line $xy$?　**the line of symmetry（axis of symmetry）**

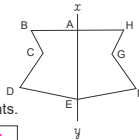❷　Find the corresponding points to the following points.

Point B　**Point H**　　Point D　**Point F**

❸　Find the corresponding sides to the following sides.

Side BC　**Side HG**　　Side CD　**Side GF**
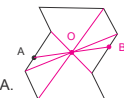
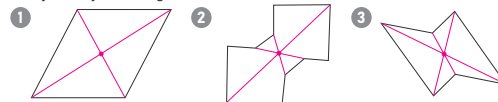❹　Find the corresponding angles to angle F　**Angle D**

**4**　The figure on the right is a point symmetric figure. Answer the following questions.
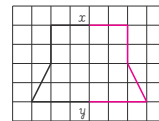
❶　Draw the point of symmetry.

❷　Draw in the figure the corresponding point B to point A.

**5**　The figure below is a point symmetric figure. Draw the point of symmetry on the figures.

❶　　❷　　❸

**6**　Draw the following figures.

❶　A line symmetric figure that has the straight line $xy$ as the line of symmetry.

❷　A point symmetric figure that has point O as the point of symmetry.
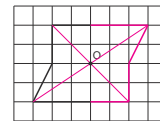
**7**　Look at the following regular polygons and answer the following questions.

❶　Write ○ and × in the table to separate line symmetric figures and point symmetric figures. In addition, for line symmetric figures how many lines of symmetry does each have?

|  | Line symmetry | Number of lines of symmetry | Point symmetry |
|---|---|---|---|
| Equilateral triangle | ○ | 3 | × |
| Pentagon | ○ | 5 | × |
| Hexagon | ○ | 6 | ○ |
| Octagon | ○ | 8 | ○ |

## 4 - 1　How to Multiply by Fractions

### Multiplying by Fractions（1）

**Example**　1 dL of paint can cover $\frac{4}{5}$ m$^2$.　How much area can $\frac{2}{3}$ dL of paint cover?

Area we can paint with 1 dL.　　Area we can paint with $\frac{2}{3}$ dL.



**Math sentence**

$$\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$$

Answer　　$\frac{8}{15}$ m$^2$

Multiply the numerators to equal the answer's numerator.　Multiply the denominators to equal the answer's denominator.

$$\frac{b}{a} \times \frac{d}{c} = \frac{b \times d}{a \times c}$$

**1**　A 1 m piece of wire weighs $\frac{4}{7}$ kg.　How much will a $\frac{3}{5}$ m piece of wire weigh?

Complete the number line diagrams and tables.



**Math sentence**

$$\frac{4}{7} \times \frac{3}{5} = \frac{4 \times 3}{7 \times 5} = \frac{12}{35}$$

Answer　　$\frac{12}{35}$ kg

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**2**　A 1 L oil weighs $\frac{5}{6}$ kg.　How much does a $\frac{5}{8}$ L of this oil weighs?



**Math sentence**

$$\frac{5}{6} \times \frac{5}{8} = \frac{5 \times 5}{6 \times 8} = \frac{25}{48}$$

Answer　　$\frac{25}{48}$ kg

**3**　A paver is a machine that can lay $\frac{5}{7}$ km$^2$ of pavement in 1 hour.　How much pavement can it lay in $\frac{3}{4}$ hour?

Complete the number line diagram and table.

**Math sentence**

$$\frac{5}{7} \times \frac{3}{4}$$
$$= \frac{5 \times 3}{7 \times 4} = \frac{15}{28}$$

Answer　　$\frac{15}{28}$ km$^2$

**4**　Calculate the following multiplication problems.　Simplify the answers when possible.　Leave the answers as improper fractions.

**1** $\frac{1}{5} \times \frac{4}{9} = \frac{1 \times 4}{5 \times 9} = \frac{4}{45}$

**2** $\frac{7}{9} \times \frac{5}{6} = \frac{7 \times 5}{9 \times 6} = \frac{35}{54}$

**3** $\frac{3}{2} \times \frac{5}{4} = \frac{3 \times 5}{2 \times 4} = \frac{15}{8}$

**4** $\frac{4}{9} \times \frac{2}{3} = \frac{4 \times 2}{9 \times 3} = \frac{8}{27}$

**5** $\frac{2}{3} \times \frac{7}{5} = \frac{2 \times 7}{3 \times 5} = \frac{14}{15}$

**6** $\frac{5}{4} \times \frac{5}{2} = \frac{5 \times 5}{4 \times 2} = \frac{25}{8}$

**7** $\frac{9}{5} \times \frac{7}{2} = \frac{9 \times 7}{5 \times 2} = \frac{63}{10}$

**8** $\frac{3}{8} \times \frac{3}{4} = \frac{3 \times 3}{8 \times 4} = \frac{9}{32}$

24　　　25

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## 4 - 2　How to Multiply by Fractions

### Multiplying by Fractions（2）

**Example 1**　There is a rectangle with a length of $\frac{5}{6}$ m and a width of $\frac{3}{10}$ m.　What is the area of this rectangle?



This rectangle has 15 small rectangles with an area of $\frac{1}{10 \times 6}$ m$^2$.　Therefore, the area of this rectangle is $\frac{15}{60}$ m$^2$ (or $\frac{1}{4}$ m$^2$).　This area can be calculated by the formula of the area of rectangle.

**Math sentence**

$$\frac{5}{6} \times \frac{3}{10} = \frac{5 \times 3}{6 \times 10} = \frac{1}{4}$$

Answer　　$\frac{1}{4}$ m$^2$

Simplifying fractions by dividing a numerator and denominator by their greatest common factors is easier to be done during the calculation.　It is also fine to simplify it after calculation.

**1**　There is a rectangle with a length of $\frac{4}{9}$ m and a width of $\frac{3}{8}$ m.　What is the area of this rectangle?　Calculate it by using the formula of the area of rectangle.

**Math sentence**

$$\frac{4}{9} \times \frac{3}{8}$$
$$= \frac{4 \times 3}{9 \times 8} = \frac{1}{6}$$

Answer　　$\frac{1}{6}$ m$^2$

How many rectangles with the area of $\frac{1}{9 \times 8}$ m$^2$ are there?

---

**Example 2**　1 L of water contains $\frac{3}{8}$ g of minerals.　How many g of minerals contained in $\frac{4}{9}$ L of water?



**Math sentence**

$$\frac{3}{8} \times \frac{4}{9} = \frac{3 \times 4}{8 \times 9} = \frac{1}{6}$$

Answer　　$\frac{1}{6}$ g

**2**　1 kg of potato contains $\frac{5}{6}$ L of water.　How many L of water is contained in $\frac{3}{10}$ kg of potatoes?

Complete the number line diagrams and tables.

**Math sentence**

$$\frac{5}{6} \times \frac{3}{10} = \frac{5 \times 3}{6 \times 10} = \frac{1}{4}$$

Answer　　$\frac{1}{4}$ L

**3**　A machine can run for $\frac{14}{15}$ of an hour on 1 L of oil.　How long can the machine run on $\frac{5}{7}$ L of oil?
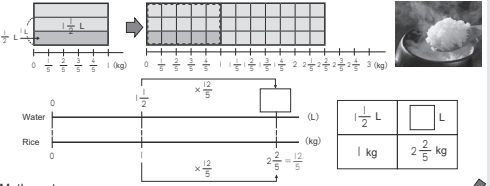
**Math sentence**

$$\frac{14}{15} \times \frac{5}{7} = \frac{14 \times 5}{15 \times 7}$$
$$= \frac{14 \times 5}{15 \times 7} = \frac{2}{3}$$

Answer　　$\frac{2}{3}$ of an hour

26　　　27

8

## 4 - 3  How to Multiply by Fractions
### Multiplying Two Mixed Numbers

**Example** $1\frac{1}{2}$ L of water is needed to cook $1$ kg of rice. How much water is needed to cook $2\frac{2}{5}$ kg of rice?



**Math sentence**

$$1\frac{1}{2} \times 2\frac{2}{5} = \frac{3}{2} \times \frac{12}{5} = \frac{3 \times \overset{6}{\cancel{12}}}{\cancel{2} \times 5} = \frac{18}{5} \text{ or } 3\frac{3}{5}$$
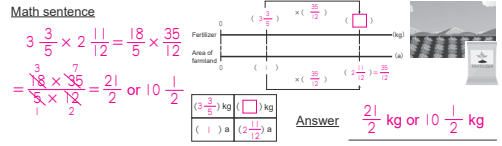
**Answer** $\frac{18}{5}$ L or $3\frac{3}{5}$ L

**Multiply two mixed numbers:**（a mixed number）×（a mixed number）
Change both mixed numbers to improper fractions. Multiply both numerators to get the answer's numerator. Multiply both denominators to get the answer's denominator.

**1** My motorcycle can run $3\frac{3}{4}$ km on $1$ L of gasoline. How many km can it go when it has $3\frac{5}{9}$ L of gasoline?
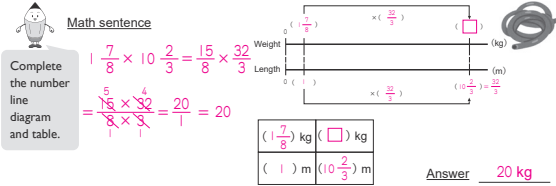
Complete the number line diagrams and tables.
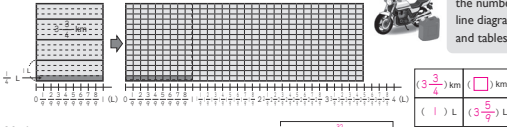
**Math sentence**

$$3\frac{3}{4} \times 3\frac{5}{9} = \frac{15}{4} \times \frac{32}{9}$$
$$= \frac{\overset{5}{\cancel{15}} \times \overset{8}{\cancel{32}}}{\cancel{4} \times \cancel{9}} = \frac{40}{3} \text{ or } 13\frac{1}{3}$$

**Answer** $\frac{40}{3}$ km or $13\frac{1}{3}$ km

**2** I have $2\frac{11}{12}$ a of farmland. I need $3\frac{3}{5}$ kg of fertilizer per a of the farmland. How many kg of fertilizer do I need to fertilize all of my farmland?

Math sentence

$$3\frac{3}{5} \times 2\frac{11}{12} = \frac{18}{5} \times \frac{35}{12}$$
$$= \frac{\overset{3}{\cancel{18}} \times \overset{7}{\cancel{35}}}{\cancel{5} \times \underset{2}{\cancel{12}}} = \frac{21}{2} \text{ or } 10\frac{1}{2}$$

**Answer** $\frac{21}{2}$ kg or $10\frac{1}{2}$ kg

**3** A $1$ m hose weighs $1\frac{7}{8}$ kg. How much does a $10\frac{2}{3}$ m hose weigh?

Complete the number line diagram and table.

Math sentence

$$1\frac{7}{8} \times 10\frac{2}{3} = \frac{15}{8} \times \frac{32}{3}$$
$$= \frac{\overset{5}{\cancel{15}} \times \overset{4}{\cancel{32}}}{\cancel{8} \times \cancel{3}} = \frac{20}{1} = 20$$
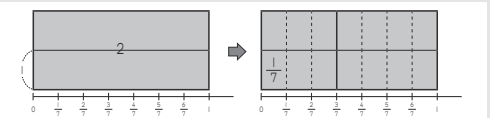
**Answer** 20 kg

**4** Calculate the following multiplication problems. Simplify the answers when possible. Convert improper fractions to mixed numbers.

**1** $1\frac{3}{4} \times 1\frac{1}{5} = \frac{7}{4} \times \frac{6}{5} = \frac{7 \times \overset{3}{\cancel{6}}}{\cancel{4} \times 5} = \frac{21}{10}$ or $2\frac{1}{10}$

**2** $1\frac{5}{6} \times 2\frac{1}{4} = \frac{11}{6} \times \frac{9}{4} = \frac{11 \times \overset{3}{\cancel{9}}}{\cancel{6} \times 4} = \frac{33}{8}$ or $4\frac{1}{8}$

**3** $4\frac{2}{3} \times 1\frac{6}{7} = \frac{14}{3} \times \frac{13}{7} = \frac{\overset{2}{\cancel{14}} \times 13}{3 \times \cancel{7}} = \frac{26}{3} = 8\frac{2}{3}$

**4** $2\frac{1}{3} \times 1\frac{2}{5} = \frac{7}{3} \times \frac{7}{5} = \frac{7 \times 7}{3 \times 5} = \frac{49}{15} = 3\frac{4}{15}$

**5** $2\frac{1}{4} \times 3\frac{1}{6} = \frac{9}{4} \times \frac{19}{6} = \frac{\overset{3}{\cancel{9}} \times 19}{4 \times \cancel{6}} = \frac{57}{8} = 7\frac{1}{8}$

**6** $2\frac{5}{8} \times 2\frac{2}{9} = \frac{21}{8} \times \frac{20}{9} = \frac{\overset{7}{\cancel{21}} \times \overset{5}{\cancel{20}}}{\cancel{8} \times \cancel{9}} = \frac{35}{6} = 5\frac{5}{6}$

**7** $1\frac{3}{7} \times 1\frac{13}{15} = \frac{10}{7} \times \frac{28}{15} = \frac{\overset{2}{\cancel{10}} \times \overset{4}{\cancel{28}}}{\cancel{7} \times \cancel{15}} = \frac{8}{3} = 2\frac{2}{3}$

Improper fractions are convenient for calculations. On the other hand, mixed numbers make it easy to imagine the size of numbers.

## 4 - 4  How to Multiply by Fractions
### Multiplying Whole Numbers by Fractions

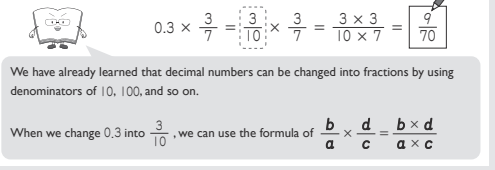**Example** Calculate $2 \times \frac{3}{7}$



$$2 \times \frac{3}{7} = \frac{2}{1} \times \frac{3}{7} = \frac{2 \times 3}{1 \times 7} = \frac{6}{7}$$

When we consider $2$ as $\frac{2}{1}$, we can use the formula of $\frac{b}{a} \times \frac{d}{c} = \frac{b \times d}{a \times c}$

If we consider it as $\frac{3}{7} \times 2$, we can use the following rule, too.
■ × ▲ = ●×▲

Calculate the following multiplication problems. Simplify the answers when possible. Leave the answers as improper fractions.

**1** $5 \times \frac{3}{8} = \frac{5}{1} \times \frac{3}{8} = \frac{5 \times 3}{1 \times 8} = \frac{15}{8}$

**2** $3 \times \frac{3}{4} = \frac{3}{1} \times \frac{3}{4} = \frac{3 \times 3}{1 \times 4} = \frac{9}{4}$

**3** $8 \times \frac{5}{7} = \frac{8}{1} \times \frac{5}{7} = \frac{8 \times 5}{1 \times 7} = \frac{40}{7}$

**4** $6 \times \frac{2}{9} = \frac{6}{1} \times \frac{2}{9} = \frac{\overset{2}{\cancel{6}} \times 2}{1 \times \cancel{9}} = \frac{4}{3}$

**5** $3 \times \frac{7}{6} = \frac{3}{1} \times \frac{7}{6} = \frac{\overset{1}{\cancel{3}} \times 7}{1 \times \underset{2}{\cancel{6}}} = \frac{7}{2}$

**6** $12 \times \frac{4}{3} = \frac{12}{1} \times \frac{4}{3} = \frac{\overset{4}{\cancel{12}} \times 4}{1 \times \cancel{3}} = \frac{16}{1} = 16$

**7** $\frac{5}{4} \times 6 = \frac{5}{4} \times \frac{6}{1} = \frac{5 \times \overset{3}{\cancel{6}}}{\cancel{4} \times 1} = \frac{15}{2}$

**8** $\frac{2}{9} \times 15 = \frac{2}{9} \times \frac{15}{1} = \frac{2 \times \overset{5}{\cancel{15}}}{\cancel{9} \times 1} = \frac{10}{3}$

**9** $3 \times 1\frac{5}{6} = \frac{3}{1} \times \frac{11}{6} = \frac{\overset{1}{\cancel{3}} \times 11}{1 \times \underset{2}{\cancel{6}}} = \frac{11}{2}$

**10** $4 \times 1\frac{1}{12} = \frac{4}{1} \times \frac{13}{12} = \frac{\overset{1}{\cancel{4}} \times 13}{1 \times \underset{3}{\cancel{12}}} = \frac{13}{3}$

## 4 - 5  How to Multiply by Fractions
### Multiplying Decimal Numbers and Fractions

**Example** Calculate $0.3 \times \frac{3}{7}$

$$0.3 \times \frac{3}{7} = \frac{3}{10} \times \frac{3}{7} = \frac{3 \times 3}{10 \times 7} = \frac{9}{70}$$

We have already learned that decimal numbers can be changed into fractions by using denominators of $10$, $100$, and so on.

When we change $0.3$ into $\frac{3}{10}$, we can use the formula of $\frac{b}{a} \times \frac{d}{c} = \frac{b \times d}{a \times c}$

**1** Calculate the following multiplication problems. Simplify the answers when possible. Leave the answers as improper fractions.

**1** $0.3 \times \frac{1}{7} = \frac{3}{10} \times \frac{1}{7} = \frac{3}{10} \times \frac{1}{7} = \frac{3}{70}$

**2** $0.9 \times \frac{5}{6} = \frac{9}{10} \times \frac{5}{6} = \frac{\overset{3}{\cancel{9}} \times \overset{1}{\cancel{5}}}{\underset{2}{\cancel{10}} \times \underset{2}{\cancel{6}}} = \frac{3}{4}$

**3** $0.8 \times \frac{2}{3} = \frac{8}{10} \times \frac{2}{3} = \frac{8 \times 2}{\underset{5}{\cancel{10}} \times 3} = \frac{8}{15}$

**4** $1.6 \times \frac{1}{2} = \frac{16}{10} \times \frac{1}{2} = \frac{\overset{4}{\cancel{16}} \times 1}{\cancel{10} \times \cancel{2}} = \frac{4}{5}$

**5** $0.9 \times 2\frac{2}{3} = \frac{9}{10} \times \frac{8}{3} = \frac{\overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}}}{\underset{5}{\cancel{10}} \times \cancel{3}} = \frac{12}{5}$

**6** $1.2 \times 1\frac{1}{6} = \frac{12}{10} \times \frac{7}{6} = \frac{\overset{2}{\cancel{12}} \times 7}{\underset{5}{\cancel{10}} \times \cancel{6}} = \frac{7}{5}$

**7** $1\frac{1}{5} \times 1.5 = \frac{6}{5} \times \frac{15}{10} = \frac{\overset{3}{\cancel{6}} \times \overset{3}{\cancel{15}}}{\cancel{5} \times \underset{5}{\cancel{10}}} = \frac{9}{5}$

**8** $3\frac{1}{8} \times 0.8 = \frac{25}{8} \times \frac{8}{10} = \frac{25 \times \overset{1}{\cancel{8}}}{\cancel{8} \times \underset{2}{\cancel{10}}} = \frac{5}{2}$

**2** Explain the following calculation errors and calculate them correctly.

**1** $2.5 \times \frac{3}{5} = \frac{25}{1} \times \frac{3}{5} = \frac{25 \times 3}{1 \times 5} = \frac{15}{1} = 15$

Misconversion from decimal number to fraction.
$$= \frac{25}{10} \times \frac{3}{5} = \frac{\overset{5}{\cancel{25}} \times 3}{\underset{2}{\cancel{10}} \times \cancel{5}} = \frac{3}{2}$$

**2** $0.8 \times \frac{3}{4} = \frac{8}{10} \times \frac{3}{4} = \frac{32 \times 30}{40} = \frac{24}{1} = 24$

A way of multiplication of fractions is incorrect.
$$= \frac{\overset{4}{\cancel{8}}}{\underset{5}{\cancel{10}}} \times \frac{3}{\cancel{4}} = \frac{3}{5}$$

## 4 - 6　How to Multiply by Fractions
### Multiplying More than Two Numbers

**Example 1**　Calculate $\frac{3}{4} \times \frac{2}{5} \times \frac{1}{3}$

$$\frac{3}{4} \times \frac{2}{5} \times \frac{1}{3} = \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}} \times 1}{4 \times 5 \times \underset{1}{\cancel{3}}} = \frac{1}{10}$$

This type of multiplication is an application of the following formula: $\frac{b}{a} \times \frac{d}{c} = \frac{b \times d}{a \times c}$

When we multiply several fractions, multiply all numerators together to equal answer's numerator. Multiply all denominators together to equal the answer's denominators.

**1**　Calculate the following multiplication problems. Simplify the answers when possible. Leave the answers as improper fractions.

❶ $\frac{4}{5} \times \frac{5}{6} \times \frac{2}{3} = \frac{4 \times \cancel{5} \times 2}{5 \times \cancel{6} \times 3} = \frac{4}{9}$　❷ $\frac{3}{8} \times \frac{3}{4} \times \frac{4}{9} = \frac{\cancel{3} \times \cancel{3} \times \cancel{4}}{8 \times 4 \times \cancel{9}} = \frac{1}{8}$

❸ $\frac{7}{8} \times \frac{3}{14} \times \frac{1}{3} = \frac{\cancel{7} \times \cancel{3} \times 1}{8 \times \cancel{14} \times \cancel{3}} = \frac{1}{16}$　❹ $\frac{6}{7} \times \frac{5}{8} \times \frac{7}{15} = \frac{\cancel{6} \times \cancel{5} \times \cancel{7}}{\cancel{7} \times 8 \times \cancel{15}} = \frac{1}{4}$

**Example 2**　Calculate $0.2 \times \frac{1}{4} \times 3$

Decimal numbers and whole numbers can be changed into fractions. Then we can calculate this problem.

$$0.2 \times \frac{1}{4} \times 3 = \frac{\overset{1}{\cancel{2}} \times 1 \times 3}{\underset{5}{\cancel{10}} \times 4 \times 1} = \frac{3}{20}$$

Convert decimal numbers and whole numbers into fractions and multiply. Remember, 3 can be written as $\frac{3}{1}$ . $\frac{b}{a} \times \frac{d}{c} = \frac{b \times d}{a \times c}$

**2**　Calculate the following multiplication problems. Simplify the answers. Leave the answers as improper fractions.

❶ $1.2 \times \frac{2}{9} \times 6 = \frac{\cancel{12} \times \cancel{2} \times \cancel{6}}{\cancel{10} \times \cancel{9} \times 1} = \frac{8}{5}$　❷ $0.4 \times \frac{1}{6} \times 3 = \frac{\cancel{4} \times 1 \times \cancel{3}}{\cancel{10} \times \cancel{6} \times 1} = \frac{1}{5}$

❸ $0.3 \times \frac{2}{9} \times 4 = \frac{\cancel{3} \times \cancel{2} \times 4}{\cancel{10} \times \cancel{9} \times 1} = \frac{4}{15}$　❹ $8 \times 2\frac{3}{16} \times 0.6 = \frac{\cancel{8} \times \cancel{36} \times \cancel{6}}{1 \times \cancel{16} \times \cancel{10}} = \frac{21}{2}$

## 4 - 7　How to Multiply by Fractions
### Using the Properties of Operations (1)

**Instruction**　For multiplication, there are four important properties of operations:

① $a \times b = b \times a$
② $(a \times b) \times c = a \times (b \times c)$
③ $(a + b) \times c = a \times c + b \times c$
④ $(a - b) \times c = a \times c - b \times c$

**Example**　By using properties of operations, simplify the math sentence and calculate.

$$\left(\frac{7}{8} \times \frac{5}{6}\right) \times \frac{6}{5}$$

**Student A**
$$\left(\frac{7}{8} \times \frac{5}{6}\right) \times \frac{6}{5} = \frac{7 \times 5}{8 \times 6} \times \frac{6}{5}$$
$$= \frac{35}{48} \times \frac{6}{5} = \frac{\overset{7}{\cancel{35}} \times \cancel{6}}{\underset{8}{\cancel{48}} \times \cancel{5}} = \boxed{\frac{7}{8}}$$

He calculated it normally in order from the beginning of the math sentence.

**Student B**
$$\left(\frac{7}{8} \times \frac{5}{6}\right) \times \frac{6}{5} = \frac{7}{8} \times \left(\frac{5}{6} \times \frac{6}{5}\right)$$
$$= \frac{7}{8} \times \frac{\cancel{5} \times \cancel{6}}{\cancel{6} \times \cancel{5}} = \boxed{\frac{7}{8}}$$

She calculated it by **using the properties of operations, especially the above** ② .

The method that Student B used is easier to calculate because there are not large numbers like "35" and "45" in the math sentence.

Simplify the following calculations by using the properties of operations and calculate. Simplify the answer.

Think about which fractions should be multiplied first.

❶ $\left(\frac{1}{3} \times \frac{5}{7}\right) \times \frac{7}{5} = \frac{1}{3} \times \left(\frac{5}{7} \times \frac{7}{5}\right) = \frac{1}{3}$　❷ $\left(\frac{7}{10} \times \frac{3}{4}\right) \times \frac{4}{3} = \frac{7}{10} \times \left(\frac{3}{4} \times \frac{4}{3}\right) = \frac{7}{10}$

❸ $\frac{11}{12} \times \left(\frac{1}{5} \times \frac{12}{11}\right) = \left(\frac{11}{12} \times \frac{12}{11}\right) \times \frac{1}{5} = \frac{1}{5}$　❹ $\frac{10}{13} \times \left(\frac{2}{7} \times \frac{13}{10}\right) = \left(\frac{10}{13} \times \frac{13}{10}\right) \times \frac{2}{7} = \frac{2}{7}$

❹ $\frac{7}{9} \times \left(\frac{3}{4} \times \frac{9}{14}\right) = \left(\frac{7}{9} \times \frac{9}{14}\right) \times \frac{3}{4} = \frac{3}{8}$　❺ $\frac{5}{7} \times \left(\frac{9}{11} \times \frac{7}{10}\right) = \left(\frac{5}{7} \times \frac{7}{10}\right) \times \frac{9}{11} = \frac{9}{22}$

## 4 - 8　How to Multiply by Fractions
### Using the Properties of Operations (2)

**Example**　By using the properties of operations, simplify the math sentence and calculate.

$$\frac{3}{4} \times 5 + \frac{3}{4} \times 7$$

**Student A**
$$\frac{3}{4} \times 5 + \frac{3}{4} \times 7 = \frac{3}{4} \times (5 + 7)$$
$$= \frac{3}{4} \times 12 = \frac{3 \times \cancel{12}}{\cancel{4}} = \frac{9}{1} = \boxed{9}$$

He calculated it by **using the properties of operations, especially** ③ **in the previous page.**

**Student B**
$$\frac{3}{4} \times 5 + \frac{3}{4} \times 7 = \frac{3 \times 5}{4} + \frac{3 \times 7}{4}$$
$$= \frac{15}{4} + \frac{21}{4} = \frac{36}{4} = \frac{9}{1} = \boxed{9}$$

She calculated it normally according to the given math sentence.

The method that Student A used is easier to calculate because there are not large numbers like "21" and "39" in the math sentence.

**1**　Simplify the following calculations by using the properties of operations and calculate. Simplify the answer and leave them as improper fractions.

To solve Problems (3),(4),(7) and (8), ④ of the properties of operations can be used.

❶ $\frac{7}{6} \times 7 + \frac{7}{6} \times 11 = \frac{7}{6} \times (7 + 11) = \frac{7}{6} \times \cancel{18} = 21$

❷ $\frac{3}{5} \times 7 + \frac{3}{5} \times 8 = \frac{3}{5} \times (7 + 8) = \frac{3}{5} \times \cancel{15} = 9$

❸ $\frac{4}{9} \times 16 - \frac{4}{9} \times 7 = \frac{4}{9} \times (16 - 7) = \frac{4}{9} \times \cancel{9} = 4$

❹ $\frac{5}{7} \times 29 - \frac{5}{7} \times 15 = \frac{5}{7} \times (29 - 15) = \frac{5}{7} \times \cancel{14} = 10$

❺ $\frac{10}{9} \times \frac{2}{5} + \frac{10}{9} \times \frac{1}{5} = \frac{10}{9} \times \left(\frac{2}{5} + \frac{1}{5}\right) = \frac{10}{9} \times \frac{\cancel{3}}{\cancel{5}} = \frac{2}{3}$

❻ $\frac{15}{4} \times \frac{5}{7} + \frac{15}{4} \times \frac{3}{7} = \frac{15}{4} \times \left(\frac{5}{7} + \frac{3}{7}\right) = \frac{15}{4} \times \frac{8}{7} = \frac{30}{7}$

❼ $\frac{18}{13} \times \frac{20}{23} - \frac{18}{13} \times \frac{7}{23} = \frac{18}{13} \times \left(\frac{20}{23} - \frac{7}{23}\right) = \frac{18}{13} \times \frac{13}{23} = \frac{18}{23}$

❽ $\frac{24}{5} \times \frac{9}{11} - \frac{24}{5} \times \frac{4}{11} = \frac{24}{5} \times \left(\frac{9}{11} - \frac{4}{11}\right) = \frac{24}{5} \times \frac{5}{11} = \frac{24}{11}$

**2**　What properties of operations ( from ① to ④ on page 33 ) can be used in order to make the following problems easier to do? Write the numbers ( ① to ④ ) in the ☐ and calculate them by using that property of operations. However, there are some in which the properties of operations cannot be used. In this case, write X in the ☐ .

(a)　$\frac{10}{11} \times \frac{7}{8} \times \frac{8}{7} = \frac{10}{11} \times \left(\frac{7}{8} \times \frac{8}{7}\right) = \frac{10}{11} \times \frac{\cancel{7} \times \cancel{8}}{\cancel{8} \times \cancel{7}} = \frac{10}{11}$　☐②

(b)　$\frac{12}{13} \times \frac{15}{7} - \frac{5}{13} \times \frac{15}{7} = \left(\frac{12}{13} - \frac{5}{13}\right) \times \frac{15}{7} = \frac{7}{13} \times \frac{15}{\cancel{7}} = \frac{15}{13}$　☐④

(c)　$13 \times \frac{9}{4} + 11 \times \frac{9}{4} = (13 + 11) \times \frac{9}{4} = \overset{6}{\cancel{24}} \times \frac{9}{\cancel{4}} = 54$　☐③

(d)　$\frac{2}{3} \times \frac{4}{5} - \frac{1}{2} \times \frac{3}{5}$　☐X

(e)　$\frac{2}{3} \times \frac{7}{12} \times \frac{12}{7} = \frac{2}{3} \times \left(\frac{7}{12} \times \frac{12}{7}\right) = \frac{2}{3} \times \frac{\cancel{7} \times \cancel{12}}{\cancel{12} \times \cancel{7}} = \frac{2}{3}$　☐②

(f)　$\frac{9}{5} \times \frac{9}{8} + \frac{7}{5} \times \frac{9}{8} = \left(\frac{9}{5} + \frac{7}{5}\right) \times \frac{9}{8} = \frac{\cancel{16}}{5} \times \frac{9}{\cancel{8}} = \frac{18}{5}$　☐③

(g)　$\frac{11}{13} \times 25 - \frac{11}{13} \times 12 = \frac{11}{13} \times (25 - 12) = \frac{11}{\cancel{13}} \times \cancel{13} = 11$　☐④

(h)　$\frac{1}{6} \times \frac{5}{7} \times \frac{5}{8}$　☐X

**Let's Try!**
This property of operations: "$(a+b) \times c = a \times c + b \times c$" is easy to understand if we think about finding the area of a rectangle showed on the right;

In the future, we will also learn "$(a+b) \times (c+d)$." Let's think about what the math sentence looks like when we expand this formula.

$$(a+b) \times (c+d) = a \times c + a \times d + b \times c + b \times d$$

| | $c$ cm |
|---|---|
| $a$ cm | $a \times c$ (cm²) |
| $b$ cm | $b \times c$ (cm²) |

| | $c$ cm | $d$ cm |
|---|---|---|
| $a$ cm | $a \times c$ (cm²) | $a \times d$ (cm²) |
| $b$ cm | $b \times c$ (cm²) | $b \times d$ (cm²) |

## 4 − 9　How to Multiply by Fractions
### Reciprocal

**Instruction**　The reciprocal of a number is 1 divided by the number.

Think of a reciprocal as the numerator and denominator switching numbers.

$$\frac{b}{a} \dashrightarrow \frac{a}{b}$$

For example,
The reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$ .

The reciprocal of $4$ which can be written as $\frac{4}{1}$ is $\frac{1}{4}$ .

$$\frac{5}{6} \times \frac{6}{5} = 1$$

When multiplying a number by its reciprocal, the product is always 1.

$$\frac{1}{4} \times 4 = 1$$

**Example**　Find the reciprocals of each of the following numbers.

① $\frac{5}{7}$ 　$\boxed{\frac{7}{5}}$ 　② $0.7 = \frac{7}{10}$ 　$\boxed{\frac{10}{7}}$

To find the reciprocal of a decimal, convert it to a fraction first. In the above case, $0.7$ is converted to $\frac{7}{10}$ first.

Find the reciprocals of each of the following numbers.

① $\frac{2}{3}$ 　$\boxed{\frac{3}{2}}$ 　② $\frac{3}{5}$ 　$\boxed{\frac{5}{3}}$ 　③ $\frac{1}{7}$ 　$\boxed{7}$

④ $\frac{3}{10}$ 　$\boxed{\frac{10}{3}}$ 　⑤ $8$ 　$\boxed{\frac{1}{8}}$ 　⑥ $12$ 　$\boxed{\frac{1}{12}}$

⑦ $56$ 　$\boxed{\frac{1}{56}}$ 　⑧ $0.3 = \frac{3}{10}$ 　$\boxed{\frac{10}{3}}$ 　⑨ $0.57 = \frac{57}{100}$ 　$\boxed{\frac{100}{57}}$

⑩ $2.11 = \frac{211}{100}$ 　$\boxed{\frac{100}{211}}$

Remember, to what fractions are $0.3, 0.57$ and $2.11$ converted?

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## 4 − 10　How to Multiply by Fractions
### Various Multiplication Problems

**Example**　A faucet gives $\frac{2}{3}$ m³ of water in 1 hour. How much water will the faucet give in $\frac{1}{6}$ of an hour?
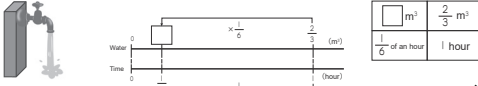
**Math sentence**
$$\frac{2}{3} \times \frac{1}{6} = \frac{2 \times 1}{3 \times 6} = \frac{1}{9}$$

Answer　$\frac{1}{9}$ m³

**1**　1 m of metal pipe weighs $\frac{3}{5}$ kg. How much does a $\frac{5}{6}$ m pipe weigh?

**Math sentence**
$$\frac{3}{5} \times \frac{5}{6} = \frac{3 \times 5}{5 \times 6} = \frac{1}{2}$$

Complete the number line diagrams and tables.

Answer　$\frac{1}{2}$ kg

**2**　One m of cloth costs $240$ zeds*. How much does $2\frac{1}{4}$ m of this cloth cost?　(* "zed(s)" is the fictional currency unit.)

**Math sentence**
$$240 \times 2\frac{1}{4} = \frac{240}{1} \times \frac{9}{4}$$
$$= \frac{\overset{60}{240} \times 9}{1 \times \underset{1}{4}} = \frac{540}{1} = 540$$

Answer　$540$ zeds

**3**　An outdoor pump gives $\frac{17}{4}$ L of water per minute. A house faucet gives $\frac{11}{4}$ L of water per minute. If the pump and faucet are running for $10$ minutes, how many L of water will we have in total?

**Math sentence**
$$\left(\frac{17}{4} + \frac{11}{4}\right) \times 10 = \frac{\overset{7}{28}}{4} \times 10 = 70$$

The following math sentence is also fine too.
$$\frac{17}{4} \times 10 + \frac{11}{4} \times 10 = \frac{170}{4} + \frac{110}{4} = \frac{\overset{70}{280}}{1} = 70$$

Answer　$70$ L

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## 4 − 11　How to Multiply by Fractions
### Review

**1**　Calculate the following multiplication problems. Leave the answers as improper fractions.

① $\frac{2}{3} \times \frac{2}{5} = \frac{2 \times 2}{3 \times 5} = \frac{4}{15}$ 　② $\frac{14}{9} \times \frac{12}{7} = \frac{\overset{2}{14} \times \overset{4}{12}}{\underset{3}{9} \times \underset{1}{7}} = \frac{8}{3}$

③ $1\frac{1}{8} \times 1\frac{1}{6} = \frac{9}{8} \times \frac{7}{6} = \frac{\overset{3}{9} \times 7}{8 \times \underset{2}{6}} = \frac{21}{16}$ 　④ $2\frac{3}{4} \times 1\frac{3}{11} = \frac{11}{4} \times \frac{14}{11} = \frac{\overset{1}{11} \times \overset{7}{14}}{\underset{2}{4} \times \underset{1}{11}} = \frac{7}{2}$

⑤ $0.3 \times 2\frac{2}{9} = \frac{3}{10} \times \frac{20}{9} = \frac{\overset{1}{3} \times \overset{2}{20}}{\underset{1}{10} \times \underset{3}{9}} = \frac{2}{3}$ 　⑥ $1\frac{3}{7} \times 2.1 = \frac{10}{7} \times \frac{21}{10} = \frac{\overset{1}{10} \times \overset{3}{21}}{\underset{1}{7} \times \underset{1}{10}} = \frac{3}{1} = 3$

⑦ $\frac{5}{8} \times \frac{7}{10} \times \frac{3}{14} = \frac{\overset{1}{5} \times \overset{1}{7} \times 3}{8 \times \underset{2}{10} \times \underset{2}{14}} = \frac{3}{32}$ 　⑧ $\frac{7}{12} \times \frac{16}{21} \times \frac{9}{10} = \frac{\overset{1}{7} \times \overset{4}{16} \times \overset{3}{9}}{\underset{1}{12} \times \underset{3}{21} \times \underset{5}{10}} = \frac{2}{5}$

⑨ $6 \times 1\frac{1}{8} \times 1.4 = \frac{\overset{3}{6} \times 9 \times \overset{7}{14}}{1 \times \underset{4}{8} \times \underset{5}{10}} = \frac{189}{20}$ 　⑩ $0.4 \times 2\frac{1}{2} \times 1\frac{1}{3} = \frac{\overset{2}{4} \times \overset{1}{5} \times \overset{2}{4}}{\underset{1}{10} \times \underset{1}{2} \times 3} = \frac{4}{3}$

**2**　Simplify the following calculations by using the properties of operations and calculate. Leave the answer as improper fractions.

① $\left(\frac{11}{5} \times \frac{7}{9}\right) \times \frac{12}{7} = \frac{11}{5} \times \left(\frac{7}{9} \times \frac{12}{7}\right) = \frac{44}{15}$ 　② $\frac{9}{7} \times \left(\frac{14}{27} \times \frac{4}{5}\right) = \left(\frac{9}{7} \times \frac{14}{27}\right) \times \frac{4}{5} = \frac{8}{15}$

③ $\frac{3}{4} \times 5 + \frac{3}{4} \times 7 = \frac{3}{4} \times (5 + 7) = 9$ 　④ $\frac{13}{8} \times \frac{3}{7} + \frac{13}{8} \times \frac{5}{7} = \frac{13}{8} \times \left(\frac{3}{7} + \frac{5}{7}\right) = \frac{13}{7}$

⑤ $\frac{3}{7} \times 25 - \frac{3}{7} \times 11 = \frac{3}{7} \times (25 - 11) = 6$ 　⑥ $\frac{5}{13} \times \frac{17}{9} - \frac{5}{13} \times \frac{4}{9} = \frac{5}{13} \times \left(\frac{17}{9} - \frac{4}{9}\right) = \frac{5}{9}$

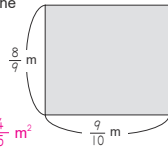**3**　Find the reciprocals of each of the following numbers.

① $\frac{3}{4}$ 　$\boxed{\frac{4}{3}}$ 　② $7$ 　$\boxed{\frac{1}{7}}$ 　③ $5.3$ 　$\boxed{\frac{10}{53}}$ 　④ $0.61$ 　$\boxed{\frac{100}{61}}$

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**4**　Find the following area of a rectangle. Write the math sentence and calculate it.

**Math sentence**　Area of a rectangle = Length × Width
$$\frac{9}{10} \times \frac{8}{9} = \frac{\overset{1}{9} \times \overset{4}{8}}{\underset{5}{10} \times \underset{1}{9}} = \frac{4}{5}$$

Answer　$\frac{4}{5}$ m²

**5**　There is a hose that weighs $\frac{9}{16}$ kg per metre. How many kg does $3\frac{1}{3}$ m of this hose weigh?

Complete the number line diagrams and tables.
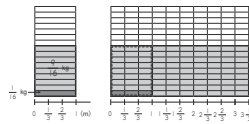
**Math sentence**
$$\frac{9}{16} \times 3\frac{1}{3} = \frac{9}{16} \times \frac{10}{3} = \frac{\overset{3}{9} \times \overset{5}{10}}{\underset{8}{16} \times \underset{1}{3}} = \frac{15}{8} = 1\frac{7}{8}$$

Answer　$\frac{15}{8}$ kg or $1\frac{7}{8}$ kg

**6**　A weaving machine can produce $1\frac{1}{5}$ m of the cloth per minute. Another machine can produce $\frac{2}{5}$ m of the cloth per minute. If we use both machines, how many m of cloth can we make in $60$ minutes?

**Math sentence**
$$\left(1\frac{1}{5} + \frac{2}{5}\right) \times 60 = \left(\frac{6}{5} + \frac{2}{5}\right) \times 60 = \frac{8}{5} \times \overset{12}{60} = 96$$

Answer　$96$ m

**7**　Calculate every math sentence when the ☐ has the number from $2$ to $20$. Then answer the following questions.

$$\frac{11}{3} \times \boxed{\phantom{x}}$$

① List the numbers whose product was a whole number.
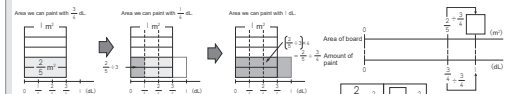
Answer　$3, 6, 9, 12, 15, 18$

② What do the numbers in your answer ① have in common?

Answer　They are all multiples of 3.

39

11

## 5 - 1  How to Divide by Fractions
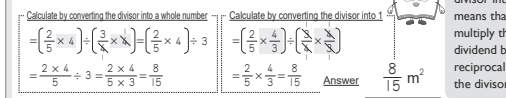### Dividing by Fractions（1）

**Example** $\frac{2}{5}$ m² can be covered with $\frac{3}{4}$ dL of paint. How much area can be covered with 1 dL of paint?



Math sentence

$\frac{2}{5} \div \frac{3}{4}$

How can we calculate this? We have two ways.

Calculating by converting the divisor into 1 means that we multiply the dividend by the reciprocal of the divisor.

Calculate by converting the divisor into a whole number

$= \left(\frac{2}{5} \times 4\right) \div \left(\frac{3}{4} \times 4\right) = \left(\frac{2}{5} \times 4\right) \div 3$

$= \frac{2 \times 4}{5} \div 3 = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$

Calculate by converting the divisor into 1

$= \left(\frac{2}{5} \times \frac{4}{3}\right) \div \left(\frac{3}{4} \times \frac{4}{3}\right)$

$= \frac{2}{5} \times \frac{4}{3} = \frac{8}{15}$

Answer $\frac{8}{15}$ m²

To divide by a fraction, we can multiply the dividend by the reciprocal of the divisor.

$$\frac{b}{a} \div \frac{d}{c} = \frac{b}{a} \times \frac{c}{d} = \frac{b \times c}{a \times d}$$

**1** $\frac{1}{3}$ m of lumber weighs $\frac{4}{5}$ kg. How much does 1 m of lumber weigh?



Complete the number line diagrams and tables.
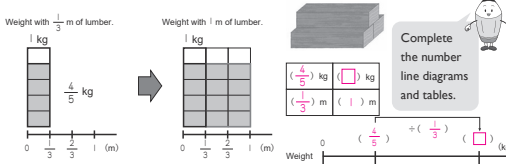
Math sentence

$\frac{4}{5} \div \frac{1}{3} = \frac{4}{5} \times \frac{3}{1} = \frac{4 \times 3}{5 \times 1} = \frac{12}{5}$ or $2\frac{2}{5}$

Answer $\frac{12}{5}$ kg or $2\frac{2}{5}$ kg

**2** There is a $\frac{3}{4}$ L liquid that weighs $\frac{5}{7}$ kg. How much is 1 L of this liquid weighs?



Math sentence

$\frac{5}{7} \div \frac{3}{4} = \frac{5}{7} \times \frac{4}{3}$

$= \frac{5 \times 4}{7 \times 3} = \frac{20}{21}$

Answer $\frac{20}{21}$ kg

**3** A printer takes $\frac{1}{6}$ of an hour to print $\frac{2}{7}$ m² of paper. How many m² of Paper can be printed in 1 hour?

Complete the number line diagrams and tables.
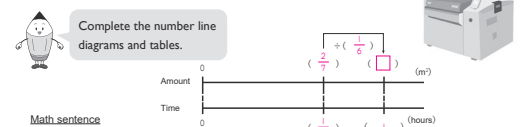


Math sentence

$\frac{2}{7} \div \frac{1}{6} = \frac{2}{7} \times \frac{6}{1}$

$= \frac{2 \times 6}{7 \times 1} = \frac{12}{7}$ or $1\frac{5}{7}$

Answer $\frac{12}{7}$ m² or $1\frac{5}{7}$ m²

**4** Calculate the following division problems. Simplify the answers when possible. Leave the answers as improper fractions.

① $\frac{3}{8} \div \frac{2}{7} = \frac{3}{8} \times \frac{7}{2} = \frac{21}{16}$

② $\frac{2}{5} \div \frac{1}{4} = \frac{2}{5} \times \frac{4}{1} = \frac{8}{5}$

③ $\frac{5}{6} \div \frac{3}{7} = \frac{5}{6} \times \frac{7}{3} = \frac{35}{18}$

④ $\frac{1}{3} \div \frac{2}{5} = \frac{1}{3} \times \frac{5}{2} = \frac{5}{6}$

⑤ $\frac{4}{3} \div \frac{5}{2} = \frac{4}{3} \times \frac{2}{5} = \frac{8}{15}$

⑥ $\frac{4}{5} \div \frac{9}{8} = \frac{4}{5} \times \frac{8}{9} = \frac{32}{45}$

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## 5 - 2  How to Divide by Fractions
### Dividing by Fractions（2）

**Example 1** A rectangle with an area of $\frac{15}{16}$ m² has a length of $\frac{5}{4}$ m. What is the width of the rectangle?



Because the area is $\frac{15}{16}$ m², there should be 15 squares whose area is $\frac{1}{16}$ m².
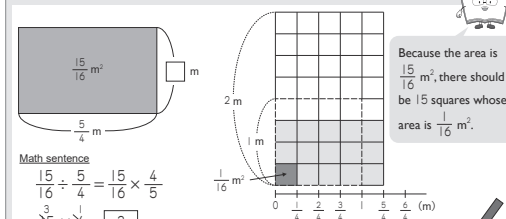
Math sentence

$\frac{15}{16} \div \frac{5}{4} = \frac{15}{16} \times \frac{4}{5}$

$= \frac{15 \times 4}{16 \times 5} = \frac{3}{4}$

Answer $\frac{3}{4}$ m

Rewrite the division math sentence as multiplying by the divisor's reciprocal. Simplify by finding the greatest common factor before multiplying.

**1** A rectangle with an area of $\frac{18}{25}$ m² has a length of $\frac{6}{5}$ m. What is the width of the rectangle?
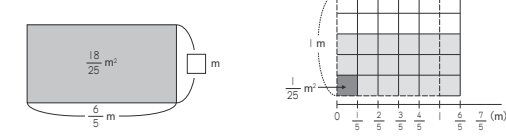


Math sentence

$\frac{18}{25} \div \frac{6}{5} = \frac{18}{25} \times \frac{5}{6} = \frac{18 \times 5}{25 \times 6} = \frac{3}{5}$

Answer $\frac{3}{5}$ m

**Example 2** I have $\frac{5}{6}$ L of milk. If I drink $\frac{5}{24}$ L of milk every day, how many days worth of milk do I have?
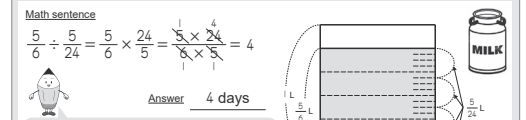
Math sentence

$\frac{5}{6} \div \frac{5}{24} = \frac{5}{6} \times \frac{24}{5} = \frac{5 \times 24}{6 \times 5} = 4$

Answer 4 days

If this problem uses the whole numbers, such as " I have 10 L of milk and I drink 2 L every day," we can make a math sentence easily.



**2** Our office has $\frac{25}{3}$ L of gasoline. If we use $\frac{5}{6}$ L of gasoline every day, how many days worth of gasoline our office have?

Math sentence

$\frac{25}{3} \div \frac{5}{6} = \frac{25}{3} \times \frac{6}{5} = \frac{25 \times 6}{3 \times 5} = \frac{10}{1} = 10$

Answer 10 days

**3** Calculate the following division problems. Simplify the answers when possible. Leave the answers as improper fractions.

① $\frac{6}{7} \div \frac{3}{5} = \frac{6}{7} \times \frac{5}{3} = \frac{10}{7}$

② $\frac{9}{10} \div \frac{4}{5} = \frac{9}{10} \times \frac{5}{4} = \frac{9}{8}$

③ $\frac{12}{5} \div \frac{8}{15} = \frac{12}{5} \times \frac{15}{8} = \frac{9}{2}$

④ $\frac{7}{6} \div \frac{21}{8} = \frac{7}{6} \times \frac{8}{21} = \frac{4}{9}$

⑤ $\frac{3}{8} \div \frac{9}{14} = \frac{3}{8} \times \frac{14}{9} = \frac{7}{12}$

⑥ $\frac{2}{15} \div \frac{6}{5} = \frac{2}{15} \times \frac{5}{6} = \frac{1}{9}$

⑦ $\frac{4}{15} \div \frac{2}{9} = \frac{4}{15} \times \frac{9}{2} = \frac{6}{5}$

⑧ $\frac{14}{15} \div \frac{7}{12} = \frac{14}{15} \times \frac{12}{7} = \frac{8}{5}$

**12**

## 5 - 3　How to Divide by Fractions
### Dividing Mixed Numbers by Mixed Numbers

**Example**　$1\frac{1}{3}$ kg of lemon are needed to make $3\frac{5}{9}$ L of lemonade. How many kg of lemons are needed to make we need to make 1 L of lemonade?

Amout of lemon that is needed to make $3\frac{5}{9}$ L of lemonade.　Amout of lemon that is needed to make 1 L of lemonade.

$1\frac{1}{3}$ kg
1 kg
$\frac{1}{96}$ kg

**Math sentence**

$$1\frac{1}{3} \div 3\frac{5}{9} = \frac{4}{3} \div \frac{32}{9} = \frac{4}{3} \times \frac{9}{32}$$

$$= \frac{\overset{1}{\cancel{4}} \times \overset{3}{\cancel{9}}}{3 \times \cancel{32}} = \frac{3}{8}$$

**Answer** $\frac{3}{8}$ kg

**Divide a mixed number by a mixed number: (a mixed number) ÷ (a mixed number)**
Change both mixed numbers to improper fractions then divide.

**1**　$2\frac{3}{4}$ L of milk is needed to bake a $3\frac{1}{7}$ kg cake. How much milk is needed to bake a 1 kg cake?

Amout of milk that is needed to make $3\frac{1}{7}$ kg of cake.　Amout of milk that is needed to make 1 kg of cake.

$2\frac{3}{4}$ L
$\frac{1}{88}$ L

Complete the number line diagrams and tables.

**Math sentence**

$$2\frac{3}{4} \div 3\frac{1}{7} = \frac{11}{4} \div \frac{22}{7}$$

$$= \frac{11}{4} \times \frac{7}{22} = \frac{\cancel{11} \times 7}{4 \times \cancel{22}} = \frac{7}{8}$$

**Answer** $\frac{7}{8}$ L

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**2**　There is a plywood that is $4\frac{1}{6}$ m² and $5\frac{5}{8}$ kg. If we cut out a 1 m² from this plywood, how much does it weigh?

**Math sentence**

$$5\frac{5}{8} \div 4\frac{1}{6} = \frac{45}{8} \div \frac{25}{6}$$

$$= \frac{45}{8} \times \frac{6}{25} = \frac{\overset{9}{\cancel{45}} \times \overset{3}{\cancel{6}}}{\underset{4}{\cancel{8}} \times \underset{5}{\cancel{25}}} = \frac{27}{20} = 1\frac{7}{20}$$

**Answer** $\frac{27}{20}$ kg or $1\frac{7}{20}$ kg

**3**　A $8\frac{1}{3}$ m metal pipe weighs $11\frac{1}{9}$ kg. How much does 1 m of this pipe weigh?

**Math sentence**

$$11\frac{1}{9} \div 8\frac{1}{3} = \frac{100}{9} \div \frac{25}{3} = \frac{100}{9} \times \frac{3}{25}$$

$$= \frac{\overset{4}{\cancel{100}} \times \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \times \underset{1}{\cancel{25}}} = \frac{4}{3} \text{ or } 1\frac{1}{3}$$

Complete the number line diagram and table.

**Answer** $\frac{4}{3}$ kg or $1\frac{1}{3}$ kg

**4**　Calculate the following division problems. Simplify the answers. If the answer is an improper fraction, change it to a mixed number.

① $1\frac{3}{5} \div 2\frac{1}{3} = \frac{8}{5} \div \frac{7}{3} = \frac{8}{5} \times \frac{3}{7} = \frac{24}{35}$

② $1\frac{4}{5} \div 2\frac{1}{2} = \frac{9}{5} \div \frac{5}{2} = \frac{9}{5} \times \frac{2}{5} = \frac{18}{25}$

③ $3\frac{1}{3} \div 1\frac{7}{9} = \frac{10}{3} \div \frac{16}{9} = \frac{\overset{5}{\cancel{10}}}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{\underset{8}{\cancel{16}}} = \frac{15}{8} = 1\frac{7}{8}$

④ $1\frac{5}{9} \div 1\frac{1}{6} = \frac{14}{9} \div \frac{7}{6} = \frac{\overset{2}{\cancel{14}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{2}{\cancel{6}}}{\cancel{7}} = \frac{4}{3} = 1\frac{1}{3}$

⑤ $4\frac{1}{2} \div 3\frac{3}{4} = \frac{9}{2} \div \frac{15}{4} = \frac{\overset{3}{\cancel{9}}}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{\underset{5}{\cancel{15}}} = \frac{6}{5} = 1\frac{1}{5}$

⑥ $1\frac{1}{9} \div 7\frac{1}{2} = \frac{10}{9} \div \frac{15}{2} = \frac{\overset{2}{\cancel{10}}}{9} \times \frac{2}{\underset{3}{\cancel{15}}} = \frac{4}{27}$

⑦ $3\frac{1}{5} \div 1\frac{5}{7} = \frac{16}{5} \div \frac{12}{7} = \frac{\overset{4}{\cancel{16}}}{5} \times \frac{7}{\underset{3}{\cancel{12}}} = \frac{28}{15} = 1\frac{13}{15}$

⑧ $1\frac{1}{2} \div 1\frac{1}{8} = \frac{3}{2} \div \frac{9}{8} = \frac{\cancel{3}}{\cancel{2}} \times \frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{9}}} = \frac{4}{3} = 1\frac{1}{3}$

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## 5 - 4　How to Divide by Fractions
### Dividing with Whole Numbers and Fractions

**Example 1**　Calculate $2 \div \frac{3}{7}$

Remember, 2 can also be written as $\frac{2}{1}$.

$$2 \div \frac{3}{7} = \frac{2}{1} \div \frac{3}{7} = \frac{2}{1} \times \frac{7}{3} = \frac{2 \times 7}{1 \times 3} = \frac{14}{3}$$

Simplify the answers. But you can leave them as improper fractions.

**1**　Calculate the following division problems.

① $6 \div \frac{5}{7} = \frac{6}{1} \div \frac{5}{7} = \frac{6}{1} \times \frac{7}{5} = \frac{42}{5}$

② $4 \div \frac{9}{2} = \frac{4}{1} \div \frac{9}{2} = \frac{4}{1} \times \frac{2}{9} = \frac{8}{9}$

③ $15 \div \frac{10}{3} = \frac{15}{1} \div \frac{10}{3} = \frac{\overset{3}{\cancel{15}}}{1} \times \frac{3}{\underset{2}{\cancel{10}}} = \frac{9}{2}$

④ $8 \div \frac{6}{5} = \frac{8}{1} \div \frac{6}{5} = \frac{\overset{4}{\cancel{8}}}{1} \times \frac{5}{\underset{3}{\cancel{6}}} = \frac{20}{3}$

⑤ $9 \div \frac{12}{7} = \frac{9}{1} \div \frac{12}{7} = \frac{\overset{3}{\cancel{9}}}{1} \times \frac{7}{\underset{4}{\cancel{12}}} = \frac{21}{4}$

⑥ $16 \div \frac{24}{5} = \frac{16}{1} \div \frac{24}{5} = \frac{\overset{2}{\cancel{16}}}{1} \times \frac{5}{\underset{3}{\cancel{24}}} = \frac{10}{3}$

⑦ $20 \div 1\frac{3}{7} = \frac{20}{1} \div \frac{10}{7} = \frac{\overset{2}{\cancel{20}}}{1} \times \frac{7}{\underset{1}{\cancel{10}}} = \frac{14}{1} = 14$

⑧ $12 \div 1\frac{1}{3} = \frac{12}{1} \div \frac{4}{3} = \frac{\overset{3}{\cancel{12}}}{1} \times \frac{3}{\underset{1}{\cancel{4}}} = \frac{9}{1} = 9$

**Example 2**　Calculate $\frac{3}{7} \div 2$

The whole number 2 can be written as $\frac{2}{1}$. Its reciprocal is $\frac{1}{2}$.

$$\frac{3}{7} \div 2 = \frac{3}{7} \div \frac{2}{1} = \frac{3}{7} \times \frac{1}{2} = \frac{3 \times 1}{7 \times 2} = \frac{3}{14}$$

**2**　Calculate the following division problems.

① $\frac{7}{8} \div 4 = \frac{7}{8} \div \frac{4}{1} = \frac{7}{8} \times \frac{1}{4} = \frac{7}{32}$

② $\frac{1}{2} \div 3 = \frac{1}{2} \div \frac{3}{1} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

③ $\frac{5}{7} \div 10 = \frac{5}{7} \div \frac{10}{1} = \frac{\overset{1}{\cancel{5}}}{7} \times \frac{1}{\underset{2}{\cancel{10}}} = \frac{1}{14}$

④ $\frac{8}{9} \div 2 = \frac{8}{9} \div \frac{2}{1} = \frac{\overset{4}{\cancel{8}}}{9} \times \frac{1}{\underset{1}{\cancel{2}}} = \frac{4}{9}$

⑤ $\frac{20}{3} \div 8 = \frac{20}{3} \div \frac{8}{1} = \frac{\overset{5}{\cancel{20}}}{3} \times \frac{1}{\underset{2}{\cancel{8}}} = \frac{5}{6}$

⑥ $5\frac{1}{3} \div 6 = \frac{16}{3} \div \frac{6}{1} = \frac{\overset{8}{\cancel{16}}}{3} \times \frac{1}{\underset{3}{\cancel{6}}} = \frac{8}{9}$

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## 5 - 5　How to Divide by Fractions
### Dividing Decimal Numbers by Fractions

**Example**　Calculate $0.7 \div \frac{2}{3}$

First change the decimal number to a fraction. Remember, 0.7 is also "seven tenths" or $\frac{7}{10}$.

$$0.7 \div \frac{2}{3} = \frac{7}{10} \div \frac{2}{3} = \frac{7 \times 3}{10 \times 2} = \frac{21}{20}$$

**1**　Calculate the following multiplication problems. Simplify the answers. Leave the answers as improper fractions.

① $0.3 \div \frac{2}{7} = \frac{3}{10} \div \frac{2}{7} = \frac{3}{10} \times \frac{7}{2} = \frac{21}{20}$

② $0.1 \div \frac{2}{3} = \frac{1}{10} \div \frac{2}{3} = \frac{1}{10} \times \frac{3}{2} = \frac{3}{20}$

③ $0.9 \div \frac{1}{2} = \frac{9}{10} \div \frac{1}{2} = \frac{9}{\underset{5}{\cancel{10}}} \times \frac{\overset{1}{\cancel{2}}}{1} = \frac{9}{5}$

④ $0.5 \div \frac{3}{2} = \frac{5}{10} \div \frac{3}{2} = \frac{\overset{1}{\cancel{5}}}{\underset{5}{\cancel{10}}} \times \frac{\overset{1}{\cancel{2}}}{3} = \frac{1}{3}$

⑤ $2.7 \div \frac{3}{5} = \frac{27}{10} \div \frac{3}{5} = \frac{\overset{9}{\cancel{27}}}{\underset{2}{\cancel{10}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{3}}} = \frac{9}{2}$

⑥ $1.5 \div 2\frac{1}{7} = \frac{15}{10} \div \frac{15}{7} = \frac{\cancel{15}}{10} \times \frac{7}{\cancel{15}} = \frac{7}{10}$

⑦ $0.2 \div 1\frac{1}{8} = \frac{2}{10} \div \frac{9}{8} = \frac{\overset{1}{\cancel{2}}}{\underset{5}{\cancel{10}}} \times \frac{\overset{4}{\cancel{8}}}{9} = \frac{8}{45}$

⑧ $0.9 \div 3\frac{3}{5} = \frac{9}{10} \div \frac{18}{5} = \frac{\overset{1}{\cancel{9}}}{\underset{2}{\cancel{10}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{18}}} = \frac{1}{4}$

**2**　Explain the following calculation errors and calculate them correctly.

① $4.6 \div 3\frac{5}{6} = \frac{46}{10} \div \frac{23}{6} = \frac{46}{10} \times \frac{23}{6} = \frac{529}{30}$

Misconversion from the division to multiplication.

$$= \frac{46}{10} \div \frac{23}{6} = \frac{\overset{23}{\cancel{46}}}{\underset{5}{\cancel{10}}} \times \frac{\overset{3}{\cancel{6}}}{\underset{1}{\cancel{23}}} = \frac{6}{5}$$

② $1.8 \div \frac{6}{5} = \frac{18}{100} \div \frac{6}{5} = \frac{18}{100} \times \frac{5}{6} = \frac{3}{20}$

Misconversion from decimal number to fraction.

$$= \frac{18}{10} \div \frac{6}{5} = \frac{\overset{3}{\cancel{18}}}{\underset{2}{\cancel{10}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{6}}} = \frac{3}{2}$$

## 5 − 6　How to Divide by Fractions
### Multiplying and Dividing by Fractions (1)

**Example**　Calculate $\dfrac{5}{6} \times \dfrac{3}{10} \div \dfrac{4}{9}$

To divide a fraction, multiply by its reciprocal. Simplify fractions by finding the greatest common factor. Multiply all numerators to equal the answer's numerator. Multiply all denominators to equal the answer's denominator.

$$\dfrac{5}{6} \times \dfrac{3}{10} \div \dfrac{4}{9} = \dfrac{5}{6} \times \dfrac{3}{10} \times \dfrac{9}{4}$$
$$= \dfrac{5 \times 3 \times 9}{6 \times 10 \times 4} = \dfrac{9}{16}$$

**1** Calculate the following. Simplify the answers. Leave them as improper fractions.

① $\dfrac{1}{6} \times \dfrac{2}{3} \div \dfrac{2}{5} = \dfrac{1}{6} \times \dfrac{2}{3} \times \dfrac{5}{2} = \dfrac{5}{18}$

② $\dfrac{1}{2} \times \dfrac{9}{2} \div \dfrac{3}{10} = \dfrac{1}{2} \times \dfrac{9}{2} \times \dfrac{10}{3} = \dfrac{15}{2}$

③ $\dfrac{7}{3} \times \dfrac{5}{9} \div \dfrac{10}{3} = \dfrac{7 \times 5 \times 3}{3 \times 9 \times 10} = \dfrac{7}{18}$

④ $\dfrac{1}{4} \times \dfrac{6}{5} \div \dfrac{9}{2} = \dfrac{1}{4} \times \dfrac{6}{5} \times \dfrac{2}{9} = \dfrac{1}{6}$

⑤ $\dfrac{9}{4} \times \dfrac{5}{2} \div \dfrac{7}{8} = \dfrac{9}{4} \times \dfrac{5}{2} \times \dfrac{8}{7} = \dfrac{45}{7}$

⑥ $\dfrac{2}{5} \times \dfrac{3}{4} \div \dfrac{7}{10} = \dfrac{2}{5} \times \dfrac{3}{4} \times \dfrac{10}{7} = \dfrac{3}{7}$

⑦ $\dfrac{5}{6} \times \dfrac{2}{15} \div \dfrac{5}{3} = \dfrac{5}{6} \times \dfrac{2}{15} \times \dfrac{3}{5} = \dfrac{1}{15}$

⑧ $\dfrac{12}{7} \times \dfrac{2}{3} \div \dfrac{8}{9} = \dfrac{12}{7} \times \dfrac{2}{3} \times \dfrac{9}{8} = \dfrac{9}{7}$

**2** Explain the following calculation errors and calculate them correctly.

*A way of addition of fraction is incorrect.*

① $\dfrac{5}{6} + \dfrac{2}{3} = \dfrac{5+2}{6+3} = \dfrac{7}{9}$

Remember the previous learning?

$= \dfrac{5}{6} + \dfrac{4}{6} = \dfrac{9}{6} = \dfrac{3}{2}$

*A way of division of fraction is incorrect.*

② $\dfrac{3}{2} \times \dfrac{1}{3} \div \dfrac{3}{4} = \dfrac{3}{2} \times \dfrac{1}{3} \times \dfrac{3}{4} = \dfrac{3}{8}$

$= \dfrac{3}{2} \times \dfrac{1}{3} \times \dfrac{4}{3} = \dfrac{2}{3}$

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## 5 − 7　How to Divide by Fractions
### Multiplying and Dividing by Fractions (2)

**Example**　Calculate $\dfrac{5}{9} \div \dfrac{7}{8} \times \dfrac{3}{4}$

Remember, to divide by a fraction, multiply by its reciprocal. Simplify fractions by finding the greatest common factor. Multiply all denominators to equal to the answer's denominator.

$$\dfrac{5}{9} \div \dfrac{7}{8} \times \dfrac{3}{4} = \dfrac{5}{9} \times \dfrac{8}{7} \times \dfrac{3}{4}$$
$$= \dfrac{5 \times 8 \times 3}{9 \times 7 \times 4} = \dfrac{10}{21}$$

**1** Calculate the following. Simplify the answers. Leave them as improper fractions.

① $\dfrac{1}{5} \div \dfrac{1}{3} \times \dfrac{6}{7} = \dfrac{1}{5} \times \dfrac{3}{1} \times \dfrac{6}{7} = \dfrac{18}{35}$

② $\dfrac{2}{3} \div \dfrac{8}{9} \times \dfrac{3}{4} = \dfrac{2}{3} \times \dfrac{9}{8} \times \dfrac{3}{4} = \dfrac{9}{16}$

③ $\dfrac{3}{8} \div \dfrac{5}{6} \times \dfrac{2}{9} = \dfrac{3}{8} \times \dfrac{6}{5} \times \dfrac{2}{9} = \dfrac{1}{10}$

④ $\dfrac{4}{5} \div \dfrac{3}{4} \times \dfrac{9}{8} = \dfrac{4}{5} \times \dfrac{4}{3} \times \dfrac{9}{8} = \dfrac{6}{5}$

⑤ $\dfrac{1}{5} \div \dfrac{2}{3} \times \dfrac{5}{6} = \dfrac{1}{5} \times \dfrac{3}{2} \times \dfrac{5}{6} = \dfrac{1}{4}$

⑥ $\dfrac{1}{2} \div \dfrac{1}{4} \times \dfrac{1}{3} = \dfrac{1}{2} \times \dfrac{4}{1} \times \dfrac{1}{3} = \dfrac{2}{3}$

⑦ $\dfrac{6}{7} \div \dfrac{5}{6} \times \dfrac{7}{8} = \dfrac{6}{7} \times \dfrac{6}{5} \times \dfrac{7}{8} = \dfrac{9}{10}$

⑧ $\dfrac{3}{4} \div \dfrac{2}{5} \times \dfrac{8}{9} = \dfrac{3}{4} \times \dfrac{5}{2} \times \dfrac{8}{9} = \dfrac{5}{3}$

**2** Explain the following calculation errors and calculate them correctly.

*A way of subtraction of fraction is incorrect.*

① $\dfrac{7}{6} - \dfrac{3}{7} = \dfrac{7}{6} - \dfrac{3}{7} = \dfrac{1}{2}$

Remember the previous learning?

$= \dfrac{49}{42} - \dfrac{18}{42} = \dfrac{31}{42}$

*Simplification at the beginning is incorrect.*

② $\dfrac{5}{7} \div \dfrac{9}{10} = \dfrac{1}{7} \times \dfrac{2}{9} = \dfrac{2}{63}$

$= \dfrac{5}{7} \times \dfrac{10}{9} = \dfrac{50}{63}$

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## 5 − 8　How to Divide by Fractions
### Dividing by More than One Fraction

**Example**　Calculate $\dfrac{1}{3} \div \dfrac{1}{4} \div \dfrac{1}{6}$

Sometimes there is more than one division in a math sentence. For each division by a fraction, multiply by its reciprocal.

$$\dfrac{1}{3} \div \dfrac{1}{4} \div \dfrac{1}{6} = \dfrac{1}{3} \times \dfrac{4}{1} \times \dfrac{6}{1}$$
$$= \dfrac{1 \times 4 \times 6}{3 \times 1 \times 1} = \dfrac{8}{1} = 8$$

Calculate the following. Simplify the answers. Leave them as improper fractions.

① $\dfrac{5}{9} \div \dfrac{5}{6} \div \dfrac{3}{7} = \dfrac{5}{9} \times \dfrac{6}{5} \times \dfrac{7}{3} = \dfrac{14}{9}$

② $\dfrac{3}{4} \div \dfrac{9}{5} \div \dfrac{5}{8} = \dfrac{3}{4} \times \dfrac{5}{9} \times \dfrac{8}{5} = \dfrac{2}{3}$

③ $\dfrac{2}{3} \div \dfrac{8}{7} \div \dfrac{2}{9} = \dfrac{2}{3} \times \dfrac{7}{8} \times \dfrac{9}{2} = \dfrac{21}{8}$

④ $\dfrac{5}{6} \div \dfrac{6}{7} \div \dfrac{7}{8} = \dfrac{5}{6} \times \dfrac{7}{6} \times \dfrac{8}{7} = \dfrac{10}{9}$

⑤ $\dfrac{1}{2} \div \dfrac{5}{6} \div \dfrac{9}{10} = \dfrac{1}{2} \times \dfrac{6}{5} \times \dfrac{10}{9} = \dfrac{2}{3}$

⑥ $\dfrac{1}{8} \div \dfrac{2}{9} \div \dfrac{3}{10} = \dfrac{1}{8} \times \dfrac{9}{2} \times \dfrac{10}{3} = \dfrac{15}{8}$

⑦ $\dfrac{4}{5} \div \dfrac{8}{7} \div \dfrac{14}{15} = \dfrac{4}{5} \times \dfrac{7}{8} \times \dfrac{15}{14} = \dfrac{3}{4}$

⑧ $\dfrac{2}{3} \div \dfrac{3}{4} \div \dfrac{4}{5} = \dfrac{2 \times 4 \times 5}{3 \times 3 \times 4} = \dfrac{10}{9}$

**Let's Try !**

Multiply the horizontal fractions and write the answer in the ☐ above.

$\dfrac{5}{36}$

$\dfrac{1}{3}$　$\dfrac{5}{12}$

$\dfrac{3}{5}$　$\dfrac{5}{9}$　$\dfrac{3}{4}$

We have to calculate not only by multiplying fractions, but also by dividing fractions.

Give a hint to those who are wondering where to calculate. You can do it from the ☐ on the left of the second row.

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## 5 − 9　How to Divide by Fractions
### Dividing by Decimal Numbers and by Fractions

**Example**　Calculate $1.5 \div \dfrac{3}{2} \div 3$

Change decimal numbers to fractions. Change mixed numbers to improper fractions. Divide fractions by multiplying by its reciprocal. Simply when possible.

$$1.5 \div \dfrac{3}{2} \div 3 = \dfrac{15}{10} \times \dfrac{2}{3} \times \dfrac{1}{3}$$
$$= \dfrac{15 \times 2 \times 1}{10 \times 3 \times 3} = \dfrac{1}{3}$$

Calculate the following. Simply the answers. Leave them as improper fractions.

① $0.4 \div \dfrac{4}{7} \div 14 = \dfrac{4}{10} \times \dfrac{7}{4} \times \dfrac{1}{14} = \dfrac{1}{20}$

② $0.6 \div \dfrac{1}{2} \div 3 = \dfrac{6}{10} \times \dfrac{2}{1} \times \dfrac{1}{3} = \dfrac{2}{5}$

③ $1.8 \div \dfrac{1}{2} \div 9 = \dfrac{18}{10} \times \dfrac{2}{1} \times \dfrac{1}{9} = \dfrac{2}{5}$

④ $1.5 \div \dfrac{1}{8} \div 5 = \dfrac{15}{10} \times \dfrac{8}{1} \times \dfrac{1}{5} = \dfrac{12}{5}$

⑤ $2.4 \div \dfrac{3}{5} \div 3 = \dfrac{24}{10} \times \dfrac{5}{3} \times \dfrac{1}{3} = \dfrac{4}{3}$

⑥ $3.6 \div \dfrac{6}{7} \div 14 = \dfrac{36}{10} \times \dfrac{7}{6} \times \dfrac{1}{14} = \dfrac{3}{10}$

⑦ $2.7 \div \dfrac{9}{10} \div 7 = \dfrac{27}{10} \times \dfrac{10}{9} \times \dfrac{1}{7} = \dfrac{3}{7}$

⑧ $5 \div \dfrac{15}{2} \div 0.5 = \dfrac{5}{1} \times \dfrac{2}{15} \times \dfrac{10}{5} = \dfrac{4}{3}$

**Let's Try !**

Put the three numbers, 2, 3, and 6 in the following ☐ to complete the math sentence. We can use each number only one time.

$$\dfrac{2}{3} \div 1\dfrac{1}{6} = \dfrac{4}{7}$$

Let's try various cases and find the correct answer.

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## 5 – 10 How to Divide by Fractions
### Division and Multiplication with Various Kinds of Numbers

**Example** Calculate $6 \times \dfrac{8}{5} \div 2.1$

$$6 \times \frac{8}{5} \div 2.1 = \frac{6}{1} \times \frac{8}{5} \div \frac{21}{10} = \frac{6}{1} \times \frac{8}{5} \times \frac{10}{21}$$

$$= \frac{\overset{2}{6} \times 8 \times \overset{2}{10}}{1 \times \overset{1}{5} \times \overset{7}{21}} = \boxed{\frac{32}{7}}$$

Remember to change whole numbers and decimal numbers to fractions. Divide by multiplying by a fraction's reciprocal. Simplify by dividing a numerator and a denominator by its greatest common factor.

Calculate the following. Simplify the answers. Leave them as improper fractions.

❶ $2 \times \dfrac{1}{5} \div 0.6 = \dfrac{2}{1} \times \dfrac{1}{5} \times \dfrac{10}{6} = \dfrac{2}{3}$

❷ $3 \times \dfrac{1}{6} \div 0.9 = \dfrac{3}{1} \times \dfrac{1}{6} \times \dfrac{10}{9} = \dfrac{5}{9}$

❸ $9 \times \dfrac{2}{3} \div 1.6 = \dfrac{9}{1} \times \dfrac{2}{3} \times \dfrac{10}{16} = \dfrac{15}{4}$

❹ $5 \times \dfrac{3}{8} \div 1.5 = \dfrac{5}{1} \times \dfrac{3}{8} \times \dfrac{10}{15} = \dfrac{5}{4}$

❺ $0.2 \times \dfrac{7}{8} \div 2.8 = \dfrac{2}{10} \times \dfrac{7}{8} \times \dfrac{10}{28} = \dfrac{1}{2}$

❻ $0.4 \times \dfrac{4}{5} \div 1.6 = \dfrac{4}{10} \times \dfrac{4}{5} \times \dfrac{10}{16} = \dfrac{1}{5}$

❼ $0.8 \times \dfrac{2}{5} \div 0.06 = \dfrac{8}{10} \times \dfrac{2}{5} \times \dfrac{100}{6} = \dfrac{16}{3}$

❽ $0.6 \times \dfrac{1}{25} \div 0.01 = \dfrac{6}{10} \times \dfrac{1}{25} \times \dfrac{100}{1} = \dfrac{12}{5}$

**Let's Try !**
As the example, multiply the three numbers side by side and write the answer in the above △ .
Complete a number pyramid.
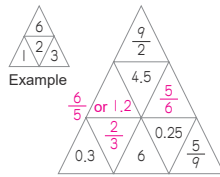
We have to calculate not only by multiplying fractions, but also by dividing fractions.

$\dfrac{6}{1} \quad \dfrac{2}{3}$

$\dfrac{9}{2}$

4.5

Example

$\dfrac{6}{5}$ or 1.2   $\dfrac{5}{6}$

$\dfrac{2}{3}$   0.25

0.3   6   $\dfrac{5}{9}$

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## 5 – 11 How to Divide by Fractions
### Division Problems

**Example** An $\dfrac{8}{9}$ m long iron bar weighs $\dfrac{4}{5}$ kg.

❶ How much does 1 m of this iron bar weigh?
❷ How long is 1 kg of this bar?

❶ 
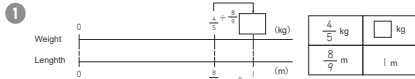
weight lenghth

$$\frac{4}{5} \div \frac{8}{9} = \frac{4}{5} \times \frac{9}{8} = \frac{9}{10}$$

**Answer** $\dfrac{9}{10}$ kg

The calculation will change depending on which amount is used as the standard.

❷ 
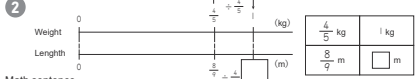
lenghth weight

$$\frac{8}{9} \div \frac{4}{5} = \frac{8}{9} \times \frac{5}{4} = \frac{10}{9} \text{ or } 1\frac{1}{9}$$

**Answer** $\dfrac{10}{9}$ m or $1\dfrac{1}{9}$ m

A carpenter can paint $\dfrac{2}{3}$ m² of a wall with $\dfrac{4}{7}$ dL of paint.

❶ How many dL of paint does he need to paint 1 m² of the wall?

Complete the number line diagram and table.

**Math sentence**
$$\frac{4}{7} \div \frac{2}{3} = \frac{4}{7} \times \frac{3}{2} = \frac{6}{7}$$

**Answer** $\dfrac{6}{7}$ dL
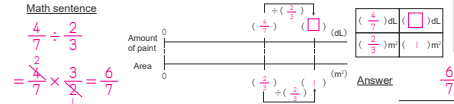
❷ If the carpenter has 1 dL of paint, how many m² of the wall can he paint?

**Math sentence**
$$\frac{2}{3} \div \frac{4}{7} = \frac{2}{3} \times \frac{7}{4} = \frac{7}{6} \text{ or } 1\frac{1}{6}$$

**Answer** $\dfrac{7}{6}$ m² or $1\dfrac{1}{6}$ m²

53
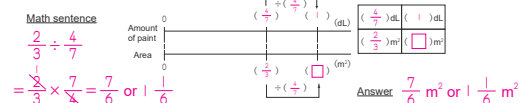
## 5 – 12 How to Divide by Fractions
### Size of the Product

**Instruction** In multiplication, when the multiplier is a whole number or decimal number, the size of the product and the size of the multiplicand are related in the following pattern:

① When multiplier > 1, product > multiplicand
② When multiplier < 1, product < multiplicand
③ When multiplier = 1, product = multiplicand

For example, ① $6 \times \dfrac{3}{2} = 9 > 6$

② $6 \times \dfrac{2}{3} = 4 < 6$

③ $6 \times 1 = 6 = 6$

**Example** Which product is less than 6? Answer the letter of the correct math sentences.

(a) $6 \times \dfrac{8}{7}$   (b) $6 \times \dfrac{12}{13}$   (c) $6 \times 1$   (d) $6 \times 1\dfrac{1}{5}$

Because the product is less than 6 which is also multiplicand, we must choose the formula whose multiplier is less than 1.

**Answer** (b)

**1** Which product is less than 12? Answer the letter of the correct math sentences.

(a) $12 \times \dfrac{8}{9}$   (b) $12 \times 1$   (c) $12 \times \dfrac{7}{5}$   (d) $12 \times \dfrac{23}{25}$

(e) $12 \times 2\dfrac{1}{3}$   (f) $12 \times \dfrac{11}{12}$

**Answer** (a), (d), (f)

**2** Which product is more than 31? Answer the letter of the correct math sentences.

(a) $31 \times \dfrac{5}{4}$   (b) $31 \times \dfrac{2}{3}$   (c) $31 \times 1\dfrac{1}{10}$   (d) $31 \times \dfrac{32}{31}$

(e) $31 \times 1$   (f) $31 \times \dfrac{17}{18}$

**Answer** (a), (c), (d)

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## 5 – 13 How to Divide by Fractions
### Size of the Quotient

**Instruction** In division, when the divisor is a whole number or decimal number, the size of the quotient and the size of the dividend are related in the following pattern:

① When divisor > 1, quotient < dividend
② When divisor < 1, quotient > dividend
③ When divisor = 1, quotient = dividend

For example, ① $6 \div \dfrac{3}{2} = 4 < 6$

② $6 \div \dfrac{2}{3} = 9 > 6$

③ $6 \div 1 = 6 = 6$

**Example** Which quotient is more than 7? Answer the letter of correct math sentences.

(a) $7 \div 1\dfrac{2}{9}$   (b) $7 \div 1$   (c) $7 \div \dfrac{9}{13}$   (d) $7 \div \dfrac{4}{3}$

Because the quotient is more than 7 (which is also dividend), we must choose the formula whose divisor is less than 1.
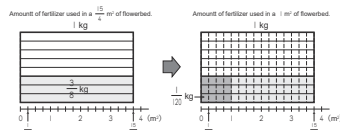
**Answer** (c)

**1** Which quotient is less than 15? Answer the letter of correct math sentences.

(a) $15 \div \dfrac{5}{9}$   (b) $15 \div \dfrac{13}{12}$   (c) $15 \div \dfrac{19}{50}$   (d) $15 \div \dfrac{7}{5}$

(e) $15 \div 1\dfrac{1}{2}$   (f) $15 \div 1$

**Answer** (b), (d), (e)

**2** Which quotient is more than 9? Answer the letter of correct math sentence.

(a) $9 \div \dfrac{3}{7}$   (b) $9 \div \dfrac{6}{5}$   (c) $9 \div 2\dfrac{3}{4}$   (d) $9 \div 1$

(e) $9 \div \dfrac{11}{10}$   (f) $9 \div \dfrac{1}{34}$

**Answer** (a), (f)

55

15

**5 - 14** How to Divide by Fractions
**Review (1)**

**1** Calculate the following division problems. Simplify the answer. Leave them as improper fractions.

**1** $\frac{3}{5} \div \frac{4}{7} = \frac{3}{5} \times \frac{7}{4} = \frac{21}{20}$

**2** $\frac{16}{25} \div \frac{12}{5} = \frac{\cancel{16}^{4}}{\cancel{25}_{5}} \times \frac{\cancel{5}^{1}}{\cancel{12}_{3}} = \frac{4}{15}$

**3** $2\frac{1}{6} \div 3\frac{1}{4} = \frac{13}{6} \div \frac{13}{4} = \frac{\cancel{13}^{1}}{\cancel{6}_{3}} \times \frac{\cancel{4}^{2}}{\cancel{13}_{1}} = \frac{2}{3}$

**4** $3\frac{1}{3} \div 1\frac{3}{7} = \frac{10}{3} \div \frac{10}{7} = \frac{\cancel{10}^{1}}{3} \times \frac{7}{\cancel{10}_{1}} = \frac{7}{3}$

**5** $18 \div 2\frac{2}{5} = \frac{18}{1} \div \frac{12}{5} = \frac{\cancel{18}^{3}}{1} \times \frac{5}{\cancel{12}_{2}} = \frac{15}{2}$

**6** $\frac{7}{9} \div 14 = \frac{7}{9} \div \frac{14}{1} = \frac{\cancel{7}^{1}}{9} \times \frac{1}{\cancel{14}_{2}} = \frac{1}{18}$

**7** $0.6 \div \frac{2}{5} = \frac{6}{10} \div \frac{2}{5} = \frac{\cancel{6}^{3}}{\cancel{10}_{2}} \times \frac{\cancel{5}^{1}}{\cancel{2}_{1}} = \frac{3}{2}$

**8** $1.8 \div 7\frac{1}{5} = \frac{18}{10} \div \frac{36}{5} = \frac{\cancel{18}^{1}}{\cancel{10}_{2}} \times \frac{\cancel{5}^{1}}{\cancel{36}_{2}} = \frac{1}{4}$

**9** $\frac{3}{4} \times \frac{2}{5} \div \frac{15}{8} = \frac{\cancel{3}^{1}}{\cancel{4}} \times \frac{\cancel{2}^{1}}{5} \times \frac{\cancel{8}^{4}}{\cancel{15}_{5}} = \frac{4}{25}$

**10** $4 \times \frac{5}{6} \div 0.8 = \frac{4}{1} \times \frac{5}{\cancel{6}} \times \frac{\cancel{10}^{5}}{\cancel{8}_{2}} = \frac{25}{6}$

**11** $\frac{8}{3} \div \frac{4}{9} \div 1.6 = \frac{\cancel{8}^{1}}{\cancel{3}} \times \frac{\cancel{9}^{3}}{\cancel{4}} \times \frac{\cancel{10}^{5}}{\cancel{16}} = \frac{15}{4}$

**12** $2.1 \div \frac{7}{8} \div 6 = \frac{\cancel{21}^{3}}{\cancel{10}} \times \frac{\cancel{8}^{4}}{\cancel{7}_{1}} \times \frac{1}{\cancel{6}} = \frac{2}{5}$

**2** Which product is more than 5? Answer the letter of correct math sentence.

(a) $5 \times 1\frac{1}{2}$   (b) $5 \times \frac{2}{3}$   (c) $5 \times 1$   (d) $5 \times \frac{17}{18}$

(e) $5 \times \frac{10}{7}$   (f) $5 \times \frac{6}{5}$

Answer ____(a), (e), (f)____

**3** Which quotient is less than 8? Answer the letter of correct math sentence.

(a) $8 \div \frac{24}{25}$   (b) $8 \div \frac{4}{5}$   (c) $8 \div \frac{4}{3}$   (d) $8 \div 1\frac{1}{10}$

(e) $8 \div 1$   (f) $8 \div \frac{7}{8}$

Answer ____(c), (d)____

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**4** $\frac{3}{8}$ kg of fertilizer is used in a $\frac{15}{4}$ m² flowerbed. Answer the following questions.

**1** How many kg of fertilizer do we need for an 1 m² of flowerbed?

Complete the number line diagrams and tables.

Math sentence
$\frac{3}{8} \div \frac{15}{4} = \frac{\cancel{3}^{1}}{8} \times \frac{\cancel{4}^{1}}{\cancel{15}_{5}} = \frac{1}{10}$

Answer $\frac{1}{10}$ kg

**2** How many m² of flowerbed can be fertilized with 1 kg of fertilizer?

Math sentence
$\frac{15}{4} \div \frac{3}{8} = \frac{\cancel{15}^{5}}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{\cancel{3}_{1}} = \frac{10}{1} = 10$

Answer 10 m²

**5** A lawnmower can mow 35 m² of grass in 20 minutes. How many m² of grass can this lawnmower mow in 1 hour?

Math sentence
20 minutes $= \frac{20}{60} = \frac{1}{3}$ of an hour
$35 \div \frac{20}{60} = \frac{35}{1} \div \frac{60}{20} = \frac{35}{1} \times \frac{\cancel{60}^{3}}{\cancel{20}_{1}} = 105$

Answer 105 m²

**6** Which of the following word problems equals the math sentence: $\frac{1}{3} \div \frac{2}{5}$? Write the letter to the correct word problem.

(a) We painted a $\frac{2}{5}$ m² of board with $\frac{1}{3}$ dL of paint. How many m² of board can we paint with 1 dL of this paint?

(b) There is a rectangular piece of paper with an area of $\frac{1}{3}$ m². The length of the paper is $\frac{2}{5}$ m. How many m is the width of this paper?

(c) There is oil that weighs $\frac{1}{3}$ kg per L. How much kg is $\frac{2}{5}$ L of this oil?

Answer ____(b)____

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**5 - 15** How to Divide by Fractions
**Times as Much with Fractions (1)**

**Example** A blue ribbon is $\frac{5}{4}$ m. A red ribbon is $\frac{1}{2}$ m. How many times longer is the blue ribbon?

If a red ribbon is 2 m and blue ribbon is 6 m, we can calculate it easily. This problem is also the same, but it uses fractions.

We can consider the length of the red ribbon as 1.
Once we draw a diagram, it is easier to understand.
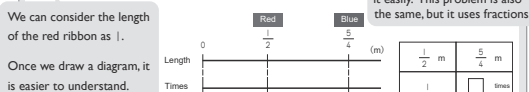
Math sentence
$\frac{5}{4} \div \frac{1}{2} = \frac{5}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{1} = \frac{5 \times 1}{4 \times 1} = \frac{5}{2}$ or $2\frac{1}{2}$

Answer $\frac{5}{2}$ times or $2\frac{1}{2}$ times

Even when fractions are involved, we use division to find out how many times as much something is as a base quantity.

**1** A black ribbon is $\frac{2}{3}$ m. A white ribbon is $\frac{5}{9}$ m. How many times longer is the black ribbon?

Complete the number line diagrams and tables.
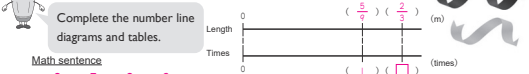
Math sentence
$\frac{2}{3} \div \frac{5}{9} = \frac{2}{3} \times \frac{9}{5}$
$= \frac{2 \times \cancel{9}^{3}}{\cancel{3} \times 5} = \frac{6}{5}$ or $1\frac{1}{5}$

Answer $\frac{6}{5}$ times or $1\frac{1}{5}$ times

**2** It takes $\frac{2}{7}$ of an hour to walk to the market. It takes $\frac{2}{25}$ of an hour to go to the market by bicycle. How many times longer does it take to walk than bicycle to the market?
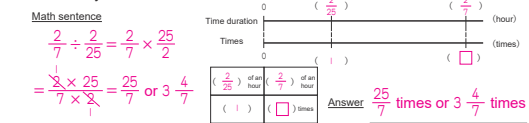
Math sentence
$\frac{2}{7} \div \frac{2}{25} = \frac{2}{7} \times \frac{25}{2}$
$= \frac{\cancel{2}^{1} \times 25}{7 \times \cancel{2}_{1}} = \frac{25}{7}$ or $3\frac{4}{7}$

Answer $\frac{25}{7}$ times or $3\frac{4}{7}$ times

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**2** Answer the following questions.

**1** How many times heavier is $\frac{5}{12}$ kg than $\frac{7}{6}$ kg?

Complete the number line diagrams and tables.

Math sentence
$\frac{7}{6} \div \frac{5}{12} = \frac{7}{6} \times \frac{12}{5}$
$= \frac{7 \times \cancel{12}^{2}}{\cancel{6}_{1} \times 5} = \frac{14}{5}$ or $2\frac{4}{5}$

Answer $\frac{14}{5}$ times or $2\frac{4}{5}$ times

**2** How many times more is $\frac{5}{12}$ L than $\frac{8}{9}$ L?

Math sentence
$\frac{5}{12} \div \frac{8}{9} = \frac{5}{12} \times \frac{9}{8}$
$= \frac{5 \times \cancel{9}^{3}}{\cancel{12}_{4} \times 8} = \frac{15}{32}$

Answer $\frac{15}{32}$ times

**3** A rectangle piece of paper is $\frac{4}{3}$ cm long and $\frac{4}{5}$ cm wide. How many times longer is the length of the rectangle than the width?

Math sentence
$\frac{4}{3} \div \frac{4}{5} = \frac{4}{3} \times \frac{5}{4}$
$= \frac{\cancel{4}^{1} \times 5}{3 \times \cancel{4}_{1}} = \frac{5}{3}$ or $1\frac{2}{3}$

Answer $\frac{5}{3}$ times or $1\frac{2}{3}$ times

**4** Package A weighs $\frac{3}{5}$ kg. Package B weighs $\frac{3}{2}$ kg. How many times heavier is Package A than Package B?

Math sentence
$\frac{3}{5} \div \frac{3}{2} = \frac{3}{5} \times \frac{2}{3}$
$= \frac{\cancel{3}^{1} \times 2}{5 \times \cancel{3}_{1}} = \frac{2}{5}$

Package A    Package B

Answer $\frac{2}{5}$ times

59

## 5 – 16 How to Divide by Fractions
### Times as Much with Fractions (2)

**Example** A blue ribbon is $\frac{5}{2}$ times longer than the red ribbon. If the red ribbon is $3$ m, how long is the blue ribbon?

Sometimes the word problem can be easier to understand if we use a whole number. If the blue ribbon is $2$ times longer than the red ribbon, the blue ribbon is $6$ m ($3 \times 2$).

Red  Blue

Length  
Times

3 m  □ m

1  $\frac{5}{2}$ times
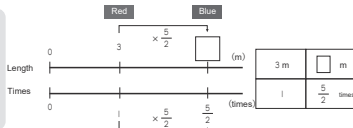
**Math sentence**

$3 \times \frac{5}{2} = \frac{3 \times 5}{1 \times 2} = \frac{15}{2}$ or $7\frac{1}{2}$

Answer $\frac{15}{2}$ m or $7\frac{1}{2}$ m

Even when fractions are involved, we use multiplication to find out the amount to be compared. A formula is: (a base amount) × (how many times as much)

**1** The tree in front of my house is $\frac{4}{3}$ times taller than my house. If my house is $9$ m tall, how tall is the tree?

Complete the number line diagrams and tables.

**Math sentence**

$9 \times \frac{4}{3} = \frac{9}{1} \times \frac{4}{3}$

$= \frac{\overset{3}{9} \times 4}{1 \times \overset{}{3}} = \frac{12}{1} = 12$

House  Tree

Height  
Times

( 9 ) m  ( □ ) m
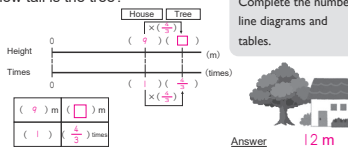
( 1 )  ( $\frac{4}{3}$ ) times

Answer $12$ m

**2** A small water tank holds $50$ L of water. A bigger water tank holds $\frac{6}{5}$ times as much water as the small tank. How much water can the bigger water tank hold?

**Math sentence**

$50 \times \frac{6}{5} = \frac{50}{1} \times \frac{6}{5}$

$= \frac{\overset{10}{50} \times 6}{1 \times \overset{}{5}} = 60$

Small  Large

Amount of water  
Times

( 50 ) L  ( □ ) L

( 1 )  ( $\frac{6}{5}$ ) times
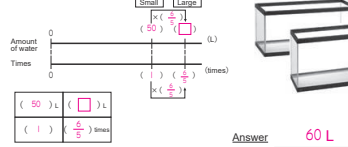
Answer $60$ L

**2** Answer the following questions.

**1** What is $\frac{3}{7}$ of $35$ kg?

Complete the number line diagrams and tables.

**Math sentence**

$35 \times \frac{3}{7} = \frac{35}{1} \times \frac{3}{7}$

$= \frac{\overset{5}{35} \times 3}{1 \times \overset{}{7}} = \frac{15}{1} = 15$

Weight  
Times

( □ ) kg  ( 35 ) kg

( $\frac{3}{7}$ ) times  ( 1 )
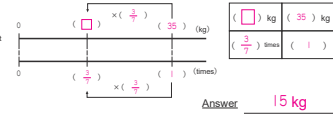
Answer $15$ kg

**2** What is $\frac{9}{4}$ of $20$ L?

**Math sentence**

$20 \times \frac{9}{4} = \frac{20}{1} \times \frac{9}{4}$

$= \frac{\overset{5}{20} \times 9}{1 \times \overset{}{4}} = \frac{45}{1} = 45$

Amount  
Times

( 20 ) L  ( □ ) L

( 1 )  ( $\frac{9}{4}$ ) times
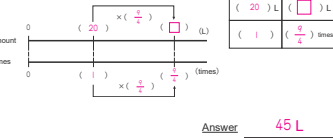
Answer $45$ L

**3** My family consumes $25$ kg of rice every month. My friend's family consumes $\frac{4}{5}$ as much. How much rice does my friend's family consume every month?

**Math sentence**

$25 \times \frac{4}{5} = \frac{25}{1} \times \frac{4}{5}$

$= \frac{\overset{5}{25} \times 4}{1 \times \overset{}{5}} = \frac{20}{1} = 20$

My friend's family  My family

Rice  
Times

( □ ) kg  ( 25 ) kg

( $\frac{4}{5}$ ) times  ( 1 )
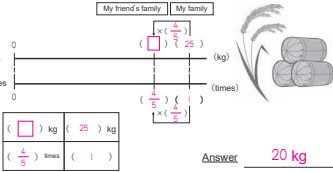
Answer $20$ kg

**4** The garden has an area of $240$ m². The flowerbed's area is $\frac{3}{16}$ of the garden. What is the area of the flowerbed?

**Math sentence**

$240 \times \frac{3}{16} = \frac{240}{1} \times \frac{3}{16}$

$= \frac{\overset{15}{240} \times 3}{1 \times \overset{}{16}} = \frac{45}{1} = 45$

Flowerbed  Garden

Area  
Times

( □ ) m²  ( 240 ) m²

( $\frac{3}{16}$ ) times  ( 1 )
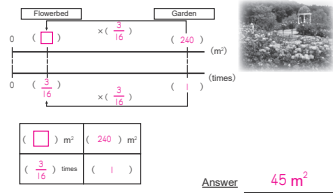
Answer $45$ m²

## 5 – 17 How to Divide by Fractions
### Times as Much with Fractions (3)

**Example** A blue ribbon is $\frac{5}{2}$ times longer than the red ribbon. If the blue ribbon is $\frac{5}{4}$ m, how long is the red ribbon?

Sometimes the word problem can be easier to understand if we use a whole number. If the blue ribbon is $5$ times longer, then divide $\frac{5}{4}$ (m) by 5.

Red  Blue

Length  
Times

□ m  $\frac{5}{4}$ m

1  $\frac{5}{2}$ times
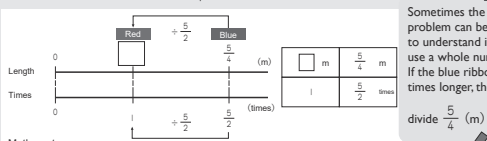
**Math sentence**

$\frac{5}{4} \div \frac{5}{2} = \frac{\overset{1}{5}}{4} \times \frac{\overset{1}{2}}{\overset{}{5}} = \frac{1}{2}$

Answer $\frac{1}{2}$ m

Even when fractions are involved, we use division to find out the amount to be a base. A formula is: (amount to be compared) ÷ (how many times as much)

**1** A juice bottle can hold $\frac{4}{3}$ times more liquid than a milk bottle. If the juice bottle can hold $\frac{6}{5}$ L, how much liquid can a milk bottle hold?

**Math sentence**

$\frac{6}{5} \div \frac{4}{3} = \frac{6}{5} \times \frac{3}{4}$

$= \frac{\overset{3}{6} \times 3}{5 \times \overset{}{4}} = \frac{9}{10}$

Complete the number line diagrams and tables.

Milk  Juice

Amount  
Times

( □ ) L  ( $\frac{6}{5}$ ) L
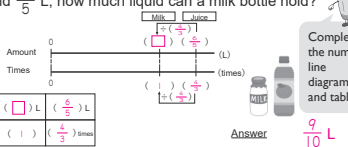
( 1 )  ( $\frac{4}{3}$ ) times

Answer $\frac{9}{10}$ L

**2** A big package is $\frac{5}{3}$ times heavier than a small package. If the big package weighs $\frac{8}{9}$ kg, how much does the small package weigh?

**Math sentence**

$\frac{8}{9} \div \frac{5}{3} = \frac{8}{9} \times \frac{3}{5}$

$= \frac{8 \times \overset{1}{3}}{\overset{3}{9} \times 5} = \frac{8}{15}$

Small box  Big box

Weight  
Times

( □ ) kg  ( $\frac{8}{9}$ ) kg

( 1 )  ( $\frac{5}{3}$ ) times
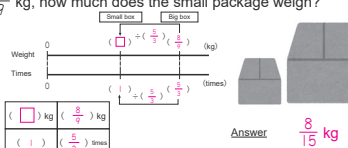
Answer $\frac{8}{15}$ kg

**3** I am $\frac{4}{5}$ as tall as my father. If I am $140$ cm tall, how tall is my father?

**Math sentence**

$140 \div \frac{4}{5} = \frac{140}{1} \times \frac{5}{4}$

$= \frac{\overset{35}{140} \times 5}{1 \times \overset{}{4}} = \frac{175}{1} = 175$

Complete the number line diagrams and tables.

My height  Father's height

Height  
Times

( 140 ) cm  ( □ ) cm

( $\frac{4}{5}$ ) times  ( 1 )
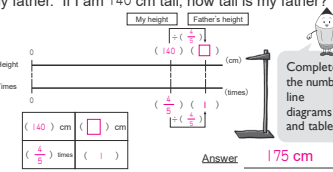
Answer $175$ cm

**4** The can of black paint holds $\frac{2}{3}$ as much as the can of yellow paint. If the can of black paint holds $\frac{25}{6}$ dL, how much does the yellow paint can hold?

**Math sentence**

$\frac{25}{6} \div \frac{2}{3} = \frac{25}{6} \times \frac{3}{2}$

$= \frac{25 \times \overset{1}{3}}{\overset{2}{6} \times 2} = \frac{25}{4}$

Black paint  Yellow paint

Paints  
Times

( $\frac{25}{6}$ ) dL  ( □ ) dL

( $\frac{2}{3}$ ) times  ( 1 )
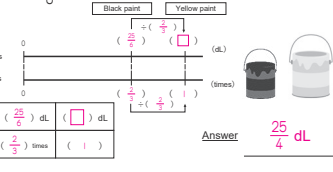
Answer $\frac{25}{4}$ dL

**5** My classmate is $\frac{3}{4}$ as heavy as mine. If my classmate weighs $42$ kg, How much do I weigh?

**Math sentence**

$42 \div \frac{3}{4} = \frac{42}{1} \times \frac{4}{3}$

$= \frac{\overset{14}{42} \times 4}{1 \times \overset{}{3}} = \frac{56}{1} = 56$

My friend's weight  My weight

Weight  
Times

( 42 ) kg  ( □ ) kg

( $\frac{3}{4}$ ) times  ( 1 )
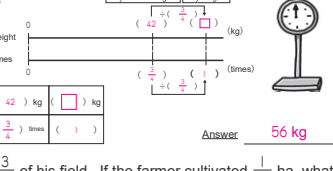
Answer $56$ kg

**6** A farmer cultivated $\frac{3}{8}$ of his field. If the farmer cultivated $\frac{1}{6}$ ha, what is the size of his field?

**Math sentence**

$\frac{1}{6} \div \frac{3}{8} = \frac{1}{6} \times \frac{8}{3}$

$= \frac{1 \times \overset{4}{8}}{\overset{3}{6} \times 3} = \frac{4}{9}$

Cultivated area  Area of the entire field

Field  
Times

( $\frac{1}{6}$ ) ha  ( □ ) ha

( $\frac{3}{8}$ ) times  ( 1 )
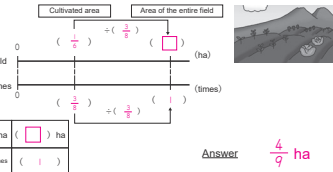
Answer $\frac{4}{9}$ ha

## 5 – 18　How to Divide by Fractions
### Review (2)

**1** Calculate the following.

① 2 times of $\frac{3}{7}$ m … $\frac{6}{7}$ m  ② $\frac{6}{5}$ times of $\frac{3}{4}$ m … $\frac{9}{10}$ m

③ $\frac{2}{3}$ times of $\frac{9}{8}$ m … $\frac{3}{4}$ m  ④ $\frac{16}{15}$ times of $\frac{5}{8}$ m … $\frac{2}{3}$ m

**2** How many times more L is $\frac{13}{15}$ L than $\frac{4}{5}$ L?

Math sentence
$$\frac{13}{15} \div \frac{4}{5} = \frac{13}{15} \times \frac{5}{4}$$
$$= \frac{13 \times 5}{15 \times 4} = \frac{13}{12} \text{ or } 1\frac{1}{12}$$

Complete the number line diagrams and tables.

Answer $\frac{13}{12}$ times or $1\frac{1}{12}$ times

**3** What is $\frac{7}{5}$ of 20 kg?

Math sentence
$$20 \times \frac{7}{5} = \frac{20}{1} \times \frac{7}{5}$$
$$= \frac{20 \times 7}{1 \times 5} = 28$$

Answer 28 kg

**4** My brother has $\frac{3}{4}$ as many pieces of candy as I have. If my brother has 12 pieces of candy, how many do I have?

Math sentence
$$12 \div \frac{3}{4} = \frac{12}{1} \times \frac{4}{3}$$
$$= \frac{12 \times 4}{1 \times 3} = \frac{16}{1} = 16$$

Answer 16 pieces of candy

**5** I have $\frac{8}{5}$ kg of sugar and $\frac{3}{4}$ kg of salt. How many times more kg of sugar than salt do I have?

Math sentence
$$\frac{8}{5} \div \frac{3}{4} = \frac{8}{5} \times \frac{4}{3}$$
$$= \frac{8 \times 4}{5 \times 3} = \frac{32}{15} \text{ or } 2\frac{2}{15}$$

Complete the number line diagrams and tables.

Answer $\frac{32}{15}$ times or $2\frac{2}{15}$ times

**6** My younger sister has $\frac{3}{5}$ as many as stamps as my older sister. If my older sister has 75 stamps, how many stamps does my younger sister have?

Math sentence
$$75 \times \frac{3}{5} = \frac{75}{1} \times \frac{3}{5}$$
$$= \frac{75 \times 3}{1 \times 5} = \frac{45}{1} = 45$$

Answer 45 stamps

**7** $\frac{4}{9}$ of the students at school participate in athletic clubs. If 100 students in athletic clubs, how many students are there at school?

Math sentence
$$100 \div \frac{4}{9} = \frac{100}{1} \times \frac{9}{4}$$
$$= \frac{100 \times 9}{1 \times 4} = \frac{225}{1} = 225$$

Answer 225 students

**8** I used $\frac{1}{9}$ of a piece of tape. The amount of tape left over is 160 cm long. What was the length of the original piece of tape?

Math sentence
Because I used $\frac{1}{9}$ of a piece of tape, the rest of the tape is $\frac{8}{9}$.
$$160 \div \frac{8}{9} = \frac{160}{1} \times \frac{9}{8}$$
$$= \frac{160 \times 9}{1 \times 8} = \frac{180}{1} = 180$$

To solve this question, calculate how much tape I used.

Answer 180 cm

## 6 – 1　How to Analyze Data
### Average and Spread (1)

**Instruction**
Representative values.
The following table summarizes the results for the cultivation of two sets of tomatoes. Compare which record is heavier.

Set 1: Tomato Cultivation Record

| Number | Weight | Number | Weight | Number | Weight | Number | Weight | Number | Weight |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 56 | 6 | 49 | 11 | 44 | 16 | 38 | 21 | 50 |
| 2 | 49 | 7 | 32 | 12 | 49 | 17 | 37 | 22 | 58 |
| 3 | 42 | 8 | 52 | 13 | 58 | 18 | 46 | 23 | 38 |
| 4 | 46 | 9 | 39 | 14 | 53 | 19 | 57 | 24 | 46 |
| 5 | 36 | 10 | 53 | 15 | 46 | 20 | 38 | 25 | 56 |

Set 2: Tomato Cultivation Record

| Number | Weight | Number | Weight | Number | Weight | Number | Weight | Number | Weight |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 54 | 6 | 45 | 11 | 37 | 16 | 36 | 21 | 39 |
| 2 | 44 | 7 | 54 | 12 | 50 | 17 | 49 | 22 | 40 |
| 3 | 49 | 8 | 49 | 13 | 49 | 18 | 44 | 23 | 54 |
| 4 | 39 | 9 | 60 | 14 | 40 | 19 | 58 | 24 | 49 |
| 5 | 37 | 10 | 38 | 15 | 38 | 20 | 49 | | |

① Find the **mean** for the records of each set. The mean for Set 1. The mean for Set 2.

Average you learned is called mean.

$1168 \div 25 = 46.72$　　　$1101 \div 24 = 45.875$

The mean learned so far is called the mean value.
（Mean value）＝（Total of data values）÷（Number of data）

From point of mean values, Set 1 is heavier than Set 2. Is there any other way to compare?

② Which is the **maximum** score for each set? Also, find the **minimum** score.

| | Maximum | Minimum |
|---|---|---|
| Set 1 | 58 | 32 |
| Set 2 | 60 | 36 |

Set 1 has a larger mean value but has smaller maximum and minimum weight.

③ Think about the graph which shows the record of Set 1.

The graph shown on the previous page is called a **dot plot**. Vertically, you can see the amount of data. Horizontally, you can understand how the data is distributed. The number corresponds to the data numbers.

**Example**
The following plot shows the results of the language class exam. A girl got 70 scores and she compares the score with classmates. Answer the following questions

① Find the mean
$(0 + 10 + 30 + 70 + 3 \times 80 + 90 + 2 \times 100) \div 10 = 64$

② Which score is the most frequent value?　80　64

③ When the data is aligned in size order, which value is located in the middle?
$(80 + 80) \div 2 = 80$　80

Find the middle value counting from top and bottom and calculate—the 5th value in this case.

Her score is higher than the mean value, but she is $7^{th}$ from the top. The mean value is not appropriate when she compares the scores this time.

● Within the data, the most frequent value is called the **mode** value.
● When the data is aligned in size order, the value that is located in the middle is called the **median** value. When the number of data is odd, the value is in the exact middle. On the other hand, when the number of data is even, the mean value of the two values is in the middle.
● Values that represent data such as the mean, mode, median are called **representative values**. Depending on data use, it is chosen among the three values.

The following data plot shows the results of a survey on students' shoe size in a class. A shoemaker wants to know which size of shoes he should make to sell more shoes. Answer the following questions.

Mean: $(21.5 + 3 \times 22 + 5 \times 22.5 + 2 \times 23 + 2 \times 23.5 + 24 + 2 \times 25 + 2 \times 25.5 + 2 \times 26) \div 20 = 23.5$
Median: $(23 + 23) \div 2 = 23$

① Find the mean, mode and median value

Mean　23.5　　Mode　22.5　　Median　23

② Which size of shoes he should make to sell more shoes? Choose an appropriate representative value for the situation.

Mode

Not many students have the same shoe size as the mean.

18

## 6 - 2　How to Analyze Data
### Average and Spread (2)

**Example** The records of Set 2 on page 66 are summarized in the dot plot below. Answer the following questions.

❶ Complete the table.

❷ From what number to what number represents the largest number of tomatoes?
More than or equal to **35 g / 45 g** and less than **40 g / 50 g**

❸ How many tomatoes are more than or equal to 35 g and less than 40 g?
**7**

❹ Which class has 4 tomatoes?
More than or equal to **40 g** and less than **45 g**

| Weight (g) | | | Number of tomatoes |
|---|---|---|---|
| more than or equal to 30 | ～ | less than 35 | 0 |
| 35 | ～ | 40 | 7 |
| 40 | ～ | 45 | 4 |
| 45 | ～ | 50 | 7 |
| 50 | ～ | 55 | 3 |
| 55 | ～ | 60 | 3 |
| Total | | | 24 |

● A section (delimiter) such as "more than or equal to 30 g and less than 35 g" is called a **class**, and the size of the section is called a **class interval**.
● The number of data counted for each class is called **frequency**. A table that shows the distribution by class or frequency is called a **frequency distribution table**.
● You can decide the class interval and use a frequency distribution table.

**1** The records of Set 1 on page 66 are summarized in the dot plot below. Answer the following questions.
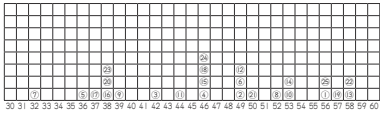
❶ Complete the table where there are blank spaces.
❷ From what number to what number represents the largest number of tomatoes?
More than or equal to **45 g** and less than **50 g**
❸ How many tomatoes are more than or equal to 35 g and less than 40 g?
**6**
❹ Which class has 4 tomatoes?
More than or equal to **50 g** and less than **55 g**

| Weight (g) | | | Number of tomatoes |
|---|---|---|---|
| more than or equal to 30 | ～ | less than 35 | 1 |
| 35 | ～ | 40 | 6 |
| 40 | ～ | 45 | 2 |
| 45 | ～ | 50 | 7 |
| 50 | ～ | 55 | 4 |
| 55 | ～ | 60 | 5 |
| Total | | | 25 |

**2** The following table summarizes the results of mathematics class exam for boys and girls. Compare which record is better.

Score of Mathematics Class Exam in Boy

| Number | Score | Number | Score | Number | Score | Number | Score | Number | Score |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 92 | 3 | 88 | 5 | 78 | 7 | 70 | 9 | 88 |
| 2 | 84 | 4 | 100 | 6 | 94 | 8 | 86 | 10 | 98 |

Score of Mathematics Class Exam in Girl

| Number | Score | Number | Score | Number | Score | Number | Score | Number | Score |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 84 | 3 | 72 | 5 | 94 | 7 | 84 | 9 | 90 |
| 2 | 100 | 4 | 100 | 6 | 92 | 8 | 78 | | |

| Score | | | Number of students |
|---|---|---|---|
| more than or equal to 70 | ～ | less than 75 | 1 |
| 75 | ～ | 80 | 1 |
| 80 | ～ | 85 | 1 |
| 85 | ～ | 90 | 3 |
| 90 | ～ | 95 | 2 |
| 95 | ～ | | 2 |
| Total | | | 10 |

| Score | | | Number of students |
|---|---|---|---|
| more than or equal to 70 | ～ | less than 75 | 1 |
| 75 | ～ | 80 | 1 |
| 80 | ～ | 85 | 2 |
| 85 | ～ | 90 | 0 |
| 90 | ～ | 95 | 3 |
| 95 | ～ | | 2 |
| Total | | | 9 |

68

69

## 6 - 3　How to Analyze Data
### Histograms (1)

**Example** The graph below was drawn based on the distribution of records of Set 2 on page 66 and its corresponding frequency distribution table. Answer the following questions.

| Weight (g) | | | Number of tomatoes |
|---|---|---|---|
| more than or equal to 0 | ～ | less than 35 | 0 |
| 35 | ～ | 40 | 7 |
| 40 | ～ | 45 | 4 |
| 45 | ～ | 50 | 7 |
| 50 | ～ | 55 | 3 |
| 55 | ～ | 60 | 3 |
| Total | | | 24 |

❶ How many tomatoes are heavier than or equal to 50 g and less than 55 g?
**3**
❷ Which class has 4 tomatoes?
More than or equal to **40 g** and less than **45 g**

● A graph like the one above is called a **histogram**. If you look at the histogram it is easy to understand the distribution.
● In a histogram, the horizontal axis represents the class interval and the vertical axis represents how many tomatoes were produced by that class.

**Instruction** Difference with a bar graph.
Bar graph and histogram use bars. What differences are there?

There is no gap between bars on histogram.

This is because it shows how many data there are within a certain interval.

The horizontal axis of a histogram is always a numerical value.

**1** The table below shows commuting time of classmates. Answer the following questions.

| Time (min) | | | Number of student |
|---|---|---|---|
| more than or equal to 0 | ～ | less than 3 | 2 |
| 3 | ～ | 6 | 6 |
| 6 | ～ | 9 | 9 |
| 9 | ～ | 12 | 7 |
| 12 | ～ | 15 | 3 |
| 15 | ～ | 18 | 1 |
| Total | | | 28 |
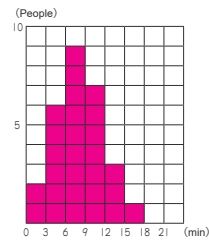
❶ Complete histogram.
❷ Which time frame represents the most students?
More than or equal to **6 min** and less than **9 min**
❸ How many students take more than or equal to 12 minutes?
**4 students**

**2** The following table summarizes the results of a language class test for girls. Complete the histogram.

| Time (score) | | | Number of student |
|---|---|---|---|
| more than or equal to 0 | ～ | less than 5 | 1 |
| 5 | ～ | 10 | 4 |
| 10 | ～ | 15 | 2 |
| 15 | ～ | 20 | 1 |
| 20 | ～ | 25 | 0 |
| 25 | ～ | 30 | 1 |
| Total | | | 10 |

70

71

19
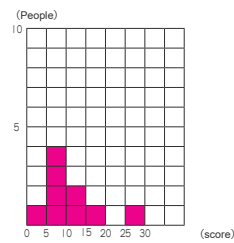
## 6 - 4　How to Analyze Data
### Histograms（2）

**Example** The following frequency distribution tables below were from the same records at throwing a softball of a class.

interval is 3 m

| Distance（m） | | | Number of students |
|---|---|---|---|
| more than or equal to | ~ | less than | |
| 9 | ~ | 12 | 3 |
| 12 | ~ | 15 | 13 |
| 15 | ~ | 18 | 16 |
| 18 | ~ | 21 | 25 |
| 21 | ~ | 24 | 21 |
| 24 | ~ | 27 | 17 |
| 27 | ~ | 30 | 5 |
| Total | | | 100 |

interval is 2 m

| Distance（m） | | | Number of students |
|---|---|---|---|
| more than or equal to | ~ | less than | |
| 9 | ~ | 11 | 1 |
| 11 | ~ | 13 | 5 |
| 13 | ~ | 15 | 10 |
| 15 | ~ | 17 | 16 |
| 17 | ~ | 19 | 19 |
| 19 | ~ | 21 | 6 |
| 21 | ~ | 23 | 11 |
| 23 | ~ | 25 | 17 |
| 25 | ~ | 27 | 10 |
| 27 | ~ | 29 | 5 |
| Total | | | 100 |

Complete the histogram on the left.



(People)
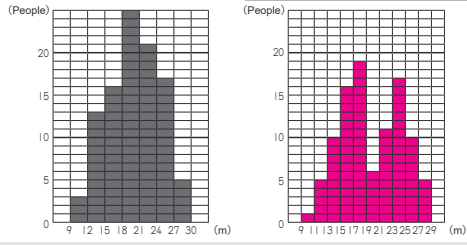
Answer the questions on the data above.

❶ Complete the histogram for 2 m interval on the right.

❷ How many mountain shapes are there?

3 m interval　**1**　　2 m interval　**2**
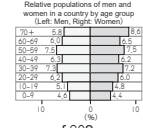
> Even using the same data, set of a class makes a shape of graph different. For example, while you can observe the most people throw a ball from 19 m to 20 m on the left graph, we can not say so on the right graph.

## 6 - 5　How to Analyze Data
### Various Graphs

**Example** The graph on the right shows the relative populations of men and women in a country by age group in 2005



Relative populations of men and women in a country by age group (Left: Men, Right: Women)

❶ Which group has the highest number of people? Answer the question for both men and women.

Men　**50-59**　　Women　**70+**

❷ What % of the total population is younger than age of 20?

Math sentence　　$5.1 + 4.6 + 4.8 + 4.4 = 18.9$

Answer　　**18.9** %

**1** The graph below shows the relative population of men and women in a country by age group for 1945 and 1975.

Relative populations of men and women in a country by age group for 1945（Left）and 1975（Right）

Total population: 71990000　　（Left: Men, Right: Women）　　Total population: 111940000



❶ For each year, which group has the highest number of people? Answer the question for both men and women

| | 1945 | 1975 |
|---|---|---|
| Men | 0-9 | 20-29 |
| Women | 0-9 | 20-29 |

❷ Choose appropriate observation.

A: The proportion of the population younger than age of 20 doesn't change from 1945 to 1975.

B: Comparing graphs in 1945 and 1975, the proportion of the population in the age of 0-9 and 10-19 turns small.

**B**

## 6 - 6　How to Analyze Data
### Review

**1** The following table summarizes the records of the number of books students in two classes borrowed. Compare which record is better.

Record of Class A Student's Borrowed Books

| Number | Books | Number | Books | Number | Books | Number | Books |
|---|---|---|---|---|---|---|---|
| 1 | 15 | 6 | 14 | 11 | 11 | 16 | 12 |
| 2 | 7 | 7 | 18 | 12 | 18 | 17 | 10 |
| 3 | 4 | 8 | 8 | 13 | 16 | 18 | 13 |
| 4 | 24 | 9 | 2 | 14 | 20 | 19 | 15 |
| 5 | 18 | 10 | 14 | 15 | 13 | 20 | 3 |

Record of Class B Student's Borrowed Books

| Number | Books | Number | Books | Number | Books | Number | Books |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 6 | 12 | 11 | 15 | 16 | 15 |
| 2 | 22 | 7 | 15 | 12 | 8 | 17 | 10 |
| 3 | 18 | 8 | 14 | 13 | 17 | 18 | 9 |
| 4 | 14 | 9 | 18 | 14 | 16 | 19 | 13 |
| 5 | 23 | 10 | 6 | 15 | 20 | | |

❶ Represent the records using data plot.

Class A
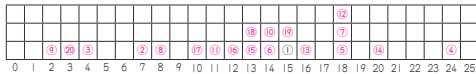


Class B
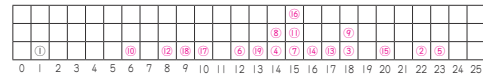


❷ Complete the table below to find the mean, mode and median values of Class A and Class B.

| | Mean | Mode | Median |
|---|---|---|---|
| Class A | 12.75 | 18 | 13.5 |
| Class B | 14 | 15 | 15 |

Finding mean value for Class A and Class B

Math sentence　　$255 ÷ 20 = 12.75$　　Math sentence　　$266 ÷ 19 = 14$

**2** The table below shows the result of a survey on learning time for G5 and G6 students at home.

Grade 5　　　　　　　　　　　　　Grade 6



Grade 5 Students　　　　　　　　Grade 6 Students

| Learning time（minutes） | | | Number of students |
|---|---|---|---|
| more than or equal to | ~ | less than | |
| 0 | ~ | 20 | 0 |
| 20 | ~ | 40 | 4 |
| 40 | ~ | 60 | 7 |
| 60 | ~ | 80 | 4 |
| 80 | ~ | 100 | 0 |
| Total | | | 15 |

| Learning time（minutes） | | | Number of students |
|---|---|---|---|
| more than or equal to | ~ | less than | |
| 0 | ~ | 20 | 1 |
| 20 | ~ | 40 | 5 |
| 40 | ~ | 60 | 1 |
| 60 | ~ | 80 | 3 |
| 80 | ~ | 100 | 6 |
| Total | | | 16 |

Learning time for Grade 5 Students　　Learning time for Grade 6 Students
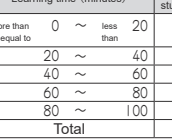


❶ Complete histogram above.

❷ Complete the table below to find the mean, mode and median values of Grade 5 and Grade 6.

| | Mean | Mode | Median |
|---|---|---|---|
| Grade 5 | 44 | 40 | 40 |
| Grade 6 | 55 | 80 | 70 |

> What can you observe from the table on the left and histogram?

❸ Choose appropriate observation.

A: Since mean, mode, and median values of Grade 5 students is lower than that of Grade 6 students, all the Grade 5 students have shorter learning time than that of Grade 6 students.
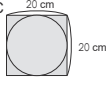
B: While mean, mode, and median values of Grade 6 students is higher than that of Grade 5 students, about half of Grade 6 students have less learning time than the mean.

**B**

## 7 − 1　Area of a Circle

### Area of a Circle (1)

**Instruction**　How to find the area of circle B.

Let's find the area of a circle with a radius of 10 cm, comparing to the area of the following figures.

A　　B　　C　20 cm

20 cm　10 cm　20 cm

The area of ◇ is composed of 4 ◁.
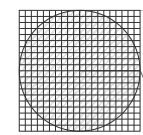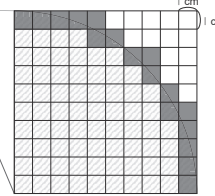
The area of square A:
Math sentence　$(10 × 10 ÷ 2) × 4 = 200$
Answer　200 cm²

The area of square C:
Math sentence　$20 × 20 = 400$
Answer　400 cm²

The area of circle B is larger than 200 cm² and smaller than 400 cm².

How we can calculate the area?

How about laying graph paper with 1 cm square sides and counting the number of squares?

**Example**　Separate the circle into 4 parts and look at one of them. Answer the following questions to find the area of the figure below.

1 cm
) 1 cm

Red squares pass through the circumference of the circle and green squares overlap the circle.

1　How many green and red squares are there?

Patterned squares　69　　Coloured squares　17

2　If we consider the red squares, as squares with an area of 0.5 cm², about how many cm² is the area of a quarter of this circle?

Green squares　69　× 1 ＝　69　(cm²)
Red squares　17　× 0.5 ＝　8.5　(cm²)

Red squares do not overlap with the circle completely. To estimate the area, we assume the red squares have 0.5 cm²

The area of a quarter of the circle is about　77.5　(cm²)

3　About how many cm² is the area of the entire circle?

Math sentence　$(77.5) × 4 = 310$　　Answer　310 cm²

1　The following diagram shows a quarter of the circle with a radius of 12 cm.

1 cm
) 1 cm

Answer the following questions.
Red squares pass through the circumference of the circle and blue squares overlap the circle.
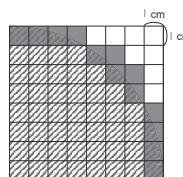
1　How many blue and red squares are there?

Patterned squares　43　　Coloured squares　13

2　If we consider the red squares as squares with an area of 0.5 cm², about how many cm² is the area of a quarter of this circle?

Blue squares　43　× 1 ＝　43　(cm²)
Red squares　13　× 0.5 ＝　6.5　(cm²)

The area of a quarter of the circle is about　49.5　(cm²)

3　About how many cm² is the area of the entire circle?

Math sentence　$(49.5) × 4 = 198$　　Answer　198 cm²

76　　　　77

## 7 − 2　Area of a Circle

### Area of a Circle (2)

**Instruction**　How to find the area of a circle (2).

Let's think about how to find the area of a circle with a radius of 5 cm.

Remember how we learned to find the area of parallelograms. Cut a part of the figure and change it to a known figure, rectangle.　Cut & Change

I see. Let us think cut the circle and rearrange a known figure. Not only rectangle, can we rearrange other shapes?

The circle was separated into 16 parts.

**Idea 1**

It is rearranged into a parallelogram.

Also remember
(Circumference) = (Diameter) × 3.14

radius
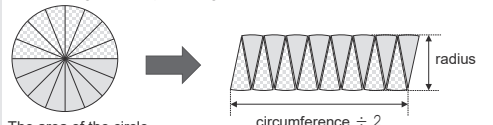
circumference ÷ 2

The area of the circle
Math sentence　$(circumference ÷ 2) × (radius)$
　　　$= 31.4 ÷ 2 × 5$
　　　$= 78.5$　　78.5 cm²

When we look at it carefully, it has curves and doesn't look like a rectangle…

As we divide the circle into small sections of equal size, it resembeles the shape of a rectangle.

**Idea 2**

In the case of dividing a circle into 16 parts, they were rearranged into a triangle.

4 times the radius

radius
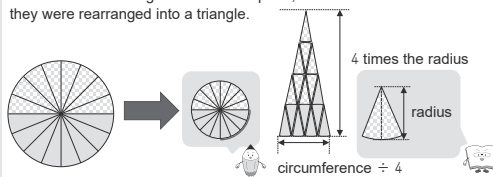
circumference ÷ 4

Since the radius is 5 cm,
The circumference is
Math sentence　$5 × 2 × 3.14 = 31.4$　　31.4 cm

The area of the circle
Math sentence　$(circumference ÷ 4) × (4 times of radius) ÷ 2$
　　　$= 31.4 ÷ 4 × 4 × 5 ÷ 2$
　　　$= 78.5$　　78.5 cm²

Idea 2 looks good, but in case you divide a circle into 8. Can you make a triangle?

Idea 1 is more applicable.

In short,
(Area of Circle) ＝ (Radius) × (Circumference ÷ 2)
　　　　　　　＝ (Radius) × (Diameter × 3.14 ÷ 2)
　　　　　　　＝ (Radius) × (Diameter ÷ 2 × 3.14)
　　　　　　　＝ (Radius) × (Radius) × 3.14

The area of a circle can be found with the following formula:
(Area of Circle) ＝ (Radius) × (Radius) × 3.14

**Example**　Find the area of the circle with a radius of 4 cm.

Since (Area of Circle) ＝ (Radius) × (Radius) × 3.14
Math sentence　$4 × 4 × 3.14 = 50.24$　　Answer　50.24 cm²　　4 cm

Find the area of the circles with the following radiuses.

1　with 10 cm　　　　　　2　with 8 cm

10 cm　　　　　　　　　8 cm

Math sentence　$10 × 10 × 3.14 = 314$　　Math sentence　$8 × 8 × 3.14 = 200.96$

Answer　314 cm²　　　　Answer　200.96 cm²

78　　　　79

21
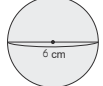
## 7 - 3　Area of a Circle

### Area of a Circle (3)

**Example 1**　Find the area of the following figures.

❶

6 cm

Find the length of the radius.
(Radius) = (Diameter) ÷ 2
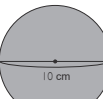
Since the diameter is 6 cm, the radius is 3 cm.

Math sentence　$3 × 3 × 3.14 = 28.26$

Answer　28.26 cm²

❷

4 cm
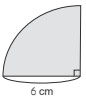
This is a half circle with a radius of 4 cm. We can find the area by calculating the area of the circle and divide it into 2.

Math sentence　$(4 × 4 × 3.14) ÷ 2 = 25.12$

Answer　25.12 cm²

**1**　Find the area of the following figures.

❶
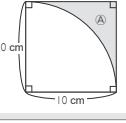
10 cm

Since the diameter is 10 cm, the radius is 5 cm.
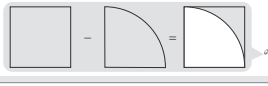
Math sentence　$5 × 5 × 3.14 = 78.5$

Answer　78.5 cm²

❷

6 cm

Since the figure is a quarter of a circle.

Math sentence　$(6 × 6 × 3.14) ÷ 4 = 28.26$

Answer　28.26 cm²

**Example 2**　Find the area of the A parts.

Ⓐ

10 cm

10 cm

What kind of figures do you observe? Also, what does it consist of?

□ − ◿ = ◺

---
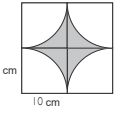
The area of the square

Math sentence　$10 × 10 = 100$

The area of the quarter of the circle

Math sentence　$(10 × 10 × 3.14) ÷ 4 = 314 ÷ 4 = 78.5$

The area of figure A

Math sentence　$100 − 78.5 = 21.5$

Answer　21.5 cm²

**2**　Find the area of the coloured parts.

10 cm

10 cm

How many are the part A above?

Separate the circle into 4 parts and look at one of them.
The radius of the quarter circle is 10 cm.
The area is 4 times of the area of the quarter of the circle

Math sentence　$4 × 21.5 = 86$

Answer　86 cm²

Alternatively, The area of the figure is as follows:
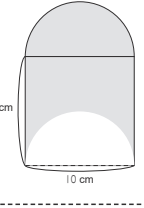
Math sentence　$20 × 20 = 400$
$10 × 10 × 3.14 = 314$
$400 − 314 = 86$

Let's Try !

Find the area of the following figure below.
If we move the half circle to the bottom of the figure, it is easy to find the area.

Math sentence　$10 × 10 = 100$

Answer　100 cm²

10 cm

10 cm

80

81

---
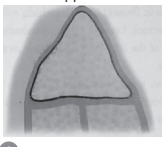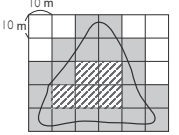
## 7 - 4　Area of a Circle

### Approximate Area

**Example 1**　The illustration below shows a field that lies between rivers.

Find the approximate area of the field surrounded by the black line.

Can we do the same as when we calculated the approximate area of a circle?

10 m

10 m

❶　In the diagram on the right, how many blue and red squares are there?

Blue squares　5　▨

Red squares　18　▨

❷　If we consider the red squares as squares with an area of 50 m², about how many m² is the area of a field that lies between rivers?

Blue squares　5　× 100 = 500　(m²)
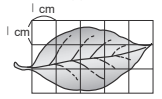
Red squares　18　× 50 = 900　(m²)

We assume Red squares have the half area of Blue squares.

Approximate area of the field is 1400 (m²)

**1**　Find the approximate area of the leaf below.

1 cm

1 cm

Colour the squares which pass through the edge of the leaf in red and colour the squares that overlap the leaf in blue.

❶　In the diagram on the right, how many blue and red squares are there?

Blue squares　3　▨

Red squares　11　▨

1 cm

1 cm
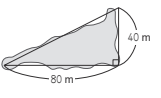
**2**　If the red squares have an area of 0.5 cm², how many cm² is the area of a quarter of this circle?

Blue squares　3　× 1 = 3　(cm²)

Red squares　11　× 0.5 = 5.5　(cm²)

Approximate area of the field is　8.5　(cm²)

**Example 2**　The illustration below shows a lake. Find the approximate area by considering it as a triangle.

40 m

80 m

Math sentence　$80 × 40 ÷ 2 = 1600$

Answer　1600 m²

By considering the object as a known figure, we can find approximate area.

**2**　The illustration below shows an island. Find the approximate area by considering it as a parallelogram.

24 km

36 km

Math sentence　$36 × 24 = 864$

Answer　864 km²

**3**　When you find the approximate area of the lake. If the actual area is 11 km², which approximate area is closer to the actual one?

5 km

2 km

3 km

2 km

Considering it as a circle

Math sentence　$2 × 2 × 3.14 = 12.56$

Answer　12.56 km²

Considering it as a trapezoid

Math sentence　$(5 + 2) × 3 ÷ 2 = 10.5$
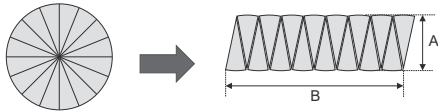
Answer　10.5 km²

Considering it as a　Trapezoid　is closer to the actual one.

82

83

22

## 7 - 5　Area of a Circle

**Review**

**1**　Thinking about how to find the area of a circle.

As we divide the circle into small sections of equal size, it approaches the shape of a rectangle.

Choose appropriate words and fill in the blanks.

The length of A is the same as the 　**Radius**　 of a circle.

Since the length of B is the same as a half of the 　**Circumference**　,

（Length of B）＝ 　**Diameter**　 ×（Ratio of circumference）÷ 2

　　　　　　　＝ 　**Radius**　 ×（Ratio of circumference）

| Circumference | Radius | Diameter | Centre |
|---|---|---|---|

**2**　Find the area of the following figures.

**1** Since the diameter is 20 cm, the radius is 10 cm.

Math sentence　$10 × 10 × 3.14 = 314$

Answer　$314 \text{ cm}^2$

**2** Since the figure is a half of a circle.

Math sentence　$(6 × 6 × 3.14) ÷ 2 = 56.52$

Answer　$56.52 \text{ cm}^2$

**3**　Find the area of the circles with the following length.

**1** with a radius of 7 cm

Math sentence　$7 × 7 × 3.14 = 153.86$

Answer　$153.86 \text{ cm}^2$

**2** with a diameter of 10 cm

Math sentence　$5 × 5 × 3.14 = 78.5$

Answer　$78.5 \text{ cm}^2$

**4**　Find the area of the coloured parts.

A　　　　B

Figure A

Math sentence　$3 × 3 × 3.14 = 28.26$

Answer　$28.26 \text{ cm}^2$

Figure B

Math sentence

Area of the larger circle
$5 × 5 × 3.14 = 78.5$

Area of the smaller circle
$3 × 3 × 3.14 = 28.26$

Area of figure A
$78.5 − 28.26 = 50.24$

Answer　$28.26 \text{ cm}^2$

**5**　The illustration below shows a lake. Answer the thefollowing questions.

**1** What kind of shape can you use it to find the area?

**Triangle**

**2** Find the approximate area.

Math sentence　$116 × 90 ÷ 2 = 5220$

Answer　$5220 \text{ m}^2$

## 8 - 1　Proportion and Inverse Proportion

**Meaning of Proportion**

**Example 1**　The diagram below shows the depth of water in a rectangular shaped tank at one minute intervals.
Let's think about the relationship between the time it takes to fill the tank with water and the depth of the water.

**1**　If you double, triple, or quadruple the time to fill the water in the tank, how does the depth of the water change accordingly?

| Time（minutes） | 1 | 2 | 3 | 4 | 5 | 6 | … |
|---|---|---|---|---|---|---|---|
| Depth（cm） | 2 | 4 | 6 | 8 | 10 | 12 | … |
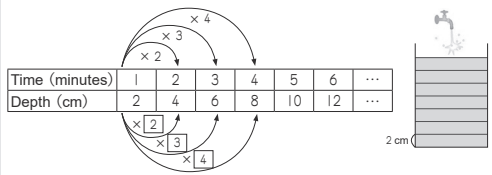
**2**　If you one-half, one-third, or one-fourth the time to fill the water in the tank, how does the depth of the water change accordingly?

| Time（minutes） | 1 | 2 | 3 | 4 | 5 | 6 | … |
|---|---|---|---|---|---|---|---|
| Depth（cm） | 2 | 4 | 6 | 8 | 10 | 12 | … |
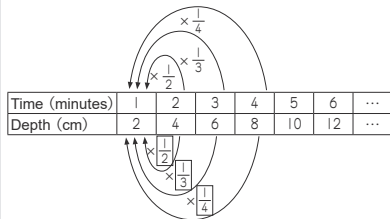
If we have two numbers $x$ and $y$, and if $x$ is doubled, tripled or quadrupled, or 1/2, 1/3 or 1/4 times as many, and $y$ is also doubled, tripled or quadrupled, or 1/2, 1/3 or 1/4 as many, we say that $x$ and $y$ are proportional. In other words, we say that "$y$ **is proportional to** $x$".

The table below shows the number of nails and their weights. Each nail weighs 6 g.

**1**　If the number of nails doubles, triples or quadruples, how does the weight change? Fill in the □ .
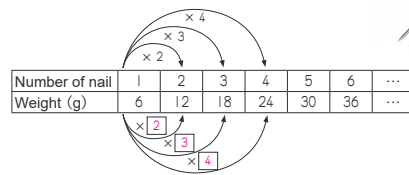
| Number of nail | 1 | 2 | 3 | 4 | 5 | 6 | … |
|---|---|---|---|---|---|---|---|
| Weight（g） | 6 | 12 | 18 | 24 | 30 | 36 | … |

**2**　If you make the number of nails one-half, one-third, or one-fourth, how does the weight change? Fill in the □ .

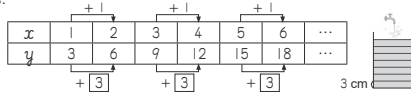| Number of nail | 1 | 2 | 3 | 4 | 5 | 6 | … |
|---|---|---|---|---|---|---|---|
| Weight（g） | 6 | 12 | 18 | 24 | 30 | 36 | … |
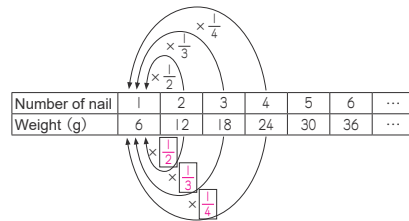
**3**　What is the relationship between the number of nails and their weight?

Answer　**Weight is proportional to number of nail**

## 8 - 2 Proportion and Inverse Proportion
### Expressing Proportion with a Math Sentence

**Example** The table below shows the depth of water in a rectangular shaped tank at one minute intervals.

| Time $x$ (minutes) | 1 | 2 | 3 | 4 | 5 | 6 | … |
|---|---|---|---|---|---|---|---|
| Depth $y$ (cm) | 3 | 6 | 9 | 12 | 15 | 18 | … |

❶ Find out how many values of $y$ increase as the value of $x$ increases by one.

| | | +1 | | +1 | | +1 | | |
|---|---|---|---|---|---|---|---|---|
| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | … | |
| $y$ | 3 | 6 | 9 | 12 | 15 | 18 | … | |
| | | +3 | | +3 | | +3 | | |

3 cm

❷ What is the relationship between the value for time $x$ and the corresponding value for the depth $y$ of the water?

Answer  $y$ is proportional to $x$

❸ Using the fact that the depth of water per 1 minute is 3 cm, investigate the relationship between the amount of water and its depth, and express the relationship between $x$ and $y$ in an math sentence.

| Depth of water $y$ (cm) | Depth of water per minute (cm) | time $x$ (minute) |
|---|---|---|
| 3 | 3 × | 1 |
| 6 | 3 × | 2 |
| 9 | 3 × | 3 |
| 12 | 3 × | 4 |
| $y$ | 3 × | $x$ |

Answer  $y = 3 × x$

If we have two quantities $x$ and $y$, and $y$ is proportional to $x$, we can express this relationship by the following math sentence
$$y = \text{Fix number} × x$$

❹ What is the depth of the water if it is filled for 15 minutes?

Math sentence  $y = 3 × 15 = 45$    Answer  45 cm

❺ how many minutes will it take for the depth of the water to reach 60 cm?    $60 = 3 × x$

Math sentence  $x = 60 ÷ 3 = 20$    Answer  20 minutes

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For the square on the right, here is a table showing the relationship between the length of one side, $x$ cm, and the length of the perimeter.

$x$ cm
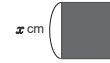
| Length of 1 side $x$ (cm) | 1 | 2 | 3 | 4 | 5 | 6 | … |
|---|---|---|---|---|---|---|---|
| Perimeter $y$ (cm) | 4 | 8 | 12 | 16 | 20 | 24 | … |

❶ Find out how many values of $y$ increase as the value of $x$ increases by one.

| | | +1 | | +1 | | +1 | | |
|---|---|---|---|---|---|---|---|---|
| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | … | |
| $y$ | 4 | 8 | 12 | 16 | 20 | 24 | … | |
| | | +4 | | +4 | | +4 | | |
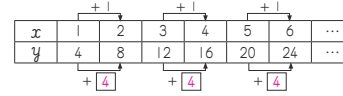
❷ What is the relationship between the value for the length of 1 side $x$ and the corresponding value for the perimeter $y$?

Answer  $y$ is proportional to $x$

❸ Using the fact that number of the side is 4 cm, investigate the relationship between the length of 1 side and its perimeter, and express the relationship between $x$ and $y$ in an math sentence.

| Perimeter $y$ (cm) | Number of the side | Length of 1 side $x$ (cm) |
|---|---|---|
| 4 | 4 × | 1 |
| 8 | 4 × | 2 |
| 12 | 4 × | 3 |
| 16 | 4 × | 4 |
| $y$ | 4 × | $x$ |

Answer  $y = 4 × x$

❹ If the value of $x$ is 8, what is the value of $y$?

Math sentence  $y = 4 × 8$    Answer  $y = 32$

❺ What is the perimeter if the length of 1 side is 12 cm?

Math sentence  $y = 4 × 12 = 48$    Answer  48 cm

❻ How long the length of 1 side is if the perimeter is 60 cm?

Math sentence  $60 = 4 × x$    Answer  15 cm
$x = 60 ÷ 4 = 15$

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## 8 - 3 Proportion and Inverse Proportion
### Drawing a Graph of Proportion

**Example** The table below shows the depth of water in a rectangular shaped tank at one minute intervals.

| Time $x$ (minutes) | 0 | 1 | 2 | 3 | 4 | 5 | … |
|---|---|---|---|---|---|---|---|
| Depth $y$ (cm) | 0 | 2 | 4 | 6 | 8 | 10 | … |

1 cm

❶ For a time of $x$ minutes and a corresponding depth of water of $y$ cm, draw a point on the graph below that represents a pair of values of $x$ and $y$.
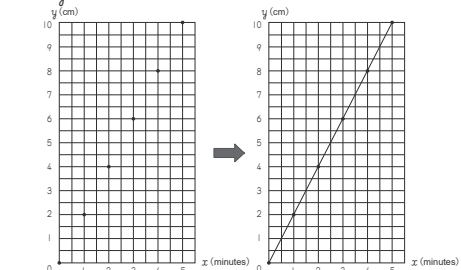


❷ What is the relationship between $x$ (minutes) and $y$ (depth)?

Answer  $y$ is proportional to $x$

❸ Using the fact that depth of water per minute is 2 cm, investigate the relationship between time and depth of water, and express the relationship between $x$ and $y$ in an math sentence.

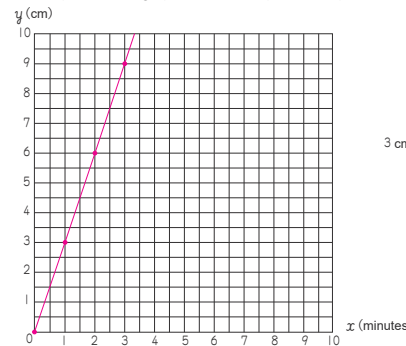| Depth of water $y$ (cm) | Depth of water per minute (cm) | Time $x$ (minutes) |
|---|---|---|
| 2 | 2 × | 1 |
| 4 | 2 × | 2 |
| 8 | 2 × | 3 |
| $y$ | 2 × | $x$ |

Answer  $y = 2 × x$

If we connect the points representing the corresponding pairs of $x$ and $y$ values with a line, we get a line like the one above. This line is the graph of $y = 2 × x$.

90

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The table below shows the depth of water in a rectangular shaped tank at one minute intervals.

| Time $x$ (minutes) | 0 | 1 | 2 | 3 | 4 | 5 | … |
|---|---|---|---|---|---|---|---|
| Depth $y$ (cm) | 0 | 3 | 6 | 9 | 12 | 15 | … |

❶ For a time of $x$ minutes and a corresponding depth of water of $y$ cm, draw a point on the graph below that represents a pair of values of $x$ and $y$.



3 cm

❷ What is the relationship between $x$ (minutes) and $y$ (depth)?

Answer  $y$ is proportional to $x$

❸ Using the fact that depth of water per minute is 3 cm, investigate the relationship between time and depth of water, and express the relationship between $x$ and $y$ in a math sentence.

| Depth of water $y$ (cm) | Depth of water per minute (cm) | Time $x$ (minutes) |
|---|---|---|
| 3 | 3 × | 1 |
| 6 | 3 × | 2 |
| 9 | 3 × | 3 |
| $y$ | 3 × | $x$ |

Answer  $y = 3 × x$

❹ Write the graph represent the relationship between $x$ and $y$.
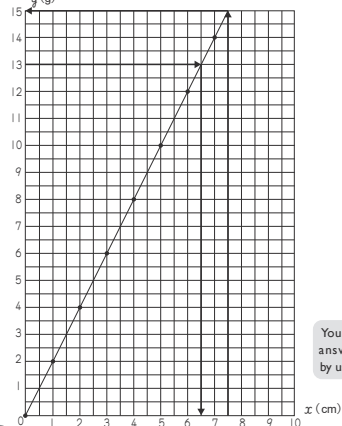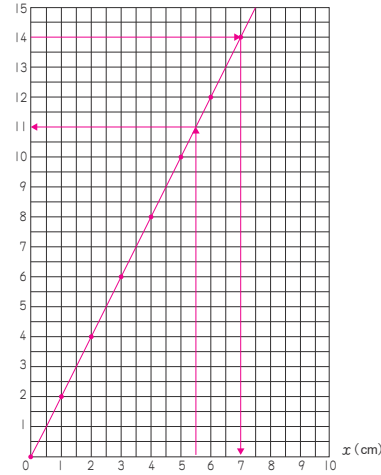
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**8 - 4** Proportion and Inverse Proportion
**Reading a Graph of Proportion**（1）

**Example** The table below shows the relationship between the length of the wire $x$ cm and the weight $y$ g.

| Length $x$ (cm) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | … |
|---|---|---|---|---|---|---|---|---|---|
| Weight $y$ (g) | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | … |

❶ Draw a graph to show the relationship between $x$ and $y$.



You can find the answer of ③, ④ by using the graph!

❷ Using the fact that weight of wire per cm is 2 g, investigate the relationship between length of wire and its weight, and express the relationship between $x$ and $y$ in an math sentence
Answer $y = 2 \times x$

❸ When a wire is 7.5 cm long, how many g does it weigh?
Answer 15 g

❹ How many cm of wire will it take if it weighs 13 g?
Answer 6.5 cm

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The table below shows the relationship between the length of the wire $x$ cm and the weight $y$ g.

| Length $x$ (cm) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | … |
|---|---|---|---|---|---|---|---|---|
| Weight $y$ (g) | 0 | 2 | 4 | 6 | 8 | 10 | 12 | … |

❶ Draw a graph to show the relationship between $x$ and $y$.



❷ Using the fact that weight of wire per cm is 2 g, investigate the investigate the relationship between length of the wire and weight and express relationship between $x$ and $y$ in an math sentence.
Answer $y = 2 \times x$

❸ When a wire is 5.5 cm long, how many g does it weigh?
Answer 11 g

❹ How many cm of wire weighs 14 g?
Answer 7 cm

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**8 - 5** Proportion and Inverse Proportion
**Reading a Graphs of Proportion**（2）

**Example** The graph below shows the relationship between the time ($x$ hours) and the journey ($y$ km) since the bike and the car set off at the same time. Read the graph.
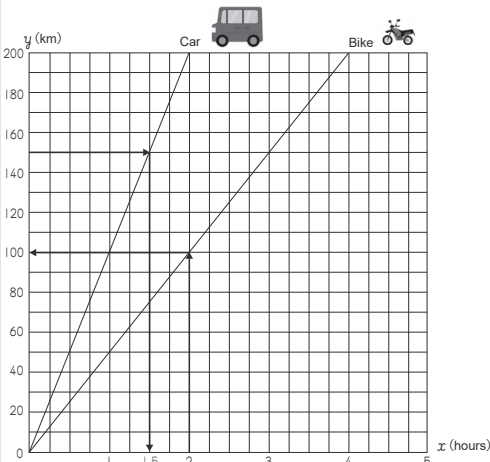


❶ How many km does a bike go in two hours?
Answer 100 km

❷ How many hours does it take for a car to travel 150 km?
Answer 1.5 hours

❸ Looking at the graph, what are the speeds of the bike and the car?

Speed is km per one hour, so…

Answer Car 100 km/hours Bike 50 km/hours

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The graph below shows the relationship between the time ($x$ hours) and the journey ($y$ km) since the bicycle and the truck set off at the same time. Read the graph.
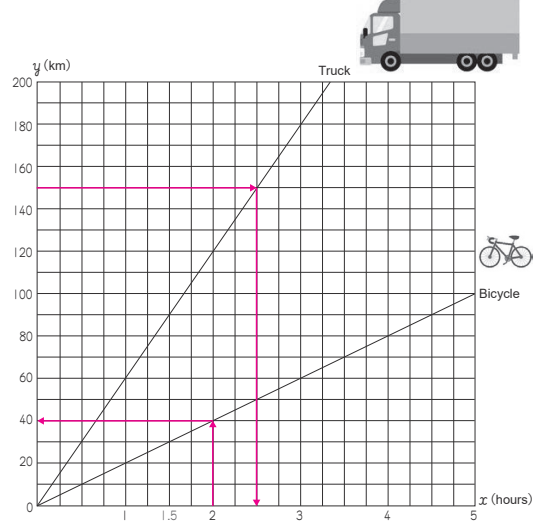


❶ How many km does a bicycle go in two hours?
Answer 40 km

❷ How many hours does it take for a truck to travel 150 km?
Answer 2.5 hours

❸ Looking at the graph, what are the speeds of the truck and the car?
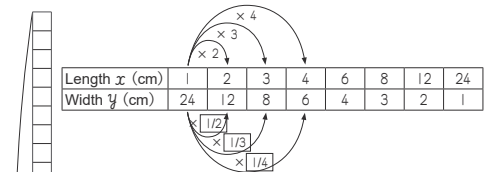Answer Truck 60 km/hour Bicycle 20 km/hour

95

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## 8 - 6

Proportion and Inverse Proportion

**Review** (1)

**1** The table below shows the relationship between the price of the wire $x$ cm and the price $y$ zeds.

| Length $x$ ($x$ cm) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | … |
|---|---|---|---|---|---|---|---|---|
| Price $y$ ($y$ zeds) | 0 | 30 | 60 | 90 | 120 | 150 | 180 | … |

**1** What is the relationship between the length ($x$ cm) and the price ($y$ zeds)?

Answer  $y$ is proportional to $x$

**2** How can you express the relationship between $x$ and $y$ in the math sentence if the length of wire is $x$ cm and the price of wire is $y$ zeds?

Answer  $y = 30 \times x$

**3** When the value of $x$ is 8, what is the value of $y$?

Math sentence  $y = 30 \times 8$　　　Answer  $y = 240$

**4** What is the price of an 8 cm wire?

Math sentence  $y = 30 \times 8 = 240$　　　Answer  240 zeds

**5** How long is a wire whose price is 360 zeds?

Math sentence  $360 = 30 \times x$ ($360 \div 30 = x$)　　Answer  12 cm

**2** The graph below shows the relationship between the time ($x$ hours) and the journey ($y$ km) since the bus and the motorbike set off at the same time. Read the graph.
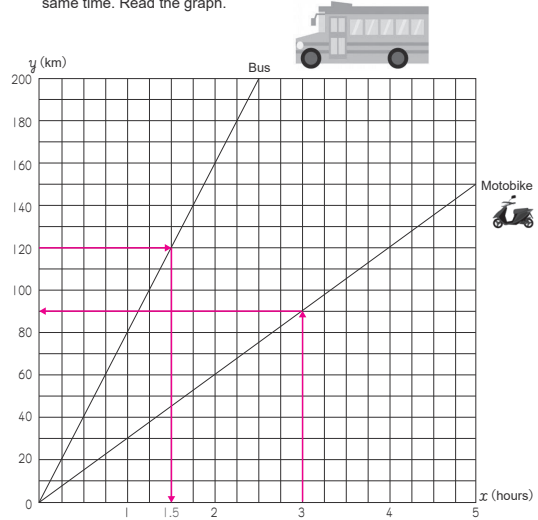


**1** How many km does a motorbike go in three hours?

Answer  90 km

**2** How many hours does it take for a bus to travel 120 km?

Answer  1.5 hours

**3** Looking at the graph, what are the speeds of the bus and the motorbike?

Answer

Bus  80 km/hour

Motorbike  30 km/hour

## 8 - 7

Proportion and Inverse Proportion

**Expressing Inverse Proportion with a Math Sentence**

**Example** The table summarizes the relationship between the length and width of a rectangle with an area of 24 cm². Let's think about the relationship between the length and the width of the rectangle.

**1** If you double, triple, or quadruple the length of the rectangle, how does the width of the rectangle change accordingly?

| Length $x$ (cm) | 1 | 2 | 3 | 4 | 6 | 8 | 12 | 24 |
|---|---|---|---|---|---|---|---|---|
| Width $y$ (cm) | 24 | 12 | 8 | 6 | 4 | 3 | 2 | 1 |

(× 2, × 3, × 4 above; × 1/2, × 1/3, × 1/4 below)

**2** If you one-half, one-third, or one-fourth the length of the rectangle, how does the width of the rectangle accordingly?

| Length $x$ (cm) | 1 | 2 | 3 | 4 | 6 | 8 | 12 | 24 |
|---|---|---|---|---|---|---|---|---|
| Width $y$ (cm) | 24 | 12 | 8 | 6 | 4 | 3 | 2 | 1 |

$\times \frac{1}{4}$, $\times \frac{1}{3}$, $\times \frac{1}{2}$ above; × 2, × 3, × 4 below
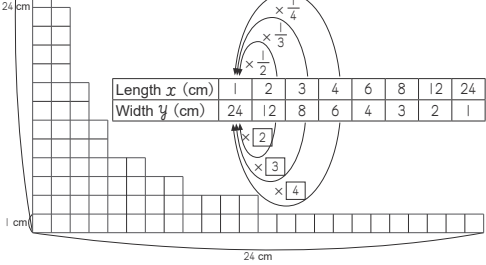


If we have two numbers $x$ and $y$, and If $x$ is doubled, tripled or quadrupled, $y$ is 1/2, 1/3 or 1/4 as many, we say that $x$ and $y$ are inversely proportional. In other words, "**y is inversely proportional to $x$**"

The table summarizes the relationship between length and width of the rectangle with 72 cm². Let's think about the relationship between the length and the width of the rectangle.

**1** If you double, triple, or quadruple the length of the rectangle, how does the width of the rectangle change accordingly?

| Length $x$ (cm) | 1 | 2 | 3 | 4 | 5 | 6 | … |
|---|---|---|---|---|---|---|---|
| Width $y$ (cm) | 72 | 36 | 24 | 18 | 14.4 | 12 | … |

(× 2, × 3, × 4 above; × 1/2, × 1/3, × 1/4 below)

**2** If you make the number of nails one-half, one-third, or one-fourth, how does the weight change? Fill in the □.

| Length $x$ (cm) | 1 | 2 | 3 | 4 | 5 | 6 | … |
|---|---|---|---|---|---|---|---|
| Width $y$ (cm) | 72 | 36 | 24 | 18 | 14.4 | 12 | … |

($\times \frac{1}{4}$, $\times \frac{1}{3}$, $\times \frac{1}{2}$ above; × 2, × 3, × 4 below)
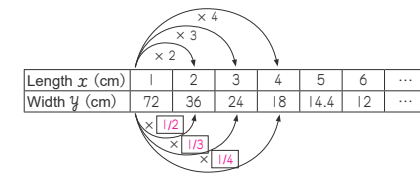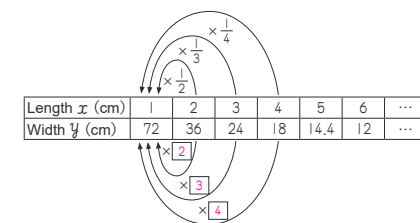
**3** What is the relationship between $x$ and $y$?

Answer  $y$ is inversely proportional to $x$

## 8 – 8 Proportion and Inverse Proportion
### Math Sentence of Inverse Proportion

**Example** The table summarizes the relationship between length and width of the rectangle with 24 cm².

| Length $x$ (cm) | 1 | 2 | 3 | 4 | 5 | 6 | ··· |
|---|---|---|---|---|---|---|---|
| Width $y$ (cm) | 24 | 12 | 8 | 6 | 4.8 | 4 | ··· |

**1** Find out how many values of $y$ increase as the value of $x$ increases by one.

×2          ×3

| Length $x$ (cm) | 1 | 2 | 3 | 4 | 5 | 6 | ··· |
|---|---|---|---|---|---|---|---|
| Width $y$ (cm) | 24 | 12 | 8 | 6 | 4.8 | 4 | ··· |

÷ 2          ÷ 3

**2** What is the relationship between $x$ and $y$ is?

Answer $y$ is inversely proportional to $x$

**3** Using the fact that the area of rectangle is 24 m², investigate the relationship between the length and width, and express the relationship between $x$ and $y$ in an math sentence.

| Length $x$ (cm) | | Width $y$ (cm) | | Area (m²) |
|---|---|---|---|---|
| 1 | × | 24 | = | 24 |
| 2 | × | 12 | = | 24 |
| 3 | × | 8 | = | 24 |
| $x$ | × | $y$ | = | 24 |

If there are two quantities $x$ and $y$, and $y$ is inversely proportional to $x$, then this relationship can be expressed by the math sentence.
**"$x × y =$ Fixed number"**

**4** If the value of $x$ is 8, what is the value for $y$?
Math sentence
$8 × y = 24$
$y = 24 ÷ 8$
Answer $y = 6$

**5** If the width of rectangle is 12 cm, what is the length of the rectangle?
Math sentence
$12 × y = 24$
$y = 24 ÷ 12$
Answer 2 cm

100

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The table summarizes the relationship between speed of walking and time for a 36 km walk.

| Speed $x$ (km/hour) | 1 | 2 | 3 | 4 | 5 | 6 | ··· |
|---|---|---|---|---|---|---|---|
| Time $y$ (hours) | 36 | 18 | 12 | 9 | 7.2 | 6 | ··· |

**1** Find out how many values of $y$ increase as the value for $x$ increases by one.

×2          ×3

| Speed $x$ (km/hour) | 1 | 2 | 3 | 4 | 5 | 6 | ··· |
|---|---|---|---|---|---|---|---|
| Time $y$ (hours) | 36 | 18 | 12 | 9 | 7.2 | 6 | ··· |

÷ 2          ÷ 3

**2** What is the relationship between $x$ and $y$ is?

Answer $y$ is inversely proportional to $x$

**3** Using the fact that the distance is 36 m, investigate the relationship between the speed and time, and express the relationship between $x$ and $y$ in a math sentence?

| Speed $x$ (km/hour) | | Time $y$ (hours) | | distance (km) |
|---|---|---|---|---|
| 1 | × | 36 | = | 36 |
| 2 | × | 18 | = | 36 |
| 3 | × | 12 | = | 36 |
| $x$ | × | $y$ | = | 36 |

Answer $x × y = 36$

**4** If the value of $x$ is 9, what is the value for $y$?
Math sentence
$9 × y = 36$
$y = 36 ÷ 9$
Answer $y = 4$

**5** If the time is 8 hours, what is the speed?
Math sentence
$x × 8 = 36$
$x = 36 ÷ 8$
Answer 4.5 km/hour

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## 8 – 9 Proportion and Inverse Proportion
### Drawing a Graph of Inverse Proportion (1)

**Example** The table summarizes the relationship between length and width of the rectangle with 24 cm².

| Length $x$ (cm) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Width $y$ (cm) | 24 | 12 | 8 | 6 | 4.8 | 4 | 3.42… | 3 | 2.66… | 2.4 | 2.18… |

**1** For a length of $x$ cm and a corresponding width of $y$ cm, find and draw a point on the graph below that represents a pair of values for $x$ and $y$.

| $x$ (cm) | 12 | 13 | 14 | 15 | 16 | ··· | 24 |
|---|---|---|---|---|---|---|---|
| $y$ (cm) | 2 | 1.84… | 1.71… | 1.6 | 1.5 | ··· | 1 |
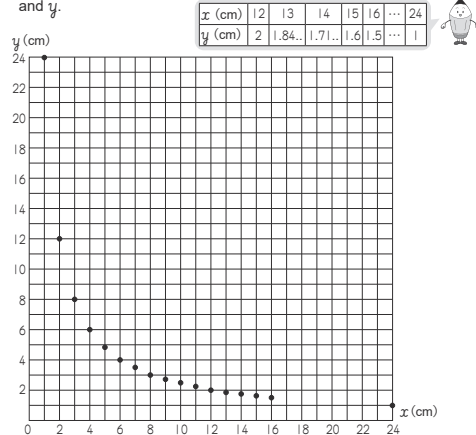


**2** What is the relationship the length ($x$ cm) and the width ($y$ cm)?
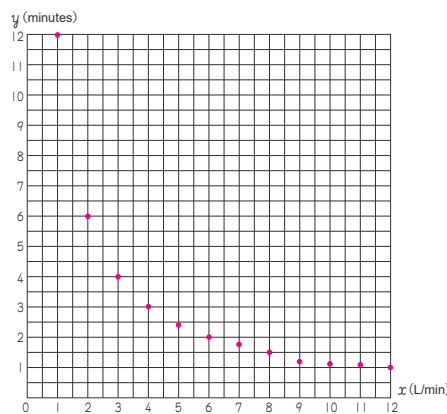Answer $y$ is inversely proportional to $x$

**3** How can you express the relationship between $x$ and $y$ in the math sentence if the length is $x$ cm and the width is $y$ cm?
$x × y =$ fix number. So $y =$ fix number $÷ x$
Answer $x × y = 24$

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There is a tank that holds 12 L of water. Find the relationship between the volume of water you fill the tank per minute and the time it takes for the tank to fill up in the table below.

| Water $x$ (L/min) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Time $y$ (minutes) | 12 | 6 | 4 | 3 | 2.4 | 2 | 1.71… | 1.5 | 1.33… |

| $x$ (L/min) | 10 | 11 | 12 |
|---|---|---|---|
| $y$ (minutes) | 1.2 | 1.09… | 1 |

**1** For a water of $x$ L/min and a corresponding time of $y$ minutes, draw a point on the graph below that represents a pair of values for $x$ and $y$.



**2** What the relationship between water ($x$ L/min) and time ($y$ minutes)?
Answer $y$ is proportional to $x$

**3** How can you express the relationship between $x$ and $y$ in the math sentence if the time to fill the water is $x$ L/min and the time is $y$ minutes?
Answer $x × y = 12$

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### 8 – 10 Proportion and Inverse Proportion
**Drawing a Graph of Inverse Proportion** (2)

**Example** The table summarizes the relationship between speed and the time it takes a car to travel between villages A and B.
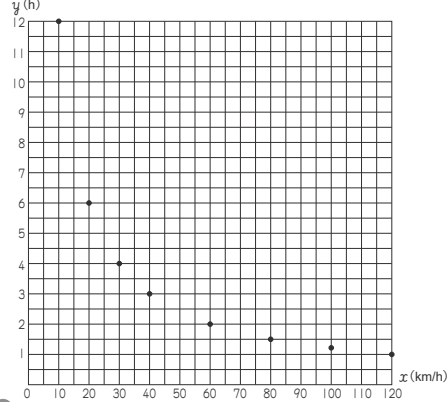
**1** How can you express the relationship between $x$ and $y$ in the math sentence if the speed is $x$ km/h and the time is $y$ hours?

Answer $x \times y = 120$

$x \times y =$ fix number. It can also be represented as $y =$ fix number $\div x$

**2** Fill in the table.

| Speed $x$ (km/h) | 10 | 20 | 30 | 40 | 60 | 80 | 100 | 120 |
|---|---|---|---|---|---|---|---|---|
| Time $y$ (hours) | 12 | 6 | 4 | 3 | 2 | 1.5 | 1.2 | 1 |

**3** For speed of $x$ and a corresponding time of $y$ draw a point on the graph below that represents the values for $x$ and $y$.



**4** Fill in the table, if $x$ is 15, 25, 75.

| Speed $x$ (km/h) | 15 | 25 | 75 |
|---|---|---|---|
| Time $y$ (hours) | 8 | 4.8 | 1.6 |

There is a job that will take 60 days if one person does the same amount of work per day. It will take $y$ days if $x$ people do this job. Let's consider the relationship between $x$ and $y$.

**1** How can you express the relationship between $x$ and $y$ in the math sentence if the number of people is $x$ and the number of days?

Answer $x \times y = 60$

$x \times y =$ fix number. It can also be represented as $y =$ fix number $\div x$

**2** Fill in the table.

| People $x$ (person) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Days $y$ (Day) | 60 | 30 | 20 | 15 | 12 | 10 |

| $x$ (person) | 8 | 10 | 12 |
|---|---|---|---|
| $y$ (Day) | 7.5 | 6 | 5 |

**3** For $x$ people and a corresponding $y$ day, draw a point on the graph below that represents a pair of values for $x$ and $y$.
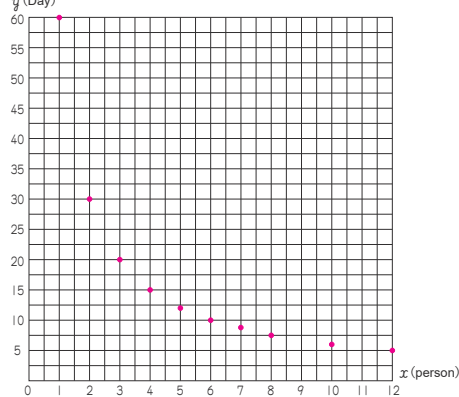


**4** Fill in the table, if $x$ is 1.5, 2.5, 7.5.

| People $x$ (person) | 1.5 | 2.5 | 7.5 |
|---|---|---|---|
| Days $y$ (Day) | 40 | 24 | 8 |

### 8 – 11 Proportion and Inverse Proportion
**Review** (2)

**1** The table below shows the relationship between the base and the height of a parallelogram with an area of 30 cm$^2$.

**1** Fill in the table.

| Base $x$ (cm) | 1 | 2 | 3 | 4 | 5 | 6 | … |
|---|---|---|---|---|---|---|---|
| Height $y$ (cm) | 30 | 15 | 10 | 7.5 | 6 | 5 | … |

**2** What is the relationship between the value for base and the corresponding value for height?

| Base $x$ (cm) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| | $\times$ ___ = 30 | $\times$ ___ = 30 | $\times$ ___ = 30 | $\times$ ___ = 30 |
| Height $y$ (cm) | 30 | 15 | 10 | 7.5 |

**3** What is the relationship between the base of the parallelogram ($x$ cm) and the height of the parallelogram ($y$ cm)?

Answer $y$ is inversely proportional to $x$

**4** How can you express the relationship between $x$ and $y$ in the math sentence if the base of the parallelogram is $x$ cm and the height of the parallelogram is $y$ cm?

Answer $x \times y = 30$

**5** If the value of $x$ is 10, what is the value of $y$?

Math sentence $y = 30 \div 10$

Answer $y = 3$

**6** If the height of the parallelogram is 8 cm, what is the base of the parallelogram?

Math sentence $x \times 8 = 30$
$x = 30 \div 8$

Answer 3.75 cm

**2** The table below shows the relationship between the amount of water $x$ L per minute and the time $y$ minutes it takes to fill a water bottle with 18 L of water.

**1** How can you express the relationship between $x$ and $y$ in the math sentence if the amount of water $x$ L per minutes and the time is $y$ minutes?
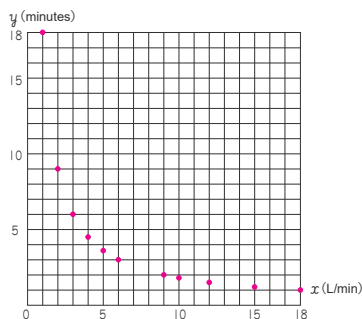
Answer $x \times y = 18$

**2** Fill in the table.

| Water $x$ (L/min) | 1 | 2 | 3 | 4 | 5 | 6 | 9 |
|---|---|---|---|---|---|---|---|
| Time $y$ (minutes) | 18 | 9 | 6 | 4.5 | 3.6 | 3 | 2 |

| $x$ (L/min) | 10 | 12 | 15 | 18 |
|---|---|---|---|---|
| $y$ (minutes) | 1.8 | 1.5 | 1.2 | 1 |

**3** For $x$ L/min and a corresponding $y$ minutes, draw a point on the graph below that represents a pair of values of $x$ and $y$.



**4** Fill in the table, if $x$ is 1.5, 2.5, 7.5.

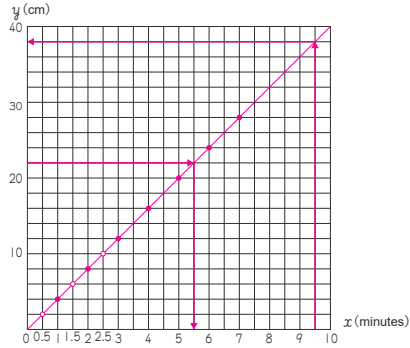| Water $x$ (L/min) | 1.5 | 2.5 | 4.5 | 7.5 |
|---|---|---|---|---|
| Time $y$ (minutes) | 12 | 7.2 | 4 | 2.4 |

**28**

## 8 – 12　Proportion and Inverse Proportion
### Review (3)

**1** The table below shows the depth of water in a rectangular shaped tank at one minutes intervals.

| Time (minutes) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | … |
|---|---|---|---|---|---|---|---|---|
| Depth (cm) | 4 | 8 | 12 | 16 | 20 | 24 | 28 | … |

① Draw a graph to show the relationship between $x$ and $y$.



② Using the math sentence $y = 4 × x$ to show the relationship between $x$ and $y$, find the value of $y$ when the value of $x$ is 0, 0.5, 1.5 and 2.5. Draw a graph showing the points that represent the values of $x$ and $y$.

③ Draw the graph representing the time ($x$ minutes) to fill the water and the depth of the water ($y$ cm).

④ How many cm is it if the time is 9.5 minutes?

Answer ___38 cm___

⑤ How many minutes does it take if the depth is 22 cm?

Answer ___5.5 minutes___

**2** This table summarizes the relationship between the length $x$ cm and the width $y$ cm of a rectangle with an area of 60 cm². Let's consider the relationship between $x$ and $y$.

① How can you express the relationship between $x$ and $y$ in the math sentence if the length is $x$ cm and the width is $y$ cm?

Answer $x × y = 60$　$x × y$ = fix number. So $y$ = fix number ÷ $x$

② Fill in the table.

| Length $x$ (cm) | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|---|---|
| Width $y$ (cm) | 60 | 30 | 20 | 15 | 12 | 10 | 7.5 | 6 | 5 |

③ For the length of $x$ cm and a corresponding width of $y$ cm, draw a point on the graph below that represents a pair of values of $x$ and $y$.
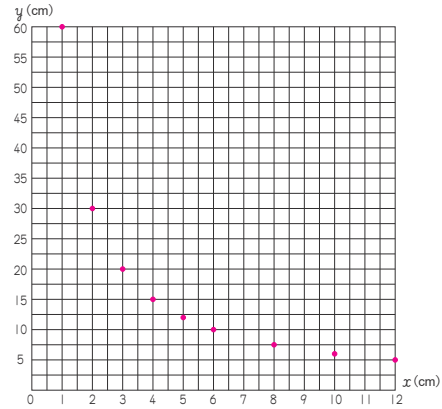


④ Fill in the table, if $x$ is 1.5, 2.5, 7.5.

| Length $x$ (cm) | 1.5 | 2.5 | 7.5 |
|---|---|---|---|
| Width $y$ (cm) | 40 | 24 | 8 |

## 9 – 1　Volume of a Prism and Cylinder
### Volume of a Prism

We learnt how to find the volume of the quadrangular prism as follow:

$$3 × 2 × 4 = 24 \text{ cm}^3$$
Length × Width × Height = Volume

Find the volume of the quadrangular prism when the height is 1 cm.
Math sentence $3 × 2 × 1 = 6$　Answer 6 cm³

**Instruction** Area of the base

The bottom area is called the **area of the base**. Find it and compare the volume of the quadrangular prism with a height of 1 cm.
Math sentence $3 × 2 = 6$　Answer 6 cm²

The volume may also be calculated by : (The area of the base) × (Height)

**Example 1** Find the volume of the triangular prism on the right.

Idea 1: It is half of a cuboid.
Math sentence $6 × 3 × 7 ÷ 2 = 63$　Answer 63 cm³
Volume of a cuboid

Idea 2: Use the area of the base.

Math sentence $6 × 3 ÷ 2 × 7 = 63$　Answer 63 cm³
Area of the base
Both answers are the same.

The volume of the triangular prism can also be found using the formula (Volume of Prism) = (Area of the base) × (Height)

**1** Find the volume of the following prisms.

① Math sentence $10 × 4 × 7 ÷ 2 = 140$
Alternatively, $10 × 4 ÷ 2 × 7 = 140$
Answer 140 cm³
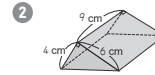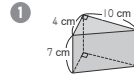
② Where is the base?
Math sentence $6 × 4 × 9 ÷ 2 = 108$
Alternatively, $6 × 4 ÷ 2 × 9 = 108$
Answer 108 cm³

**Example 2** Find the volume of the prism below.

Find the area of the base.
Math sentence $(5 + 10) × 6 ÷ 2 × 12 = 540$　Answer 540 cm³
Area of the base

**2** Find the volume of the prism below.
Math sentence $10 × 6 ÷ 2 × 3 = 90$
Answer 90 cm³

## 9 - 2

Volume of a Prism and Cylinder

### Volume of a Cylinder

**Instruction** How to find the volume of a cylinder.

$(3 × 3 × 3.14) × 5 = 141.3$

Volume | Find the volume like how you find the volume of a prism.

Area of the base | Height

How do you find the area of the base?

3 cm
5 cm

（Volume of Cylinder）＝（Radius）×（Radius）× 3.14 ×（Height）

＝（Area of the base）×（Height）

**Example 1** Find the volume of the cylinder on the right.

❶ Fill in the ☐ with words.

（Volume of cylinder）＝（Area of the base）×（Height）

❷ Fill in the ☐ with numbers.

（ 2 × 2 × 3.14 ）× 5 = 62.8

2 cm
5 cm

**1** Find the volume of the cylinder on the right.

❶ Fill in the ☐ with words.

（Area of the base）＝（Radius）×（Radius）× 3.14

❷ Fill in the ☐ with numbers.

（ 6 × 6 × 3.14 ）× 4 = 452.16

6 cm
4 cm

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**Example 2** Find the volume of the following cylinders.

❶ Since the diameter is 6 cm, the radius is 3 cm.

Math sentence $(3 × 3 × 3.14) × 12 = 339.12$

Answer 339.12 cm$^3$

6 cm
12 cm

❷ Since the figure is a half of the cylinder with 4 cm of radius and 10 cm of height.

Math sentence $(4 × 4 × 3.14 ÷ 2) × 10 = 251.2$

Alternatively, $(4 × 4 × 3.14) × 10 ÷ 2 = 251.2$

Answer 251.2 cm$^3$

First, you find the area of the base. Or you can also find the volume by halving the volume of the cylinder with a 4 cm radius and a 10 cm height.

4 cm
10 cm

**2** Find the volume of the following cylinders.

❶
14 cm
10 cm

Math sentence $(7 × 7 × 3.14) × 10 = 1538.6$

Answer 1538.6 cm$^3$

❷
20 cm
10 cm

Math sentence $(5 × 5 × 3.14) × 20 = 1570$

Answer 1570 cm$^3$

❸
5 cm
8 cm

Math sentence $(5 × 5 × 3.14 ÷ 2) × 8 = 314$

Alternatively, $(5 × 5 × 3.14) × 8 ÷ 2 = 314$

Answer 314 cm$^3$

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## 9 - 3

Volume of a Prism and Cylinder

### Review

**1** Find the volume of the following prisms and cylinders.

❶
6 cm
10 cm
8 cm

Math sentence $10 × 6 × 8 = 480$

Answer 480 cm$^3$

❷
2 cm
10 cm

Math sentence $(2 × 2 × 3.14) × 10 = 125.6$

Answer 125.6 cm$^3$

❸
4 cm
4 cm
7 cm

Math sentence $4 × 4 × 7 = 112$

Answer 112 cm$^3$

❹
4 cm
6 cm

Math sentence $(4 × 4 × 3.14) × 6 = 301.44$

Answer 301.44 cm$^3$

**2** Fill in the blank with numbers to find the volume of the triangular prism on the right.

7 cm
3 cm
4 cm

❶ The triangle prism is half of a cuboid. The volume is found by

（Length）×（Width）×（Height）÷ 2,

Volume of a cuboid

7 × 4 × 3 ÷ 2 = 42 cm$^3$

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❷ Find the volume using the area of the base, The volume is found by

（Length）×（Width）÷ 2 ×（Height）

Area of a base

7 × 4 ÷ 2 × 3 = 42 cm$^3$

**3** Find the volume of the following prisms.

❶
2 cm
3 cm
5 cm

Math sentence $5 × 2 × 3 ÷ 2 = 15$

Alternatively, $5 × 2 ÷ 2 × 3 = 15$

Answer 15 cm$^3$

❷
5 cm
6 cm
7 cm

Math sentence $6 × 5 × 7 ÷ 2 = 105$

Alternatively, $6 × 5 ÷ 2 × 7 = 105$

Answer 105 cm$^3$

❸
8 cm
3 cm
4 cm
6 cm

Math sentence $(4 + 8) × 3 ÷ 2 × 6$
$= 12 × 3 ÷ 2 × 6$
$= 108$

Answer 108 cm$^3$

❹
3 cm
7 cm
5 cm

Math sentence $(3 + 7) × 7 ÷ 2 × 5$
$= 10 × 7 ÷ 2 × 5$
$= 175$

Answer 175 cm$^3$

❺
27 cm$^2$
6 cm

Math sentence $27 × 6 = 162$

Answer 162 cm$^3$

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**30**

## 10 - 1 Ratio and Its Application
### Ratio and Value of Ratio

**Instruction** To make soup, the quantity of ingredients needed is as follows;
Soup……………… Water（4 cups）, Sauce（1 cup）

❶ We need 4 times more water than sauce.
❷ We can represent the relationship as follow.    Water    Sauce

When the quantity of water is 4 then the quantity of sauce is 1, the ratio can be represented by using the ":" symbol as "4 : 1".
4 : 1 is read as "Four is to one". This way of representation is called **ratio**

❸ We can also represent the relationship by a fraction.

When a ratio is represented as "a : b", the quotient of "a" divided by "b" is called the **value of ratio**. The value of ratio represents how many times "b" is "a"    Value of ratio a : b is the quotient of a ÷ b
In particular, when "a" and "b" are whole numbers, the value of ratio "a : b" can be represented as $\dfrac{a}{b}$

Math sentence    $4 \div 1 = \dfrac{4}{1}$    There are various ways to express ratios!

**Example 1** To make salad dressing, the quantity of ingredients needed is as follows;
Salad dressing…Salad oil（3 spoons）, Vinegar（2 spoons）

❶ Fill in the blank.    Math sentence  $3 \div 2 = 1.5$    Salad oil    Vinegar
we need 1.5 times more salad oil than vinegar.
❷ How can the ratio between the quantity of vinegar and the quantity of salad oil be represented using two numbers?    Answer    3 : 2
❸ How can the ratio between the quantity of salad oil and vinegar be represented by a fraction?    Math sentence
$3 \div 2 = \dfrac{3}{2}$    Ratio

If the base quantity is the vinegar, how many times more salad oil is needed than the vinegar?    Compared quantity    Base quantity

To make the rice, the quantity of ingredients needed is as follow;
Rice…… Water（360 mL）, Rice（300 mL）    Math sentence    Water
❶ Fill in the blank.    $360 \div 300 = 1.2$
we need 1.2 times more water than Rice.    Rice
❷ How can the ratio between the quantity of water and the quantity of rice be represented using two numbers?    Answer    360 : 300
❸ How many times the quantity of the water is that of the rice? Let's represent it with a fraction.
Math sentence  $360 \div 300 = \dfrac{360}{300} = \dfrac{6}{5}$

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## 10 - 2 Ratio and Its Application
### Equal Ratio（1）

**Example 1** You made the soup using a small cup as shown below.
Soup……………… Water（4 cups）and Sauce（1 cup）

❶ What is the ratio of ingridient of the soup represented as "a : b"
The ratio of ingredients = 4 : 1
❷ Your friend has made two portions of soup in the same cup as you. What is the ratio of ingridient of the soup represented as "a : b"
The ratio of ingredients = 8 : 2

❸ Is the concentration of the soup you and your friend made the same?
The value of the ratio 4 : 1    $4 \div 1 = \dfrac{4}{1} = 4$    Answer    4

The value of the ratio 8 : 2    $8 \div 2 = \dfrac{8}{2} = 4$    Answer    4
Answer    same

The ratio of the ingredients in your friend's soup is the same as the ratio of yours, 4 : 1.    $4 : 1 \overset{\times 2}{\underset{\times 2}{=}} 8 : 2$

When the value of the ratio is equal, such as 4 : 1 and 8 : 2, the two ratios are said to be equal, and are written as follows.
$4 : 1 = 8 : 2$

What is the concentration（ratio）of coffee to milk?
❶ What is a ratio of ingredient of the cafe represented as "a : b"
（a） 120 mL of coffee and 80 mL of milk
The value of the ratio 120 : 80    ❶ Coffee / Milk    ❷
（b） 9 cups of coffee and 6 cups of milk
The value of the ratio 9 : 6    Coffee

❷ Is the concentration（ratio）of these café the same?
（a） The value of the ratio = $120 \div 80 = \dfrac{120}{80} = \dfrac{3}{2}$
（b） The value of the ratio = $9 \div 6 = \dfrac{9}{6} = \dfrac{3}{2}$    Milk
Answer    same

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## 10 - 3 Ratio and Its Application
### Equal Ratio（2）

**Instruction** There are two combinations of rice and water. The relationship between the rice and the water is the same.

（a）… Rice（60 mL）, water（72 mL）    （b）… Rice（300mL）, water（360 mL）

❶ Calculate the ratio of（a）to（b）. Write the answers in fractions.
（a） The ratio 60 : 72    Value of ratio = $60 \div 72 = \dfrac{60}{72} = \dfrac{5}{6}$
（b） The ratio 300 : 360    Value of ratio = $300 \div 360 = \dfrac{300}{360} = \dfrac{5}{6}$
❷ Find the relationship of 2 equal ratio above.
$60 : 72 = (60 \times 5) : (72 \times 5) = 300 : 360$

$\times 5$
$60 : 72 = 300 : 360$
$\times 5$

$300 : 360 = (300 \div 5) : (360 \div 5) = 60 : 72$
$\div 5$
$360 : 300 = 60 : 36$
$\div 5$

The ratio "a : b" is equal to other ratio that is created by multiplying or dividing "a" and "b" by the same number.

**Example** Among the ratios below, Which are the same?
（a）$2 : 6 = (2 \div 2) : (6 \div 2) = 1 : 3$    Let's simplify the ratio to find same ratio with smallest whole number!!
（b）$4 : 8 = (4 \div 4) : (8 \div 4) = 1 : 2$
（c）$6 : 18 = (6 \div 6) : (18 \div 6) = 1 : 3$
（d）$16 : 32 = (16 \div 16) : (32 \div 16) = 1 : 2$
（e）$4 : 12 = (4 \div 4) : (12 \div 4) = 1 : 3$
Answer    （a）,（c）,（e）and（b）,（c）

Among the ratio below, What is the same ratio?
（a）$8 : 10 = (8 \div 2) : (10 \div 2) = 4 : 5$
（b）$40 : 50 = (40 \div 10) : (50 \div 10) = 4 : 5$
（c）$6 : 18 = (6 \div 6) : (18 \div 6) = 1 : 3$
（d）$16 : 32 = (16 \div 16) : (32 \div 16) = 1 : 2$
（e）$4 : 12 = (4 \div 4) : (12 \div 4) = 1 : 3$
Answer    （a）,（b）and ,（c）,（e）

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## 10 - 4 Ratio and Its Application
### Equal Ratio（3）

**Example** To make juice, we need the ingredients below.

Water（120 mL）, Juice base（30 mL）    Water    Juice base

If there is 60 mL of juice base, how many mL of water should be added to make the same concentraion of juice?

$x$ mL of water is needed to get the same concentration. We can make an equal ratio using $x$.

Math sentence    $120 : 30 = x : 60$    We can use the diagram on the left to find $x$!

$\times 2$
$120 : 30 = x : 60$
$\times 2$

$x = 120 \times 2$
$= 240$    Answer    240 mL

**1** To make pancakes we need the ingredient below.

Pancake mix（200 g）, Milk（160 mL）

If there is 100 g of pancake mix, how many mL of milk should be added to the mixture to make?
Make the math sentence and find the answer using $x$ mL of milk.

Math sentence    $200 : 160 = 100 : x$    100 g    100 mL

$\div 2$
$200 : 160 = 100 : x$
$\div 2$

$x = 160 \div 2$
$= 80$    Pancake ingredient    Milk

Answer    80 mL of milk

**2** To make café latte we need the ingredient below.

Coffee（120 mL）, Milk（360 mL）

If there is 40 mL of coffee, how many mL of milk should be added to make?
Make the math sentence and find the answer using $x$ mL of milk.

Math sentence    $120 : 360 = 40 : x$

$\div 3$
$120 : 360 = 40 : x$
$\div 3$

$x = 160 \div 3$
$= 120$    Coffee    Milk

Answer    120 mL of milk

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## 10 - 5  Ratio and Its Application
### Simple Ratio

**Example** Use equal ratios to express ratios with the smallest whole number if it is possible. Then find the same ratios.

> Decimal number make the ratios difficult to understand, so it's better to represent as whole number!

(a) $1.2 : 2.8 = (1.2 \times \boxed{10}) : (2.8 \times \boxed{10})$
$= \boxed{12} : \boxed{28} = (\boxed{12} \div \boxed{4}) : (\boxed{28} \div \boxed{4}) = \boxed{3} : \boxed{7}$

(b) $1.6 : 3.2 = (1.6 \times \boxed{10}) : (3.2 \times \boxed{10})$
$= \boxed{16} : \boxed{32} = (\boxed{16} \div \boxed{16}) : (\boxed{32} \div \boxed{16}) = \boxed{1} : \boxed{2}$

(c) $1.5 : 3.5 = (1.5 \times \boxed{10}) : (3.5 \times \boxed{10})$
$= \boxed{15} : \boxed{35} = (\boxed{15} \div \boxed{5}) : (\boxed{35} \div \boxed{5}) = \boxed{3} : \boxed{7}$

(d) $2.4 : 4.8 = (2.4 \times \boxed{10}) : (4.8 \times \boxed{10})$
$= \boxed{24} : \boxed{48} = (\boxed{24} \div \boxed{24}) : (\boxed{48} \div \boxed{24}) = \boxed{1} : \boxed{2}$

Answer ___(a), (c) and (b), (d)___

Use equal ratios to express ratios with the smallest whole number if it is possible. Then find the same ratios.

(a) $0.2 : 0.6 = (0.2 \times \boxed{10}) : (0.6 \times \boxed{10})$
$= \boxed{2} : \boxed{6} = (\boxed{2} \div \boxed{2}) : (\boxed{6} \div \boxed{2}) = \boxed{1} : \boxed{3}$

(b) $0.9 : 1.2 = (0.9 \times \boxed{10}) : (1.2 \times \boxed{10})$
$= \boxed{9} : \boxed{12} = (\boxed{9} \div \boxed{3}) : (\boxed{12} \div \boxed{3}) = \boxed{3} : \boxed{4}$

(c) $4.5 : 6.0 = (4.5 \times \boxed{10}) : (6.0 \times \boxed{10})$
$= \boxed{45} : \boxed{60} = (\boxed{45} \div \boxed{15}) : (\boxed{60} \div \boxed{15}) = \boxed{3} : \boxed{4}$

(d) $1.3 : 3.9 = (1.3 \times \boxed{10}) : (3.9 \times \boxed{10})$
$= \boxed{13} : \boxed{39} = (\boxed{13} \div \boxed{13}) : (\boxed{39} \div \boxed{13}) = \boxed{1} : \boxed{3}$

Answer ___(a), (d) and (b), (c)___

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**Example 2** Use equal ratios to express ratios with the smallest whole number if it is possible. Then find the same ratios

> Fraction make the ratios difficult to understand, so it's better to represent as whole number!

(a) $\frac{3}{4} : \frac{2}{3} = \frac{9}{12} : \frac{8}{12}$
$= \left(\frac{9}{12} \times \boxed{12}\right) : \left(\frac{8}{12} \times \boxed{12}\right)$
$= \boxed{9} : \boxed{8}$

> To get whole number, Let's convert to one fraction!

(b) $\frac{4}{9} : \frac{2}{3} = \frac{12}{27} : \frac{18}{27}$
$= \left(\frac{12}{27} \times \boxed{27}\right) : \left(\frac{18}{27} \times \boxed{27}\right)$
$= \boxed{12} : \boxed{14} = \boxed{6} : \boxed{7}$

Use equal ratios to express ratios with the smallest whole number if it is possible. Then find the same ratios

(a) $\frac{2}{5} : \frac{3}{8} = \frac{16}{40} : \frac{15}{40}$
$= \left(\frac{16}{40} \times \boxed{40}\right) : \left(\frac{15}{40} \times \boxed{40}\right)$
$= \boxed{16} : \boxed{15}$

(b) $\frac{4}{5} : \frac{3}{4} = \frac{16}{20} : \frac{15}{20}$
$= \left(\frac{16}{20} \times \boxed{20}\right) : \left(\frac{15}{20} \times \boxed{20}\right)$
$= \boxed{16} : \boxed{15}$
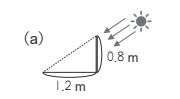
(c) $\frac{3}{6} : \frac{6}{5} = \frac{15}{30} : \frac{36}{30}$
$= \left(\frac{15}{30} \times \boxed{30}\right) : \left(\frac{36}{30} \times \boxed{30}\right)$
$= \boxed{15} : \boxed{36} = \boxed{5} : \boxed{12}$

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## 10 - 6  Ratio and Its Application
### Ratio Application
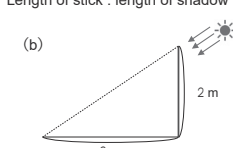
**Example** Let's find the height of the tree from the length of its shadow. For sticks A and B, compare the ratio of the length of the stick to the length of the shadow.

Stick A··· 0.8 m, its shadow 1.2 m
Stick B··· 2 m, its shadow 3 m

Length of stick : length of shadow $= \boxed{0.8} : \boxed{1.2}$

(a)
0.8 m
1.2 m

$= (\boxed{0.8} \times 10) : (\boxed{1.2} \times 10)$
$= \boxed{8} : \boxed{12} = \boxed{2} : \boxed{3}$
$= \boxed{2} : \boxed{3}$

Length of stick : length of shadow

(b)
2 m
3 m

> The sun's rays are shining in the same direction, parallel to each other, so the ratio of any sticks and shadow should be same.

❶ At this time, the shadow of the trees in the schoolyard were 15 m. What is the height of this tree?

> To find the height of tree, let's use ratio and set the height of the tree to $x$.

Height of tree : Length of shadow $= 2 : 3 = x : 15$

$2 : 3 = x : 15$ with $\times \boxed{5}$

$x = \boxed{2} \times \boxed{5} = \boxed{10}$

Answer ___10 m___

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**1** In the same situation as the example, what is the height of the tree when its shadow is 21 m? Fill in the blank.

Height of stick : Length of shadow $= 2 : 3 = x : 21$

$2 : 3 = x : 21$ with $\times \boxed{7}$

$x = \boxed{2} \times \boxed{7} = \boxed{14}$

Answer ___14 m___

**2** Make cookies with a $3 : 1$ weight of flour and sugar.

❶ What is the weight of sugar if the weight of flour is 150 g?
Weight of flour : Weight of sugar $= 3 : 1 = 150 : x$

$3 : 1 = 150 : x$ with $\times \boxed{50}$

$x = \boxed{1} \times \boxed{50} = \boxed{50}$

Flour   Sugar

Answer ___50 g___

❷ What is the weight of flour if the weight of sugar is 200 g?
Weight of flour : Weight of sugar $= 3 : 1 = x : 200$

$3 : 1 = x : 200$ with $\times \boxed{200}$

$x = \boxed{3} \times \boxed{200} = \boxed{600}$

Answer ___600 g___

**3** Draw a rectangular court in the schoolyard with a length to width ratio of $5 : 3$.
❶ What is the length if the width is 9 m?
Length : Width $= 5 : 3 = x : 9$

$5 : 3 = x : 9$ with $\times \boxed{3}$

$x = \boxed{5} \times \boxed{3} = \boxed{15}$

5
3

Answer ___15 m___

123

32

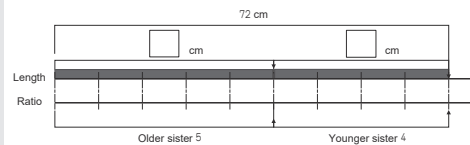## 10 - 7

Ratio and Its Application

### How to Divide by Ratio

**Example** A ribbon 72 cm long is divided into two pieces with a length ratio of 5 : 4.

What is the length of each?

72 cm

☐ cm            ☐ cm

Length

Ratio

Older sister 5            Younger sister 4

**1** What is the length of the older sister's ribbon?

The overall ratio is 5 + 4 = 9. So compare the overall ratio and older sister ratio.

Older sister ratio : Overall ratio = 5 : 9 = $x$ : 72

× 8
5 : 9 = $x$ : 72
× 8

$x$ = 5 × 8 = 40

Answer    40 cm

**2** What is the length of the older sister's ribbon? Find the answer using the ratio.

Older sister ratio : Younger sister ratio = 5 : 4 = 40 : $x$

× 8
5 : 4 = 40 : $x$
× 8

$x$ = 4 × 8 = 32

Answer    32 cm

※ You can also use the fraction to find the answer.

Example: The ratio of the ribbon of the older sister is $\frac{5}{9}$ of the total.

$72 \times \frac{5}{9} = (72 \div 9) \times 5 = 40$

124

**1** In the same situation as the example, if the ribbon is 54 cm long, how long are the ribbons for the older sister and younger sister? Find the answer using ratio.

Older sister's ratio : Overall ratio = 5 : 9 = $x$ : 54

× 6
5 : 9 = $x$ : 54
× 6

Math sentence
$x$ = 5 × 6 = 30

Answer    30 cm

Older sister's ratio : Younger sister's ratio = 5 : 4 = 30 : $x$

× 6
5 : 4 = 30 : $x$
× 6

Math sentence
$x$ = 4 × 6 = 24

Answer    24 cm

**2** You and your father share 400 mL of milk in a ratio of 3 : 2.

400 ml

☐ mL            ☐ mL      Your father

Amount

Ratio                                            You

Your father 3            You 2
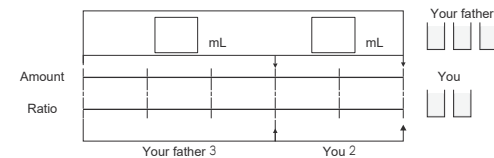
Your father's ratio : Overall ratio = 3 : 5 = $x$ : 400

× 80
3 : 5 = $x$ : 400
× 80

Math sentence
$x$ = 3 × 80 = 240

Answer    240 mL

Your father's ratio : Your ratio = 3 : 2 = 240 : $x$

× 80
3 : 2 = 240 : $x$
× 80

Math sentence
$x$ = 2 × 80 = 160

Answer    160 mL

125

## 10 - 8

Ratio and Its Application

### Problems

**Example 1** Divide a 56 m² garden into flower bed and fields so that the ratio is 3 : 5. What should be the area of the flower bed?

**1** What is the ratio of the area of the flower bed to the area of the whole garden?

The ratio of flower bed : Whole garden = 3 : 8

3    5

**2** Write an math sentence where the area of the flower bed is $x$ m² and the ratios are equal and find the answer

Math sentence    3 : 8 = $x$ : 56

$x$ = 3 × 7 = 21

Answer    21 m²

**3** Find the area of the flower bed by thinking about how much of the area of the whole garden is covered by the flower bed.

To find the proportion of the flower bed to the total area, divide the flower bed by the total area.

Ratio of flower bed…    $3 : 8 = \frac{3}{8}$

Math sentence    $56 \times \frac{3}{8} = 21$

Answer    21 m²

Cut 10 m of ribbon into 2 : 3 pieces. How many cm will they be?

**1** What is the ratio of 2 to the whole ribbon?

10 cm

The ratio of 2 : Whole ribbon = 2 : 5

2    3

**2** Write a math sentence where The ratio of 2 is $x$ cm and the ratios are equal and find the answer

Math sentence    2 : 5 = $x$ : 10

$x$ = 2 × 2 = 4

Answer    4 cm

**3** Find the length of ribbon in ratio of 2 by thinking about how much of the whole ribbon is covered by the ribbon in ratio of 2 .

To find the ribbon in ratio of 2 to the whole ribbon, divide the length of ribbon in ratio of 2 by the whole ribbon.

Ratio of ribbon in ratio of 2…    $2 : 5 = \frac{2}{5}$

Math sentence    $10 \times \frac{2}{5} = 4$

Answer    4 cm

126

**Example 2** Draw a rectangle so that the length and width of the rectangle is 7 : 4.

Let's consider the answer as $x$.

4

7

**1** If the length is 28 cm, what is the width?

× 4
7 : 4 = 28 : $x$
× 4

Math sentence
$x$ = 4 × 4 = 16

Answer    16 cm

**2** If the width is 36 cm, what is the length?

× 9
7 : 4 = $x$ : 36
× 9

Math sentence
$x$ = 4 × 9 = 36

Answer    36 cm

Draw a triangle so that the base and height of the rectangle is 8 : 5.

**1** If the base is 64 cm, what is the height.

5

8

× 8
8 : 5 = 64 : $x$
× 8

$x$ = 5 × 8 = 40

Answer    40 cm

**2** If the height is 65 cm, what is the base.

× 13
8 : 5 = $x$ : 65
× 13

$x$ = 8 × 13 = 104

Answer    104 cm

127

33

## 10 – 9  Ratio and Its Application
### Review

**1** Express the following proportions as simple ratios.

(a) Ratio of 20 ml vinegar to 30 ml salad oil.  Answer  2 : 3

(b) The ratio of the length of a rectangle 18 cm to its width 14 cm.
Answer  9 : 7

**2** Find the number that applies to $x$.

(a) $2 : 5 = 12 : x$  Answer  $x = 30$

(b) $63 : 49 = x : 7$  Answer  $x = 9$

**3** Simplify the following ratios.

(a) 16 : 24  Answer  2 : 3   (b) 52 : 13  Answer  4 : 1

(c) 1.5 : 4.5  Answer  1 : 3   (d) 0.6 : 3.8  Answer  3 : 19

(e) $\frac{3}{2} : \frac{3}{7}$  Answer  7 : 2   (f) $\frac{12}{7} : \frac{9}{4}$  Answer  16 : 21

**4** There is a 5 : 3 ratio of story books to science books in the school library and there are 240 story books. How many science books are there?

Math sentence  $5 : 3 = 240 : x$
$x = 3 × 48$  Answer  144 books

$5 : 3 = 240 : x$ (× 48)

**5** Make a flag in the shape of a rectangle so that the ratio of length to width is 3 : 4. If the length is 60 cm, how long should the width be? Find the length of the rectangle.

Length : Width $= 3 : 4 =$  60 : $x$   $x =$ $4 × 20 = 80$
Answer  80 cm

$3 : 4 = 60 : x$ (× 20)

**6** In one school, grade 5th and 6th students are in the football club. The ratio of 5th and 6th grade students in the football club is 9 : 7 and there are 32 members in the football club. How many 5th grade students are there in the football club?

**1** Find the ratio of the whole number  $9 + 7 = 16$

**2** Find the number of 5th grade students.   $9 : 16 = x : 32$ (× 2)

Ratio of 5th grade : Ratio of whole number =  9 : 16  =  $x$ : 32

$x =$  $9 × 2 = 18$

Answer  18 students

**7** Make a lottery so that the ratio of the number of winning lots to the number of losing lots is 3 : 7. If the total number of lots is 120, how many winning lots should there be? Write two different math sentences.

(a) Find the ratio of the total number of lots.   $3 + 7 = 10$

(b) Math sentence...
Ratio of winning lots : Ratio of total lots =  3 : 10  =  $x$ : 120

$3 : 10 = x : 120$ (× 12)

$x =$  $3 × 12 = 36$

Answer  36 winning lots

(c) What is the ratio of winning lots to the total lots? Express it in fraction.

Math sentence   $3 ÷ 10 = \frac{3}{10}$

(d) What is the ratio of wining lots? Calculate with fraction above.

Math sentence   $120 × \frac{3}{10} = 36$

Answer  36 winning lots

---

## 11 – 1  Enlarged and Reduced Drawings
### Introduction to Enlarged and Reduced Drawings

**Example** From the following figures Ⓑ, Ⓒ, Ⓓ, and Ⓔ which has the same shape as figure Ⓐ below?

Each size of angle B is 90°.
If the length of side AB and BC are the same, these are congruent.

**1** Complete the table below.

|  | Length of sides (Number of square) | |
|---|---|---|
|  | Side AB | Side BC |
| A | 4 | 6 |
| B | 4 | 12 |
| C | 8 | 6 |
| D | 8 | 12 |
| E | 4 | 6 |

Figure A and E are congruent. What about Figure A and D?

**2** Represent the length of the corresponding sides of Figure A to E with simplified ratios. Let the corresponding sides of Figure A be 1.

|  | Ratio of the lengths | |
|---|---|---|
|  | Side AB | Side BC |
| A | 1 | 1 |
| B | 1 | 2 |
| C | 2 | 1 |
| D | 2 | 2 |
| E | 1 | 1 |

What about the size of the corresponding angles?

**3** How many times the length of the corresponding side of Figure D is the length of the side of Figure A?

2 times

- When all the lengths of corresponding sides are extended in the same ratio and the corresponding angles are respectively equal, then it is called an **enlarged drawing**.
- If shorted in the same ratio, then it's called a **reduced drawing**.
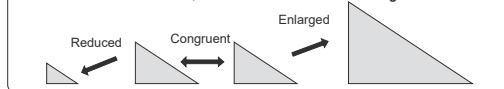
Reduced    Congruent    Enlarged

Look at the figures below and answer the following questions.

**1** Which of the following figures is an enlarged drawing of Figure A? Also, how many times is it enlarged?

Enlarged drawing  C   How many times  3 times

**2** Which of the following figures is a reduced drawing of Figure A? Also, how much is it reduced?

Reduced drawing  F   How much reduced  half

**2** Figure B is a reduced drawing of Figure A. Answer the following questions.

**1** Represent the length of the corresponding sides of BC to FG with simplified ratios.
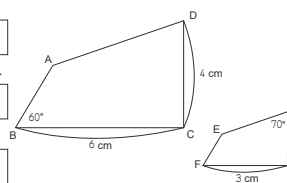
$6 : 3 =$  2  :  1

**2** Find the length of side GH.   2 cm

**3** Find the size of angle D.   70°

**34**

**11 - 2** Enlarged and Reduced Drawings
**How to Draw Enlarged Drawings**（1）

**Instruction** How to draw congruent figures.

When you draw enlarged figures, the method of drawing a congruent triangle can be helpful. Review how to draw the congruent triangle with the figure on the right.

| Method 1 Measure all the sides and draw. | Method 2 Measure two sides and angle and draw. | Method 3 Measure two angles, the side, and draw. |
|---|---|---|

**Example 1** Draw enlarged triangle DEF which is 2 times as large as triangle ABC. Point E, corresponding to point B, is already located on the grid paper below.

1. Draw point F at twice the length of side BC and connect point E and F.

2. Measure the size of the angle B and make the same size of the angle at point E.

3. Draw point D at twice the length of side AB and connect point D and E.

4. Connect point D and F.

This is an example using Method 2. You can also draw with Method 1 or 3.

**1** Draw enlarged triangle DEF which is 2 times as large as triangle ABC. Point E, corresponding to point B is already located on the grid paper below.
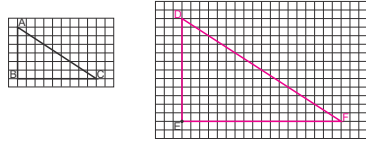
**Example 2** Draw enlarged triangle DBE which is 2 times as large as triangle ABC.
2.8 cm  1.6 cm  2.7 cm

1. Measure the length of side AB, BC and draw point D and E at twice the length of side AB, BC.

2. Extend side AB, BC to point D, E respectively and connect point D and E.

**2** Draw enlarged triangle DBE which is 2 times as large as triangle ABC.

① 2 cm  2 cm  2 cm

② 2.8 cm  1.7 cm  1.6 cm
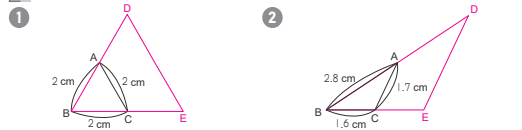
**3** Draw enlarged triangle DBE which is 1.5 times as large as triangle ABC.
2 cm  4.7 cm  4 cm
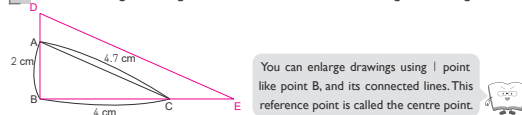
You can enlarge drawings using 1 point like point B, and its connected lines. This reference point is called the centre point.

132

133

**11 - 3** Enlarged and Reduced Drawings
**How to Draw Enlarged Drawings**（2）

**Example 1** When you draw enlarged triangle DEF 2 times as large as triangle ABC, answer the following questions.

**①** Find the length of the corresponding sides AB and BC.

Corresponding Side AB 〔 7 cm 〕 Corresponding Side BC 〔 6 cm 〕 3.5 cm 3 cm 45°

**②** Find the size of the corresponding angle B.

Corresponding Angle B 〔 45° 〕

**③** Draw the enlarged triangle DEF.

**1** Draw the enlarged triangle DEF 2 times as large as triangle ABC. Answer the following questions.

**①** Find the length of the corresponding sides AB and BC.

Corresponding Side AB 〔 6 cm 〕 Corresponding Side BC 〔 8 cm 〕

Corresponding Side AC 〔 10 cm 〕

5 cm  3 cm  4 cm

**②** Find the size of the corresponding angle A.

Corresponding Angle A 〔 90° 〕

**③** Draw the enlarged triangle DEF.

**Example 2** Draw a 2 times enlarged drawing of the following quadrilateral. Use point B as a centre point.

2.3 cm  3.2 cm  105°  2.2 cm  75°  3 cm

You can draw it using the same method when drawing an enlarged triangle.

1. Measure the length of AB, BC, and BD, then take points at twice the length of them.

2. Extend side AB, BC to the plotted points and connect them.

**2** Draw a 2 times enlarged drawing of the following quadrilateral using the point as a centre point.

① trapezoid
4 cm  3 cm  3 cm  3 cm  120°

② parallelogram
3 cm  75°  5 cm
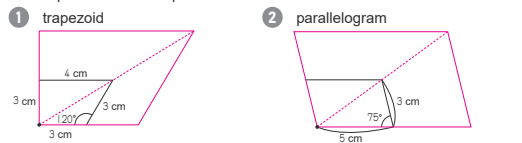
**3** Draw a 1.5 times enlarged drawing of the following quadrilateral.

① rectangle
4 cm  6 cm

You can measure the length using a ruler easily.

② trapezoid
2.8 cm  3 cm  2.8 cm  65°  5 cm

134

135
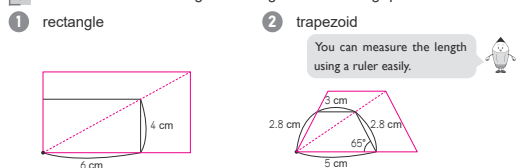
35

**11 - 4**  Enlarged and Reduced Drawings
**How to Draw Enlarged Drawings** (3)

**Example 1**  Consider point E as the centre point and draw an enlarged quadrilateral FGHJ that is 2 times as large as quadrilateral ABCD.



Previously, we consider from one of points, like point B. The ratio of the lengths of each side is 1 : 2.

1. Measure the length from point E and each point. Then, draw point G, H and J at twice the length.

2. Connect these points.



**1**  Consider point E as the centre point and draw the following enlarged figures

❶  Quadrilateral FGHJ that is 2 times larger than quadrilateral ABCD.

❷  Triangle FGH that is 2 times larger than triangle ABC.
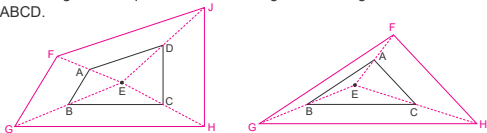


**Example 2**  Using point D, draw triangle EFG that is 2 times quadrilateral ABC.

Compared to Example 1, the point is outside of the figure.



1. Measure the length from point E and each point. Then, draw point G, H and J at twice the length.

2. Connect these points.

**2**  Using point O as the centre point, draw the following enlarged figures.

❶  Triangle EFG that is 2 times larger than triangle ABC.

❷  Pentagon FGHIJ that is 2 times larger than quadrilateral ABCDE.
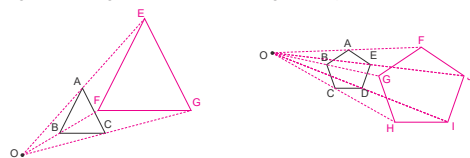


**3**  Using point O as the centre point, draw the following figure 2 times larger than the enlarged figure.
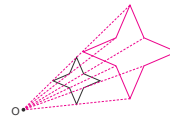


136  137

**11 - 5**  Enlarged and Reduced Drawings
**How to Draw Reduced Drawings** (1)

**Example 1**  Draw reduced triangle DEF which is half of triangle ABC. Point E, corresponding to point B, is already located on the grid paper below.



1. Draw point F at half the length of side BC and connect point E and F.

2. Measure the size of the angle B and make the same size of the angle at point E.

3. Draw point D at half the length of side AB and connect point D and E.

4. Connect point D and F.



**1**  Draw enlarged triangle DEF which is half of triangle ABC. Point E, corresponding to point B is already located on the grid paper below.
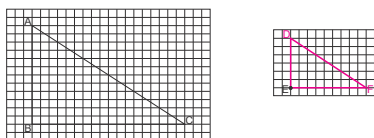


**Example 2**  Draw a $\frac{1}{2}$ reduced drawing of triangle ABC and answer the following questions.

❶  Find the length of the corresponding sides AB and BC.

Corresponding Side AB  4 cm
Corresponding Side BC  2 cm

❷  Find the size of the corresponding angle B.

Corresponding Angle B  60°

❸  Draw the reduced triangle DBE.



1. Measure the length from point B and draw points D and E.

2. Connect point D and E.



**1**  When you draw a $\frac{1}{2}$ reduced drawing of triangle ABC, answer the following questions.

❶  Find the length of the corresponding sides AB and BC.

Corresponding Side AB  6.5 cm
Corresponding Side BC  3.5 cm



❷  Using point B as a centre point, draw the reduced triangle DBE.



138  139
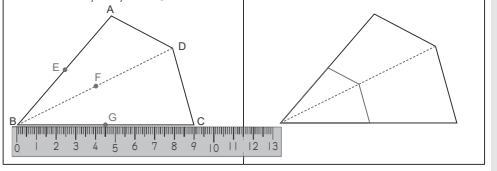
36

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## 11 − 6

Enlarged and Reduced Drawings
**How to Draw Reduced Drawings** (2)

**Example 1** Using point B as a centre point, draw a $\frac{1}{2}$ reduced drawing of quadrilateral ABCD.
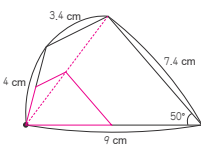
A 3.5 cm
7.5 cm
D
4.3 cm
75°
B 9 cm C
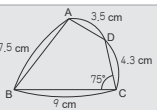
1. Measure the length from point B and draw points, E, F and G in the middle of AB, DB, and CB.

A
D
E
F
B G C
0 1 2 3 4 5 6 7 8 9 10 11 12 13

2. Connect point E, F and G.

**1** Using the point as a centre point, draw the following reduced drawings.

**1** A $\frac{1}{2}$ reduced drawing

3.4 cm
7.4 cm
4 cm
50°
9 cm

**2** A $\frac{1}{3}$ reduced drawing

8.4 cm
75°
7.8 cm
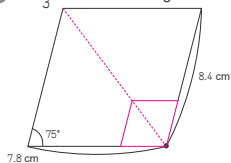
**Example 2** Using point O as a centre point, draw a reduced quadrilaterals that is 2 times as small as the quadrilateral.

A D
B C
• O
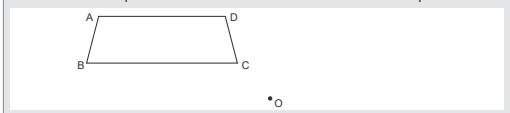
1. Draw lines from point O to each point. Then, take point E, F, G and H at $\frac{1}{2}$ of the length.

A D
B E C H G
0 1 2 3 4 5 6 7 8 9 10 11 12 13 O

2. Connect these points.

A D
B E C H G F O

**2** Using point O as a centre point, draw the following triangles.

**1** A $\frac{1}{2}$ reduced drawing

O

**2** A $\frac{1}{3}$ reduced drawing

O
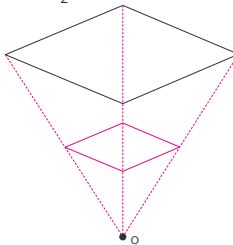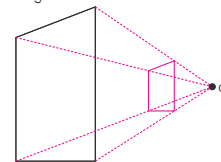
**3** A $\frac{1}{3}$ reduced drawing

O

140

141
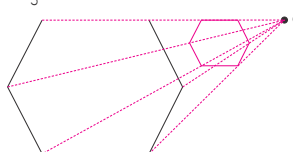
## 11 − 7

Enlarged and Reduced Drawings
**Application of Reduced Drawings** (1)

**Instruction** Reduced scale.

The figure on the right is a reduced drawing from school to a student's house. The actual distance between the school and the house is 300 m. It is shown as 3 cm on the reduced drawing.
Express the rate of reduction using a fraction. Also express the rate as a ratio.
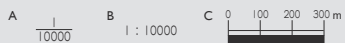Since the actual distance is 300 m and it is shown as 3 cm on the drawing,

School
House

$$\frac{3}{30000} = \frac{1}{10000}$$

| | | m | | cm |
|---|---|---|---|---|
| | | | | 3 |
| 3 | 0 | 0 | 0 | 0 |

Also, expressing the rate as a ratio,
$$3 : 30000 = 1 : 10000$$

1 m = 100 cm.
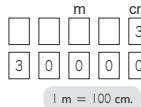
● The ratio that represents how many it is reduced by from the actual distance is called the reduced scale.
● There are 3 ways to show a reduced scale:

A $\frac{1}{10000}$  B 1 : 10000  C 0 100 200 300 m

**Example 1** The figure on the right is a reduced drawing of a park. Answer the following questions.

5 cm
4 cm
Flower bed

**1** The actual lateral length of the park is 20 m. Express the rate of reduction using a fraction.

| | | m | | cm |
|---|---|---|---|---|
| | | | | 5 |
| 2 | 0 | 0 | 0 | 0 |

$$\frac{5}{2000} = \frac{1}{400}$$

Reduced scale = $\frac{1}{400}$

**2** Find the actual width of the park.

Math sentence  $4 \times 400 = 1600$  Answer  16 m

$$4 : \frac{1}{400} = x : 1$$
×400

142
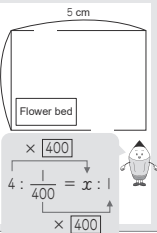
**1** The figure on the right is a reduced drawing of a field. Answer the following questions.

6 cm
4 cm
Onion
Tomato
Maize

**1** The actual lateral length of the park is 30 m. Express the rate of reduction using a fraction.

| | | m | | cm |
|---|---|---|---|---|
| | | | | 6 |
| 3 | 0 | 0 | 0 | 0 |

$$\frac{6}{3000} = \frac{1}{500}$$

Reduced scale = $\frac{1}{500}$

$$4 : \frac{1}{500} = x : 1$$
×500

**2** Find the actual width of the field.

Math sentence  $4 \times 500 = 2000$  Answer  20 m

**3** The size of a maize field is 5 cm in length and 1 cm in width. Find the actual size of the maize field. Also find the area of the field.

Length:  25 m    Width:  5 m

Area:
Math sentence  $25 \times 5 = 125$  Answer  125 m²

**2** There is a map of a school that is drawn to $\frac{1}{400}$ reduced scale. In the reduced drawing, the assembly hall has a rectangular shape with a length that is 6 cm long and a width that is 3.2 cm long. How many meters is the actual length and width of the hall?
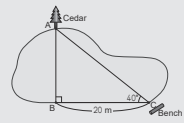
Length:
Math sentence  $6 \times 400 = 2400$  Answer  24 m

Width:
Math sentence  $3.2 \times 400 = 1280$  Answer  12.8 m

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## 11 - 8
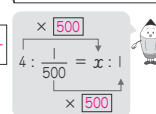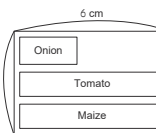
### Application of Reduced Drawings (2)

**Example** A girl walked from point C to point B as shown on the right. What should we do to find the distance from point B to point A? Point A is a cedar tree at the opposite side of the pond.
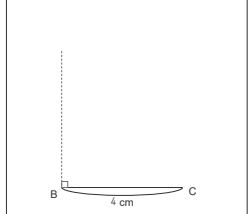
Draw a $\frac{1}{500}$ reduced drawing of the right triangle ABC.
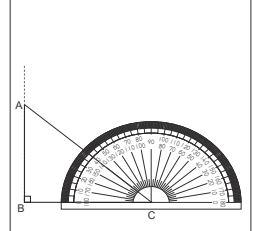
Step 1

Calculate the length of corresponding Side BC:

$20 \times \frac{1}{500} = 0.04$    4 cm

Step 2. Draw 4 cm as the corresponding side BC and draw a perpendicular line to the drawn line.

Step 3. Measure angle C at 40° and place point A. Then, connect point A and point C.

Step 4

Measure the length of side AB on the reduced drawing and find the distance between point A and point B.

Given that the length of side AB on the reduced drawing is 3.2 cm, what is the actual distance?

Math sentence    $3.2 \times 500 = 1600$    Answer    16 m

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1 In the figure shown on the right, how many meters is the actual height of the tree? Find the answer by drawing a reduced triangle in $\frac{1}{100}$ reduced scale.
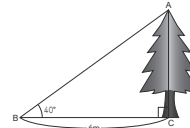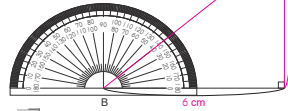
Calculate the length of the corresponding Side BC:

$6 \times \frac{1}{100} = 0.06$    6 cm

Given that the length of side AB on the reduced drawing is 4.8 cm, what is the actual distance?
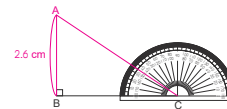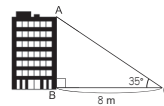
Math sentence    $4.8 \times 100 = 480$

Answer    4.8 m

2 Find the actual height of the building shown below.

1 Draw a reduced triangle in $\frac{1}{200}$ reduced scale.

Calculate the length of the corresponding Side BC:

$8 \times \frac{1}{200} = 0.04$    4 cm

2 Find the height of the building.

Given that the length of side AB on the reduced drawing is 2.6 cm, what is the actual distance?

Math sentence    $2.6 \times 200 = 520$    Answer    5.2 m

3 When you draw a reduced triangle in $\frac{1}{400}$ reduced scale, find the height of the building.

Calculate the length of the corresponding Side BC and draw the triangle,

$8 \times \frac{1}{400} = 0.02$    2 cm

Given that the length of side AB on the reduced drawing is 2.6 cm, what is the actual distance?

Math sentence    $1.3 \times 400 = 520$    Answer    5.2 m
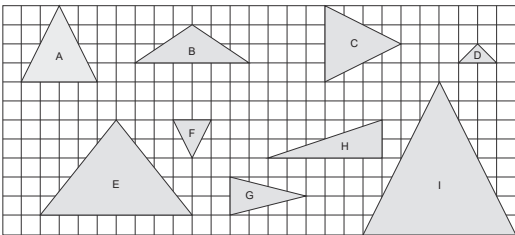
Do you have any findings when you change reduced scale?

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## 11 - 9

### Review

1 Look at the figures below and answer the following questions.

1 Which of the following figures is congruent to Figure A?

Congruent    C

2 Which of the following figures is an enlarged drawing of Figure A? Also, how many times is it enlarged?

Enlarged drawing    I    How many times    2 times

3 Which of the following figures is a reduced drawing of Figure A? Also, how much is it reduced?

Reduced drawing    F    How much reduced    $\frac{1}{2}$

2 Draw an enlarged triangle DEF 3 times larger than triangle ABC. Answer the following questions.

1 Find the length of the corresponding sides AB and BC.

Corresponding Side AB    21 cm    Corresponding Side AC    39 cm
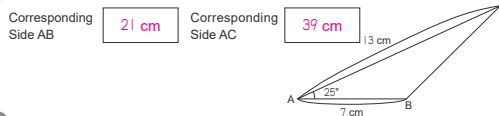
2 Find the size of the corresponding angle A.

Corresponding Angle A    25°

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3 Answer about a $\frac{1}{2000}$ reduced scale.

1 Using the scale, how many m is the actual length of 4 cm and 6.5 cm?

In the case of 4 cm:

Math sentence    $4 \times 2000 = 8000$    Answer    8 m

In the case of 6.5 cm:

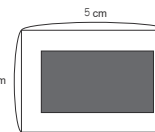Math sentence    $6.5 \times 2000 = 13000$    Answer    13 m

2 60 m are represented by how many cm using the scale?

Math sentence    $60000 \times \frac{1}{2000} = 30$    Answer    30 cm

4 The figure on the right is a reduced drawing of a swimming pool. Answer the following questions.

1 The actual lateral length of the pool is 30 m. Express the rate of reduction using a fraction.

| m | cm |
|---|---|
| 3 0 0 0 | 5 |

$\frac{5}{3000} = \frac{1}{600}$

Reduced scale $= \frac{1}{600}$

2 Find the actual width of the field.

Math sentence    $3 \times 600 = 1800$    Answer    18 m

3 The size of a maize field is 4 cm in length and 1.5 cm in width. Find the actual size of the maize field. Also find the area of the field.

Length:    24 m        Width:    9 m

Area:

Math sentence    $24 \times 9 = 216$    Answer    216 m$^2$

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**12－1**  Ways of Ordering and Combination

## Ways of Ordering（1）

**Instruction**  How to put items in order without missing an item or choosing the same item twice. Three students, Emma, John, and Mia are relay athletes.

Well, Emma, John, Mia
Then, Mia, John, Emma…Oh it isn't very easy. Do you have any good ideas to decide?
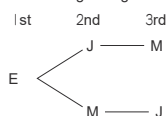
Consider the case where Emma runs first. How will the running order of John, Mia, and Steven be decided?

To write down their names easily, you can use symbols such as For example, E for Emma, J for John, M for Mia, and S for Steven.

Idea 1. Using a table.

| 1st | 2nd | 3rd |
|-----|-----|-----|
| E | J | M |
| E | M | J |

Idea 2. Using a diagram.

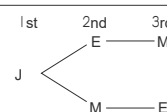| 1st | 2nd | 3rd |
|-----|-----|-----|
| | J ── M |
| E | |
| | M ── J |

Using a table or diagram is helpful when ordering items.

The following tips are effective for ordering items.
- Use symbols to write down their names easily.
- Choose the first item.
- Use tables and diagrams to avoid mistakes.

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**Example**  Draw tables and diagrams where the first runner is John or Mia respectively.
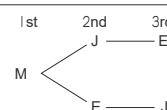
John is the first runner.

| 1st | 2nd | 3rd |
|-----|-----|-----|
| J | E | M |
| J | M | E |

| 1st | 2nd | 3rd |
|-----|-----|-----|
| | E ── M |
| J | |
| | M ── E |

Mia is the first runner.

| 1st | 2nd | 3rd |
|-----|-----|-----|
| M | J | E |
| M | E | J |

| 1st | 2nd | 3rd |
|-----|-----|-----|
| | J ── E |
| M | |
| | E ── J |

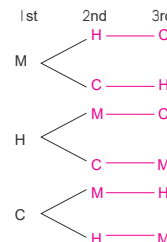❷  In total, how many ways of ordering are there?

In each case, we can find two ways.

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A farmer delivers her crops to the market, hotel, and a customer's house. As for how to decide the delivery order, let market, hotel, and customer's house be M, H, and C, respectively, and answer the following questions.

❶  Fixing the first delivery place, draw tables and diagrams.

| 1st | 2nd | 3rd |
|-----|-----|-----|
| M | H | C |
| M | C | H |
| H | M | C |
| H | C | M |
| C | M | H |
| C | H | M |

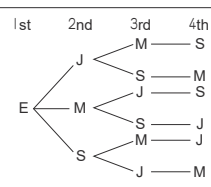| 1st | 2nd | 3rd |
|-----|-----|-----|
| | H ── C |
| M | |
| | C ── H |
| | M ── C |
| H | |
| | C ── M |
| | M ── H |
| C | |
| | H ── M |

❷  In total, how many ways of ordering are there?
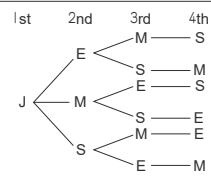
6

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**12－2**  Ways of Ordering and Combination

## Ways of Ordering（2）

**Example**  Four students, Emma, John, Mia, and Steven are relay athletes. Decide the running order. Answer the following questions.
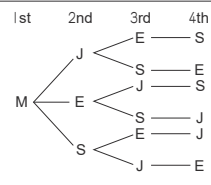
| 1st | 2nd | 3rd | 4th |
|-----|-----|-----|-----|
| E | J | M | S |
| E | J | S | M |
| E | M | J | S |
| E | M | S | J |
| E | S | M | J |
| E | S | J | M |

| 1st | 2nd | 3rd | 4th |
|-----|-----|-----|-----|
| | | M ── S |
| | J | |
| | | S ── M |
| E | M | |
| | | S ── J |
| | S | |
| | | M ── J |
| | | J ── M |

| 1st | 2nd | 3rd | 4th |
|-----|-----|-----|-----|
| J | E | M | S |
| J | E | S | M |
| J | M | E | S |
| J | M | S | E |
| J | S | M | E |
| J | S | E | M |

| 1st | 2nd | 3rd | 4th |
|-----|-----|-----|-----|
| | | M ── S |
| | E | |
| | | S ── M |
| J | M | |
| | | S ── E |
| | S | |
| | | M ── E |
| | | E ── M |

| 1st | 2nd | 3rd | 4th |
|-----|-----|-----|-----|
| M | J | E | S |
| M | J | S | E |
| M | E | J | S |
| M | E | S | J |
| M | S | E | J |
| M | S | J | E |

| 1st | 2nd | 3rd | 4th |
|-----|-----|-----|-----|
| | | E ── S |
| | J | |
| | | S ── E |
| M | E | |
| | | S ── J |
| | S | |
| | | E ── J |
| | | J ── E |

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| 1st | 2nd | 3rd | 4th |
|-----|-----|-----|-----|
| S | J | M | E |
| S | J | E | M |
| S | M | J | E |
| S | M | E | J |
| S | E | M | J |
| S | E | J | M |

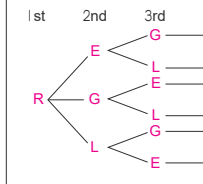| 1st | 2nd | 3rd | 4th |
|-----|-----|-----|-----|
| | | M ── E |
| | J | |
| | | E ── M |
| S | M | |
| | | E ── J |
| | | M ── J |
| | E | |
| | | J ── M |
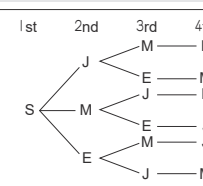
❷  In total, how many ways of ordering are there?

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When we fix the first order, there are 6 ways to order.

A student visits a zoo to see four of her favorite animals: Rhino, Elephant, Giraffe, and Leopard. Rhino, Elephant, Giraffe, and Leopard are R, E, G, and L, respectively.

❶  If you see the Rhino first, decide the visiting order by drawing tables and diagrams.

| 1st | 2nd | 3rd | 4th |
|-----|-----|-----|-----|
| R | E | G | L |
| R | E | L | G |
| R | G | E | L |
| R | G | L | E |
| R | L | G | E |
| R | L | E | G |

| 1st | 2nd | 3rd | 4th |
|-----|-----|-----|-----|
| | | G ── L |
| | E | |
| | | L ── G |
| R | G | |
| | | E ── L |
| | | L ── E |
| | L | |
| | | G ── E |
| | | E ── G |

❷  Fill in the blanks with numbers.

In case we visit the Rhino first, there are  6  ways.

There are  4  cases.

And there are  6  ways each. So in total, there are  24  ways.

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## 12 − 3  Ways of Ordering and Combination
### Ways of Ordering (3)

**Example 1** There are red, yellow, green and blue crayons. If you colour the flag below, how many ways of colouring are there?
Let red, yellow, green, and blue be R, Y, G, and B respectively.
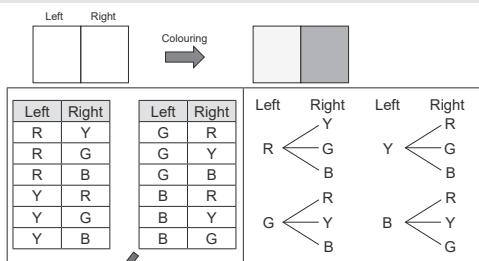


There are **12** ways in total.

**1** There is a cookie, gum, potato chips, and a donut in a box.
Let cookie, gum, potato chips, and donut be C, G, P, and D respectively.
Complete the diagram and find the total ways of ordering.
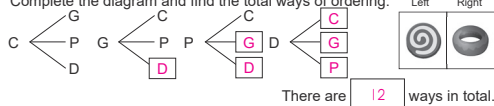


There are **12** ways in total.

**Example 2** There is one card for each of the following numbers: 1, 3, 5, 7. From these 4 cards, use 3 cards to create 3-digit whole numbers. How many whole numbers can you make in total?

| | 1 | 3 | 5 | 7 |

**1** Complete the table and diagram below when **1** is in the hundreds place.

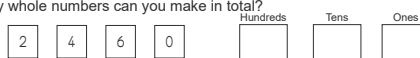| Hundreds | Tens | Ones |
|---|---|---|
| 1 | 3 | 5 |
| 1 | 3 | 7 |
| 1 | 5 | 3 |
| 1 | 5 | 7 |
| 1 | 7 | 3 |
| 1 | 7 | 5 |
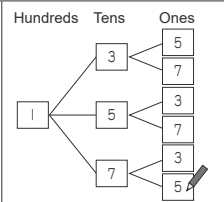


**2** How many 3-digit whole numbers can you make in total?

When 1 is in the hundreds place, there are **6** ways.
There are **4** cases where we can put the number in the hundreds place.
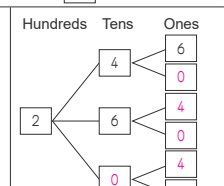And there are **6** ways each. So in total, there are **24** ways.

**2** There is one card for each of the following numbers: 2, 4, 6, 0.
From these 4 cards, use 3 cards to create 3-digit whole numbers. How many whole numbers can you make in total?

| | 2 | 4 | 6 | 0 |

**1** Complete the table and diagram below when **2** is in the hundreds place.

| Hundreds | Tens | Ones |
|---|---|---|
| 2 | 4 | 6 |
| 2 | 4 | 0 |
| 2 | 6 | 4 |
| 2 | 6 | 0 |
| 2 | 0 | 4 |
| 2 | 0 | 6 |



**2** How many 3-digit whole numbers can you make in total?

**18** numbers

If you place 0 in the hundreds place, you cannot make a 3-digit whole number.

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## 12 − 4  Ways of Ordering and Combination
### Combination (1)

**Instruction** Ways of combination.
Four teams (Team A, B, C, D) play football. If each team competes against the other team only one time, how many games will be played in total?



Let's try like we consider ways of ordering
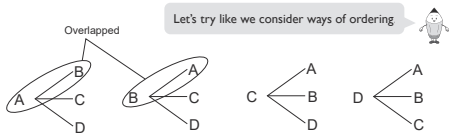
Given that each team competes only once, A – B and B – A are overlapped. Do we have any other ideas to make a table or diagram?

**Idea 1.** Using a table.
Look at the horizontal array and find pairs of teams, like A – B, A – C, and A – D. Add a check mark.

| | A | B | C | D |
|---|---|---|---|---|
| A | | ✓ | ✓ | ✓ |
| B | | | | |
| C | | | | |
| D | | | | |

Move on to the next array and continue to find pairs. For example, B – A is overlapped with A – B.

| | A | B | C | D |
|---|---|---|---|---|
| A | | ✓ | ✓ | ✓ |
| B | | | ✓ | ✓ |
| C | | | | ✓ |
| D | | | | |

**Idea 2.** Using a diagram.
Place A, B, C, D to form square

Draw lines to connect pairs and write them.
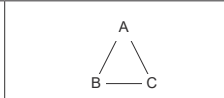A – B, A – C, A – D, B – C, C – D, and B – D



The same as when exploring ways of ordering, use a table or diagram to erase one of the repeated combinations and then count the total cases.

**Example** Three teams (Team A, B, C) play football. If each team competes with the other teams only one time, answer the following questions.

**1** Complete the table and diagram below.

| | A | B | C |
|---|---|---|---|
| A | | ✓ | ✓ |
| B | | | ✓ |
| C | | | |



When we consider pairs among three teams, we can draw a triangle.

**2** In total, how many combinations are there?
According to the table or diagram above, the ways are A – B, A – C, and B – C.  **3**

There are 5 types of pastries, Apple pie, Banana cake, Chocolate cake, Donut, and Egg tart. Two types were bought. Answer the following questions. Let Apple pie, Banana cake, Chocolate cake, Donut, and Egg tart be A, B, C, D, and E, respectively.

**1** Complete the table and diagram below.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | | ✓ | ✓ | ✓ | ✓ |
| B | | | ✓ | ✓ | ✓ |
| C | | | | ✓ | ✓ |
| D | | | | | ✓ |
| E | | | | | |
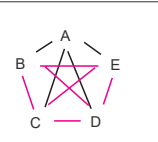


**2** Write down the combinations and find the total number.

Combinations:
A – B, A – C, A – D, A – E, B – C, B – D, B – E, C – D, C – E, D – E

The number of cases  **10**

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## 12 - 5 Ways of Ordering and Combination
### Combination (2)

**Example 1** There is one card for each of the following numbers: 2, 4, 6, 8. From these 4 cards, add 2 cards to create whole numbers. Answer the following questions.

❶ Complete the table and diagram below.

| | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| 2 | | ✓ | ✓ | ✓ |
| 4 | | | ✓ | ✓ |
| 6 | | | | ✓ |
| 8 | | | | |

2 —— 4
6 —— 8

❷ Write down the combinations and find the total number.

Combinations
$2 + 4 = 6, 2 + 6 = 8, 2 + 8 = 10$
$(4 + 6 = 10), 4 + 8 = 12, 6 + 8 = 14$

The number of cases    5

**1** There is one card for each of the following numbers: 1, 3, 5, 7. From these 4 cards, add 2 cards to create whole numbers. Answer the following questions.

❶ Complete the table and diagram below.

| | 1 | 3 | 5 | 7 |
|---|---|---|---|---|
| 1 | | ✓ | ✓ | ✓ |
| 3 | | | ✓ | ✓ |
| 5 | | | | ✓ |
| 7 | | | | |

1 —— 3
5 —— 7

❷ Write down the combinations and find the total number of cases.

Combinations
$1 + 3 = 4, 1 + 5 = 6, 1 + 7 = 8 (3 + 5 = 8),$
$3 + 7 = 10, 5 + 7 = 12$
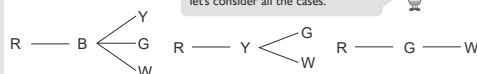
The number of cases    5

**Example 2** There are 5 coloured balls: red, blue, yellow, green, and white. From these 5 balls, pick up 3 balls and put them into a bag. Write down the combinations and find the total number of cases. Let red, blue, yellow, green, and white be R, B, Y, G, and W respectively.
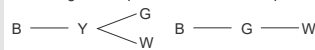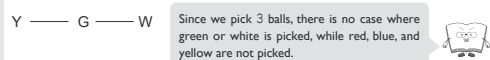
R  B  Y  G  W

Assuming red is picked,

By fixing a case and using a diagram, let's consider all the cases.

R —— B <Y/G/W    R —— Y <G/W    R —— G —— W

Assuming blue is picked, and red is not picked,

B —— Y <G/W    B —— G —— W

Assuming yellow is picked, and red and blue are not picked,

Y —— G —— W

Since we pick 3 balls, there is no case where green or white is picked, while red, blue, and yellow are not picked.

The number of cases    10

**2** There is one coin for each amount: 1 zed, 10 zeds, 50 zeds, and 100 zeds. From these 4 coins, choose 3 coins and find the total amount of money. ("zed(s)" is the fictional currency unit.)
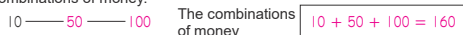
1  10  50  100

❶ Start with 1 zed and write down all the different combinations of money.

1 —— 10 <50/100    1 —— 50 —— 100

The combinations of money
$1 + 10 + 50 = 61, 1 + 10 + 100 = 111,$
$1 + 50 + 100 = 151$

❷ When you use 10 zed and do not use 1 zed, write down all the different combinations of money.

10 —— 50 —— 100    The combinations of money    $10 + 50 + 100 = 160$

❸ How many the different combinations of money are there in total?    4

## 12 - 6 Ways of Ordering and Combination
### Combination (3)

**Example 1** Four students, Emma, John, Mia, and Steven are nominees for student council. From these 4 students, 3 students will be elected. Write down the combinations of selected students. Let Emma, John, Mia, and Steven be E, J, M, and S respectively.

"Select 3 students out of 4 students" also means the same as "do not select 1 student out of 4 students".

E  J  M (Selected)    S (Not selected)

Let's consider cases where "not selected 1 student out of 4 students" is.

Combinations    E – J – M, E – J – S, E – M – S, J – M – S

The number of cases    4

**1** There are 5 different coloured flours: red, blue, yellow, purple, and white. From these 5 flowers, choose 4 flowers and make a bouquet. Write down the combinations and find the total number of cases. Let red, blue, yellow, purple, and white be R, B, Y, P, and W respectively.
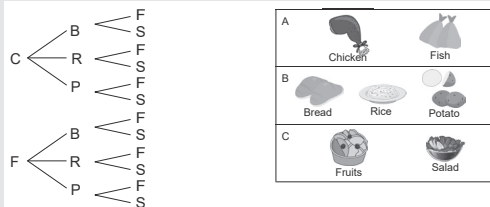
R  B  Y  P  W

Combinations
R – B – Y – P, R – B – Y – W, R – B – P – W,
R – Y – P – W, B – Y – P – W

The number of cases    5

**Example 2** You go to a school canteen for lunch. You can choose one from each category A, B, C. How many different cases are there? (Let the name of foods represent their first letter.)
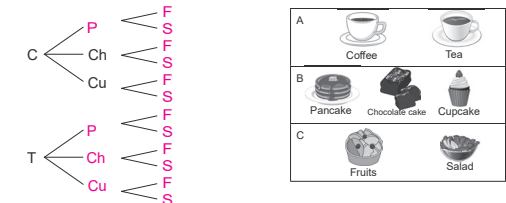
C < B <F/S  R <F/S  P <F/S
F < B <F/S  R <F/S  P <F/S

A: Chicken, Fish
B: Bread, Rice, Potato
C: Fruits, Salad

The number of cases    12

**2** You go to a café for afternoon tea. You can choose one from each category A, B, C.

❶ Complete the diagram below. Let the name of foods represent their first letter. If there are the same first letter, use second letter to distinguish.

C < P <F/S  Ch <F/S  Cu <F/S
T < P <F/S  Ch <F/S  Cu <F/S

A: Coffee, Tea
B: Pancake, Chocolate cake, Cupcake
C: Fruits, Salad

❷ Find the number of cases you can choose.

The number of cases    12

## 12 - 7  Ways of Ordering and Combination
### Review

**1** There is one card for each of the following numbers: 1, 2, 3, 4.
From these 4 cards, use 3 cards to create 3-digit whole numbers. How many whole numbers can you make in total?

| 1 | 2 | 3 | 4 | | Hundreds | Tens | Ones |
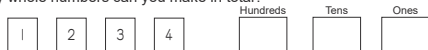
**①** Complete the table and diagram below when 1 is in the hundreds place.

| Hundreds | Tens | Ones |
|---|---|---|
| 1 | 2 | 3 |
| 1 | 2 | 4 |
| 1 | 3 | 2 |
| 1 | 3 | 4 |
| 1 | 4 | 2 |
| 1 | 4 | 3 |

Hundreds  Tens  Ones

1 — 2 < 3 / 4
1 — 3 < 2 / 4
1 — 4 < 2 / 3

**②** How many 3-digit whole numbers can you make in total?    **24**

**2** There is one card for each of the following numbers: 1, 2, 3, 0.
From these 4 cards, use 3 cards to create 3-digit whole numbers. How many whole numbers can you make in total?

| 1 | 2 | 3 | 0 | | Thousands | Hundreds | Tens | Ones |

**①** Complete the table and diagram below when 2 is in the thousands place.

| Thousands | Hundreds | Tens | Ones |
|---|---|---|---|
| 2 | 1 | 3 | 0 |
| 2 | 1 | 0 | 3 |
| 2 | 3 | 1 | 0 |
| 2 | 3 | 0 | 1 |
| 2 | 0 | 1 | 3 |
| 2 | 0 | 3 | 1 |

**②** How many 3-digit whole numbers can you make in total?

**18** numbers

---

**3** There are 5 students, A, B, C, D, E, and F. 2 students were elected as school council members.

**①** Complete the table below.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | | ✓ | ✓ | ✓ | ✓ | ✓ |
| B | | | ✓ | ✓ | ✓ | ✓ |
| C | | | | ✓ | ✓ | ✓ |
| D | | | | | ✓ | ✓ |
| E | | | | | | ✓ |
| F | | | | | | |

**②** Write down the combinations and find the total number.

Combinations:
A – B, A – C, A – D, A – E, A – F, B – C,
    B – D, B – E, B – F, C – D, C – E, C – F,
    D – E, D – F, E – F

The number of cases    **15**

**4** There is one coin for each: 1 zed, 50 zeds, 100 zeds, and 500 zeds.
From these 4 coins, choose 3 coins and find the total amount of money.
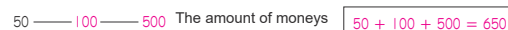("zed(s)" is the fictional currency unit.)

| 1 | 50 | 100 | 500 |

**①** When you use 1 zed, write down all the different combinations of money.

1 — 50 < 100 / 500          1 — 100 — 500

The amount of moneys:
1 + 50 + 100 = 151, 1 + 50 + 500 = 551,
1 + 100 + 500 = 601

**②** When you use 50 zed and do not use 1 zed, write down all the different combinations of money.

50 — 100 — 500    The amount of moneys    50 + 100 + 500 = 650

**③** How many the different combinations of money in total?    **4**

---

### Number & Operation
### Entire Grade-6 Review (1)

**1** Calculate the following multiplication problems.

① $\frac{2}{7} \times 3$    ② $\frac{2}{3} \times \frac{4}{5}$    ③ $\frac{7}{12} \times \frac{3}{14}$    ④ $\frac{13}{16} \times \frac{32}{13}$

⑤ $\frac{5}{9} \times 3\frac{3}{4}$    ⑥ $27 \times \frac{4}{9}$    ⑦ $0.3 \times \frac{5}{6}$    ⑧ $2\frac{2}{9} \times 0.6$

| ① $\frac{6}{7}$ | ② $\frac{8}{15}$ | ③ $\frac{1}{8}$ | ④ $2$ |
| ⑤ $\frac{25}{12}$ or $2\frac{1}{12}$ | ⑥ $12$ | ⑦ $\frac{1}{4}$ | ⑧ $\frac{4}{3}$ or $1\frac{1}{3}$ |

**2** Calculate the following division problems.

① $\frac{8}{9} \div 4$    ② $\frac{5}{6} \div \frac{5}{8}$    ③ $\frac{2}{3} \div \frac{4}{9}$    ④ $\frac{11}{18} \div \frac{11}{20}$

⑤ $\frac{1}{18} \div 1\frac{1}{12}$    ⑥ $2\frac{1}{4} \div 1\frac{1}{6}$    ⑦ $4 \div \frac{8}{5}$    ⑧ $0.7 \div \frac{1}{4}$

| ① $\frac{2}{9}$ | ② $\frac{4}{3}$ or $1\frac{1}{3}$ | ③ $\frac{3}{2}$ or $1\frac{1}{2}$ | ④ $\frac{10}{9}$ or $1\frac{1}{9}$ |
| ⑤ $\frac{2}{39}$ | ⑥ $\frac{27}{14}$ or $1\frac{13}{14}$ | ⑦ $\frac{5}{2}$ or $2\frac{1}{2}$ | ⑧ $\frac{14}{5}$ or $2\frac{4}{5}$ |

**3** Calculate the following problems.

① $\frac{3}{4} \times \left( \frac{1}{12} \times \frac{16}{17} \right)$    ② $\frac{5}{12} \times \frac{5}{6} \times \frac{18}{25}$    ③ $\frac{2}{5} \times \left( \frac{5}{3} + \frac{5}{6} \right)$

④ $\frac{3}{8} \times \left( \frac{8}{9} + \frac{16}{3} \right)$    ⑤ $\frac{12}{25} \times \left( \frac{5}{4} - \frac{1}{6} \right)$    ⑥ $\frac{7}{10} \times \left( \frac{10}{21} - \frac{5}{28} \right)$

⑦ $\frac{2}{19} \times \frac{5}{6} + \frac{2}{19} \times \frac{2}{9}$    ⑧ $\frac{1}{22} \times \frac{3}{4} - \frac{1}{22} \times \frac{1}{5}$

| ① $\frac{1}{17}$ | ② $\frac{1}{4}$ | ③ $1$ | ④ $\frac{7}{3}$ or $2\frac{1}{3}$ |
| ⑤ $\frac{13}{25}$ | ⑥ $\frac{5}{24}$ | ⑦ $\frac{1}{9}$ | ⑧ $\frac{1}{40}$ |

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**4** Which product is more than 8. Write the letter of the correct math sentence.

(a) $8 \times \frac{3}{5}$    (b) $8 \times 1$    (c) $8 \times \frac{18}{15}$    (d) $8 \times 1\frac{1}{2}$

(e) $8 \div \frac{4}{7}$    (f) $8 \div \frac{10}{9}$    Answer  **(c), (d), (e)**

**5** Read the following problems and write a math sentence using the letter $x$.

**①** Length of the perimeter of a square. One side is $x$ cm long.
Answer  **$x \times 4$ (cm)**

**②** Price of 3 notebooks. One notebook costs $x$ zeds*.
("zed(s)" is the fictional currency unit.)  Answer  **$x \times 3$ (zeds)**

**③** Total weight when $x$ g of candy is put in a 200 g box.
Answer  **$x + 200$ (g)**

**6** 1 m of rope costs 30 zeds*. How much does $\frac{1}{6}$ m of this rope cost?

Complete the number line diagrams and tables.

Math sentence
$30 \times \frac{1}{6}$
$= 5$    Answer  **5 zeds**

**7** I have a 30 m long ribbon. I used $\frac{7}{15}$ of this ribbon. How many m of ribbon do I have left?

Math sentence
$\frac{7}{15}$ of 30 m is;
$30 \times \frac{7}{15} = 14$ m
$30 - 14 = 16$    Answer  **16 m**

**8** There is $2\frac{1}{3}$ kg of strawberry jam. If you divide the jam into bottles that can hold $\frac{1}{6}$ kg each, how many bottles can you fill?

Math sentence
$2\frac{1}{3} \div \frac{1}{6} = \frac{7}{3} \div \frac{1}{6}$
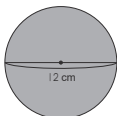$= \frac{7}{3} \times \frac{6}{1} = \frac{14}{1} = 14$    Answer  **14 bottles**

### Geometry
## Entire Grade-6 Review (2)

**1** The figure on the right is a point symmetric figure. Answer the following questions.

① Draw the point of symmetry on the figure.

② Draw in the figure the corresponding point B to point A

**2** Find the area of the following figures.

① Since the diameter is 12 cm, the radius is 6 cm.

Math sentence　$6 × 6 × 3.14 = 113.04$

Answer　113.04 cm²

② Since the figure is a quarter of a circle.

Math sentence　$(2 × 2 × 3.14) ÷ 4 = 3.14$

Answer　3.14 cm²

**3** The illustration below shows a forest. Answer the following questions.

① What kind of shape can you use it to find the area?

Triangle

② Find the approximate area.

Math sentence　$80 × 40 ÷ 2 = 1600$

Answer　1600 m²

**4** Fill in the blank with numbers to find the volume of the triangular prism on the right.

① The triangle prism is half of a cuboid. The volume of a cuboid is found by (Length) × (Width) × (Height),

$\boxed{6} × \boxed{5} × \boxed{7} ÷ \boxed{2} = \boxed{105}$ cm³

　Volume of the cuboid

② Find the volume using the area of the base, (Area of the base) × (Height)

$\boxed{6} × \boxed{5} ÷ \boxed{2} × \boxed{7} = \boxed{105}$ cm³

　Area of the base

**5** Triangle DBE is an enlarged drawing of triangle ABC. Answer the following questions.

① How many times is it enlarged?

Since side AC is 4 cm and side DE is 6 cm,

Math sentence　$\frac{6}{4} = 1.5$　1.5 times

② Find the length of side BE.

Math sentence　$1.5 × 5 = 7.5$　7.5 cm

③ Find the size of angle C.

50°

**6** There is a map of a school that is drawn in $\frac{1}{500}$ reduced scale. In the reduced drawing, the gymnasium has a rectangular shape with a length that is 10 cm long and a width that is 8 cm long. How many meters is the actual and width of the hall?

Length :

Math sentence　$10 × 500 = 5000$　Answer　50 m

Width :

Math sentence　$8 × 500 = 4000$　Answer　40 m

164

165

### Number & Operation
## Entire Grade-6 Review (3)

**1** The table below shows the time of walking and distance you go. If you walk at a speed of 40 m per minute, let $y$ m be the distance you travel in $x$ minutes. Answer the following questions.

| Time $x$ (minutes) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | … |
|---|---|---|---|---|---|---|---|---|
| Distance $y$ (m) | 0 | 40 | 80 | 120 | 160 | 200 | 240 | … |

① Write the number that fits in the blank in the table below.

② What is the relationship between time and distance ?

Answer　$y$ is proportional to $x$

③ Express the relationship between $x$ and $y$ in terms of a math sentence.

Answer　$y = 40 × x$

④ Draw a graph representing $x$ and $y$.

⑤ How many meters do you walk when you walk for 30 minutes?

Math sentence　$y = 40 × 30 = 1200$

Answer　1200 m

**2** The table below shows the time of driving a car and speed you go. Let $x$ time be the time it takes you to travel 120 km at a speed of $y$ per hour. Answer the following questions.

| Time (hours) $x$ | 1 | 2 | 3 | 4 | 5 | 6 | … |
|---|---|---|---|---|---|---|---|
| Speed (km/hour) $y$ | 120 | 60 | 40 | 30 | 24 | 20 | … |

① Write the number that fits in the blank in the table below.

② If the value of $x$ doubles or triples, how does the value of $y$ change?

Answer　$y$ becomes 1/2 or 1/3 as many

③ What is the relationship between $x$ and $y$?

Answer　$y$ is proportional to $x$

④ Express the relationship between $x$ and $y$ in an math sentence?

Answer　$x × y = 120$

⑤ If the time is 8 hours, what is the speed?

Math sentence　$8 × y = 120$　$y = 120 ÷ 8$　Answer　15 km/hours

**3** Express the following proportions as simple ratios.

(a) The ratio of the length of a rectangle 35 cm to its width 21 cm.

Answer　5 : 3

(b) Ratio of 150 mL rice to 180 mL water.

Answer　5 : 6

**4** Find the number that applies to $x$.

(a) $8 : 5 = x : 25$　Answer　$x = 40$

(b) $x : 20 = 5 : 4$　Answer　$x = 25$

**5** Simplify the following ratios.

(a) $36 : 28$　Answer　9 : 7　(b) $420 : 350$　Answer　6 : 5

(c) $5.6 : 2.1$　Answer　8 : 3　(d) $0.6 : 3.8$　Answer　3 : 19

(e) $\frac{1}{6} : \frac{3}{10}$　Answer　5 : 9　(f) $\frac{2}{7} : \frac{2}{9}$　Answer　9 : 7

**6** My brother and sister have a coloured paper. The ratio of the number of sheets is 5 : 3 accordingly. My brother has 30 sheets. How many sheets of coloured paper does my sister have?

Math sentence　$5 : 3 = 30 : x$　$x = 18$　Answer　18 sheets

**7** We measured the length of the shade of a tree in the school and found it to be 9 m. If you put a stick 0.6 m long, the length of its shade is 1.5 m. What is the height of the tree?.

Math sentence　$0.6 : 1.5 = x : 9$　Answer　3.6 m

**8** The 3,000 zeds given to your family by your father will be divided 3 : 2 between your brother and sister. How many zeds will my brother and sister get?

① Find the total ratio.　$3 + 2 = 5$

② How many zeds will my brother get?

Math sentence　$3 : 5 = x : 3000$　$x = 1800$　Answer　1800 zeds

③ How many zeds will my sister get?

Math sentence　$2 : 5 = x : 3000$　$x = 1200$　Answer　1200 zeds

166

167

43

### Data Utilization
## Entire Grade-6 Review（4）

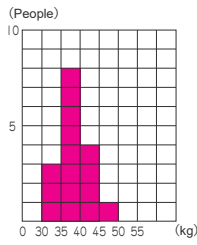**1** The following information is the records of classmates' weight.

37, 43, 41, 30, 49, 38, 36, 42, 39, 37, 32, 43, 37, 30, 35, 39 (kg)

**1** Find the mean of the weight.

Math sentence  608 ÷ 16 = 38

**2** Complete frequency table and histogram.

Record of Classmates' Weight

| Weight（kg) | | | Number of student |
|---|---|---|---|
| more than or equal to 30 | ～ | less than 35 | 3 |
| 35 | ～ | 40 | 8 |
| 40 | ～ | 45 | 4 |
| 45 | ～ | 50 | 1 |
| | Total | | 16 |

（People)
(histogram)

**3** Where does the student who is 10th of the lightest belong?

35~40

**4** How much % of the number of students who are in more than or equal to 40 kg and less than 45 kg? Round the second decimal place.

Since the total number of student is 16,

Math sentence  4 ÷ 16 × 100 = 25

25%

**2** A boy tosses a coin 3 times. In how many ways does he get heads and tails?

Consider the case where it gets head first. Complete the table and diagram below. Let head and tail be H and T, respectively.

| 1st | 2nd | 3rd |
|---|---|---|
| H | H | H |
| H | H | T |
| H | T | H |
| H | T | T |

(diagram: 1st 2nd 3rd with H, T branches)

How many ways are there in all?
Since there are 4 ways when it gets heads first,

2 × 4 = 8

8

**3** You are making four-digit passwords using numerals. For each of the following situations, figure out how many different passwords are possible.

**1** Using each numeral 1, 2, 3, and 4 once
Since there are 6 ways when you fix "1" in the thousands place,

4 × 6 = 24

24

**2** If you use the numerals 0~9, and if you can use each numerals as many times as you like.
Since there are 10 ways when you fix a number in the thousands place,

7 × 8 × 9 × 9 = 4536

4536

**4** A girl is buying 2 flavors of ice cream from 5 choices: banana, chocolate, strawberry, vanilla, and walnut. How many combinations of flavor possible?  Let banana, chocolate, strawberry, vanilla, and walnut be B, C, S, V, and W.

Combinations   B-C, B-S, B-V, B-W, C-S, C-V, C-W, S-V, S-W, V-W

The number of cases   10

168                                                                              169

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### Power up !
## Diagnostic Review 1

**1** Calculate the following problems. 【①～⑧：G1～G4, ⑨～⑫：G3～G5】

① 326＋674   ② 613－36   ③ 53×46   ④ 204÷6
⑤ 34＋9×5   ⑥ 96－81÷9   ⑦ 46－(30－19)   ⑧ 17＋(45－6×7)
⑨ 4.8＋2.3   ⑩ 50.8＋7.34   ⑪ 7.6－5.3   ⑫ 9.152－8.72

| ① | 1000 | ② | 577 | ③ | 2438 | ④ | 34 |
|---|---|---|---|---|---|---|---|
| ⑤ | 79 | ⑥ | 87 | ⑦ | 35 | ⑧ | 20 |
| ⑨ | 7.1 | ⑩ | 58.14 | ⑪ | 2.3 | ⑫ | 0.432 |

**2** Write the correct numbers in the ☐. 【G4～G6】

① $0.8+7.6=☐+0.8$
② $\frac{1}{2}×\frac{3}{5}=\frac{3}{5}×☐$
③ $(198+84)+16=198+(☐+16)$
④ $(3.2×0.25)×4=3.2×(☐×0.25)$
⑤ $9÷8+4.2÷8=(9+☐)÷8$
⑥ $10×\left(1.2-\frac{9}{10}\right)=10×1.2-10×☐$
⑦ $\left(\frac{1}{2}+\frac{1}{4}\right)×8=\frac{1}{2}×8+☐×8$
⑧ $5×\frac{8}{9}-4×\frac{8}{9}=(5-4)×☐$
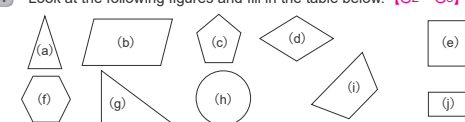
| ① | 7.6 | ② | $\frac{1}{2}$ | ③ | 84 | ④ | 4 |
|---|---|---|---|---|---|---|---|
| ⑤ | 4.2 | ⑥ | $\frac{9}{10}$ | ⑦ | $\frac{1}{4}$ | ⑧ | $\frac{8}{9}$ |

**3** Answer the following questions. 【G4～G5】

① What is 2 hundred millions, 8 one millions, and 5 ten thousands?
② How many 1000s are there in 357000?
③ What is 7 0.1's and 8 0.01's?
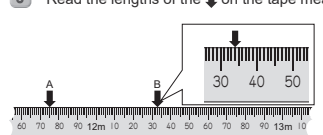④ How many 0.01 are there in 2.64?
⑤ What is $\frac{2}{3}$ times 3?

| ① | 208050000 | ② | 357 | ③ | 0.78 | ④ | 264 | ⑤ | 2 |
|---|---|---|---|---|---|---|---|---|---|

**4** Look at the following figures and fill in the table below. 【G2～G5】

(a) (b) (c) (d) (e) (f) (g) (h) (i) (j)
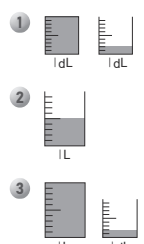
| Name | Right triangle | Isosceles triangle | Pentagon | Hexagon | Circle |
|---|---|---|---|---|---|
| Answer | (g) | (a) | (c) | (f) | (h) |
| Name | Trapezoid | Rectangle | Square | Parallelogram | Rhombus |
| Answer | (i) | (j) | (e) | (b) | (d) |

**5** Read the lengths of the ↓ on the tape measures below. 【G3】

30  40  50

A is 11 m 75 cm.
B is 12 m 34 cm.

60 70 80 90 12m 10 20 30 40 50 60 70 80 90 13m 10

**6** Match the same capacities with a line. 【G2】

① (1dL 1dL)
② (1L)
③ (1L 1dL)

500 mL
1L 20 mL
120 mL

**7** The time is 9:15 now. Write the time points that shows the following times. 【G2】

① 30 minutes after   ② 3 hours before

It is 9:45   It is 6:15

(clock diagram)

170                                                                              171

## Power up !
## Diagnostic Review 2

**1** Calculate the following problems. 【①〜⑧：G1〜G4，⑨〜⑫：G3〜G5】

① 3208 + 4895　② 8100 − 354　③ 326 × 418　④ 360 ÷ 45

⑤ 47 + 7 × 9　⑥ 84 − 36 ÷ 12　⑦ 52 − (16 − 5 × 2)　⑧ 12 + (45 − 15 ÷ 5)

⑨ 16.6 + 3.4　⑩ 8.6 + 2.53　⑪ 3 − 0.2　⑫ 2.5 − 1.86

| ① | 8103 | ② | 7746 | ③ | 136268 | ④ | 8 |
|---|---|---|---|---|---|---|---|
| ⑤ | 110 | ⑥ | 81 | ⑦ | 46 | ⑧ | 54 |
| ⑨ | 20 or 20.0 | ⑩ | 11.13 | ⑪ | 2.8 | ⑫ | 0.64 |

**2** Rewrite the following math sentences using the properties of operations and solve. 【G5〜G6】

① $6.3 + 1.75 + 3.7 = (6.3 + 3.7) + 1.75 = 10 + 1.75 = 11.75$

② $0.25 × 9 × 4 = (0.25 × 4) × 9 = 1 × 9 = 9$

③ $2.4 × 9.3 − 7.3 × 2.4 = (9.3 − 7.3) × 2.4 = 2 × 2.4 = 4.8$

④ $\left(\frac{6}{5} − \frac{3}{4}\right) × 400 = \left(\frac{6}{5} × 400\right) − \left(\frac{3}{4} × 400\right) = 480 − 300 = 180$

⑤ $1.02 × 23 = (1 + 0.02) × 23 = (1 × 23) + (0.02 × 23) = 23 + 0.46 = 23.46$

**3** Answer the following questions. 【G5】

① Which numbers are even numbers?

(a) 26,　(b) 43,　(c) 187,　(d) 6590,　(e) 192837　　**(a)，(d)**

② Write the least common multiple of the numbers in the（　）.

(a)（2，8）　(b)（3，5）　(c)（8，6）　(d)（5，6，10）

| (a) | 8 | (b) | 15 | (c) | 24 | (d) | 30 |
|---|---|---|---|---|---|---|---|

③ Write the greatest common factor of the numbers in the（　）.

(a)（14，21）　(b)（16，32）　(c)（18，24）　(d)（12，16，18）

| (a) | 7 | (b) | 16 | (c) | 6 | (d) | 2 |
|---|---|---|---|---|---|---|---|

**4** Two different set squares are used to make the following angles.　Find the size of each marked angle. 【G4】

Angle a is　**105°**　　　　Angle b is　**75°**

**5** Match solid figures and their name. 【G4，G6】

Cuboid　　Cube　　Cylinder　　Prism

**6** Convert the lengths to specified units. 【G2〜G3】

km　　　m　　cm　mm

① 3 km = **3000** m　　　② 1500 m = **1** km **500** m

③ 1 m 75 cm = **175** cm　④ 125 mm = **12** cm **5** mm

**7** Fill in the □ with the appropriate numbers. 【G3】

① Units of Weight

1 t = **1000** kg

1 kg = **1000** g

1 g = **1000** mg

② Units of Capacity

1 L = **10** dL = **1000** mL

1 dL = **100** mL

③ Units of Time

1 day = **24** hours, 1 hour = **60** minutes, 1 minute = **60** seconds

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## Power up !
## Diagnostic Review 3

**1** Calculate the following problems. 【G3〜G5】

① 2.6 × 8　② 1.7 × 3.6　③ 7.04 × 5.2　④ 24.5 × 0.34

⑤ 8.4 ÷ 7　⑥ 9.1 ÷ 2.6　⑦ 46.4 ÷ 14.5　⑧ 2.1 ÷ 0.42

⑨ (3.5 + 6.5) × 0.1　⑩ 1.05 × 100 ÷ 5　⑪ (4.5 − 2) ÷ 0.125　⑫ 20 + 10 × 3.14

| ① | 20.8 | ② | 6.12 | ③ | 36.608 | ④ | 8.33 |
|---|---|---|---|---|---|---|---|
| ⑤ | 1.2 | ⑥ | 3.5 | ⑦ | 3.2 | ⑧ | 5 |
| ⑨ | 1 | ⑩ | 21 | ⑪ | 20 | ⑫ | 51.4 |

**2** Write the correct numbers in the □. 【G5】

① $1 = \frac{1}{(a)} = \frac{(b)}{2}$　② $3 = \frac{(c)}{1} = \frac{6}{(d)}$　③ $0.3 = \frac{3}{(e)}$　④ $1.57 = \frac{157}{(f)}$　⑤ $1 ÷ 4 = \frac{(h)}{(g)}$

| (a) | 1 | (b) | 2 | (c) | 3 | (d) | 2 | (e) | 10 | (f) | 100 | (g) | 4 | (h) | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

**3** Change the following fractions to decimal numbers. 【G5】

① $\frac{1}{8}$ **0.125**　② $\frac{9}{5}$ **1.8**　③ $\frac{3}{4}$ **0.75**　④ $\frac{19}{20}$ **0.95**

**4** Change the following decimal numbers to fractions.　Simplify answers.

① 0.7 **$\frac{7}{10}$**　② 0.45 **$\frac{9}{20}$**　③ 1.5 **$\frac{3}{2}$**　④ 2.05 **$\frac{41}{20}$**

**5** Arrange the following numbers in ascending order. 【G5】

① $\frac{1}{6}$, $\frac{2}{3}$, $\frac{5}{3}$, $\frac{5}{6}$, $\frac{8}{5}$, 0, 1

**0** < $\frac{1}{6}$ < $\frac{2}{3}$ < $\frac{5}{6}$ < **1** < $\frac{8}{5}$ < $\frac{5}{3}$

② $\frac{7}{3}$, $\frac{1}{2}$, $\frac{2}{5}$, 0.3, 0.75, 1.2, 2

**0.3** < $\frac{2}{5}$ < $\frac{1}{2}$ < **0.75** < **1.2** < **2** < $\frac{7}{3}$

**6** Every position in the space is represented by a list of three numbers. The position of the pen is 2 width, 1 length, and 2 height. We express the position as (2, 1, 2) . 【G4】

① Represent the position of following items.

Sessors　**(4, 2, 1)**

Notebook　**(4, 0, 3)**

② What item is in position (0, 3, 0)?　**Eraser**

**7** Fill in the □ with appropriate units of quantities. 【G3】

① Length of a classroom.　8 **m**

③ Amount of canned juice.　350 **mL**

④ Thickness of a notebook.　4 **mm**

⑤ Weight of a tennis ball.　58 **g**

**8** There are 72 passengers on a bus with a capacity of 60 per vehicle. 【G5〜6】

① What percentage of the capacity is occupied by passengers?

Math sentence

| 60 people | 72 people |
|---|---|
| 1 | □ |

$72 ÷ 60 = 1.2$　$1.2 × 100 = 120$　Answer **120%**

② What is the smallest ratio between the number of passengers and the capacity of the vehicle?

Math sentence $72 : 60 = 6 : 5$　Answer **6 : 5**

**9** The table below shows the records of math test are summarized in the dot plot below. Answer the following questions.　① Find the mean value 【G6】

**41.5**

② Find the median value

**43**

## Power up！
## Diagnostic Review 4

**1** Calculate the quotient to the ones place and find the remainder. 【G4〜G5】

① $29.8 \div 3$　② $56.3 \div 14$　③ $70.5 \div 13.6$　④ $1.83 \div 0.52$

| ① | ② | ③ | ④ |
|---|---|---|---|
| 9　R 2.8 | 4　R 0.3 | 5　R 2.5 | 3　R 0.27 |

**2** Calculate the following problems. 【G3〜G6】

① $\frac{3}{7} + \frac{6}{7}$　② $\frac{4}{5} + \frac{1}{3}$　③ $1\frac{1}{6} + 2\frac{3}{4}$　④ $\frac{9}{8} - \frac{3}{8}$

⑤ $\frac{4}{7} - \frac{1}{5}$　⑥ $3\frac{5}{6} - 2\frac{4}{9}$　⑦ $\frac{2}{3} \times \frac{1}{4}$　⑧ $\frac{3}{5} \times \frac{10}{9}$

⑨ $\frac{7}{13} \times 1\frac{6}{7}$　⑩ $\frac{3}{4} \div \frac{1}{3}$　⑪ $\frac{6}{7} \div 3$　⑫ $1\frac{5}{8} \div 2\frac{1}{4}$

| ① | ② | ③ | ④ |
|---|---|---|---|
| $\frac{9}{7}$ | $\frac{17}{15}$ or $1\frac{2}{15}$ | $\frac{47}{12}$ or $3\frac{11}{12}$ | $\frac{3}{4}$ |
| ⑤ | ⑥ | ⑦ | ⑧ |
| $\frac{13}{35}$ | $\frac{25}{18}$ or $1\frac{7}{18}$ | $\frac{1}{6}$ | $\frac{2}{3}$ |
| ⑨ | ⑩ | ⑪ | ⑫ |
| 1 | $\frac{9}{4}$ or $2\frac{1}{4}$ | $\frac{2}{7}$ | $\frac{13}{18}$ |

**3** Round the following numbers to the nearest thousand. 【G4】

① $4351$　② $97820$　③ $40259$　④ $852541$　⑤ $129834$

| ① | ② | ③ | ④ | ⑤ |
|---|---|---|---|---|
| 4000 | 98000 | 40000 | 853000 | 130000 |

**4** What are the largest and smallest numbers that can be made when rounded to the nearest ten thousand. 【G4】

|  | | Largest | Smallest |
|---|---|---|---|
| ① | 630000 | 634999 | 625000 |
| ② | 9870000 | 9874999 | 9865000 |

**5** $2.4$ L of tea will be shared equally among $x$ people. How many L of tea can each person get? Write a math sentence and calculate when $x = 6$. 【G6】

Answer　$2.4 \div x$ (L)

When $x = 6$, the amount of juice is　$0.4$ L

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**6** Find the area and volume of the following figures. 【G4〜G6】

①
Math sentence　$4 \times 3 = 12$
Answer　$12 \text{ cm}^2$

②
Math sentence　$(8 \times 4 \div 2) + (8 \times 5 \div 2) + (10 \times 3 \div 2)$
$= 16 + 20 + 15 = 51$
Answer　$51 \text{ cm}^2$

③
Math sentence　$6 \times 3 \div 2 \times 5 = 45$
Answer　$45 \text{ cm}^3$

④
Math sentence　$(7 \times 7 \times 3.14) \times 10 = 1538.6$
Answer　$1538.6 \text{ cm}^3$

**7** From the following (a)〜(c), choose the one in which $y$ is proportional to $x$ and the one in which $y$ is inversely proportional to $x$. 【G5〜6】

(a) A square with a perimeter of $40$ cm, length $x$ cm and width $y$ cm.
(b) A rectangle with a length of $4$ cm and a width of $x$ cm and an area of $y$ cm$^2$.
(c) An area of $36$ cm$^2$ with a length of $x$ cm and a rectangle of length $y$ cm.

$y$ is proportional to $x$ is　(b)　　　$y$ is inversely proportional to $x$ is　(c)

In (c), if the length is $4$, what is the width of a rectangle?

Math sentence　$36 \div 4 = 9$　　Answer　$9$ cm

**8** There is one card for each of the following numbers: $1, 3, 8, 0$. 【G6】 From these $4$ cards, use $3$ cards to create $3$-digit whole numbers. How many whole numbers can you make in total?

| 1 | 3 | 8 | 0 |
|---|---|---|---|

| Hundreds | Tens | Ones |
|---|---|---|
|  |  |  |

How many $3$-digit whole numbers can you make in total?　$18$

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## Power up！
## Diagnostic Review 5

**1** Calculate the following problems by converting whole numbers and decimal numbers to fractions. 【G6】

① $0.6 \times \frac{2}{3}$　② $7 \div \frac{7}{8}$　③ $\frac{3}{4} \div 3$　④ $0.3 \times \frac{5}{6} + \frac{3}{4}$

⑤ $2.4 \div \frac{4}{5} + 1$　⑥ $4.5 - \frac{4}{3} \times 2$　⑦ $\frac{7}{8} - 0.7 \div \frac{14}{5}$　⑧ $\frac{7}{3} \div 28 \times 0.8$

⑨ $\frac{7}{12} \div 4.2 \times 0.6$　⑩ $7 \times \frac{3}{4} \div 1.4$　⑪ $0.8 \div 0.75 \times \frac{5}{8}$　⑫ $\frac{4}{5} \div \frac{2}{3} \div 6$

| ① | ② | ③ | ④ |
|---|---|---|---|
| $\frac{2}{5}$ | 8 | $\frac{1}{4}$ | 1 |
| ⑤ | ⑥ | ⑦ | ⑧ |
| 4 | $\frac{11}{6}$ or $1\frac{5}{6}$ | $\frac{5}{8}$ | $\frac{1}{15}$ |
| ⑨ | ⑩ | ⑪ | ⑫ |
| $\frac{1}{12}$ | $\frac{15}{4}$ or $3\frac{3}{4}$ | $\frac{2}{3}$ | $\frac{1}{5}$ |

**2** Write the correct decimal numbers from ① to ④ and fractions from ⑤ to ⑧ . 【G5】

| ① | ② | ③ | ④ |
|---|---|---|---|
| 0.2 | 1.4 | 2.5 | 3.8 |
| ⑤ | ⑥ | ⑦ | ⑧ |
| $\frac{2}{3}$ | $\frac{5}{3}$ | $\frac{7}{3}$ | $\frac{11}{3}$ |

**3** $12.5$ kg of rice is divided into bags of $1.8$ kg each. How many bags can be made and how many kg of rice is left over? 【G5】

Math sentence
$12.5 \div 1.8 = 6$ R $1.7$

Answer　6 bags can be made and $1.7$ kg of rice is left over.

**4** $1\frac{1}{3}$ kg of sugar is needed to make a $2\frac{2}{5}$ kg cake. How much sugar is needed to bake a $1$ kg cake? 【G6】

Math sentence
$1\frac{1}{3} \div 2\frac{2}{5} = \frac{4}{3} \div \frac{12}{5} = \frac{4}{3} \times \frac{5}{12} = \frac{5}{9}$

Answer　$\frac{5}{9}$ kg

---

**5** The following triangles are congruent. Answer the following questions.

① Which is the corresponding vertex to vertex C? 【G5】
Vertex　F

② Which is the corresponding angle to angle E? Also, how many degrees is the size?
Angle　B　　Size　$40°$

③ Which is the corresponding side to side DF? Also, how many cm is it?
Side　AC　　Length　$3.6$ cm

**6** Draw the following figures. 【G6】

① A line symmetric figure that has the straight line $xy$ as the line of symmetry.

② A point symmetric figure that has the point O as the point of symmetry.

**7** There are $42$ students in your class and the ratio of boys to girls is $3 : 4$. How many boys and girls are there in the class? 【G5〜G6】

① Find the ratio of total number.
Math sentence　$3 + 4 = 7$　　Answer　$7$

② How many boys are there in the class ?
Math sentence　$3 : 7 = x : 42$　　$x = 3 \times 6$　　Answer　$18$ boys

③ How many girls are there in the class?
Math sentence　$4 : 7 = x : 42$　　$x = 4 \times 6$　　Answer　$24$ girls

**8** Choose appropriate kinds of graph with the following situation. 【G3〜G6】

| Data | What you present | Kinds of graph |
|---|---|---|
| Daily temperature in a city | Changes in every hours | B |
| Planted area in crops | Ratio to the whole | C |
| Number of books borrowed by classmate | Frequency of use of the library. | A |

Kinds of graphs
A: Bar graph, B: Line graph, C: Pie chart or strip chart, D Histogram