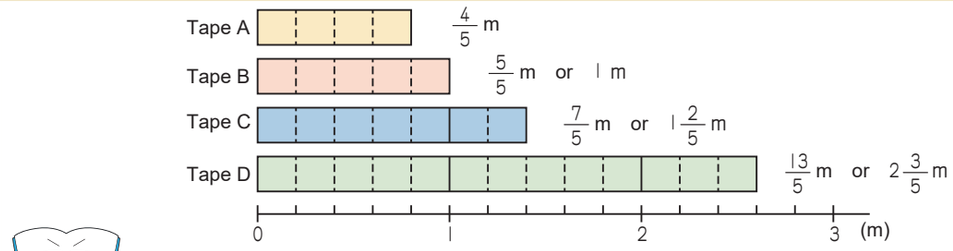
15 - 1 Fractions

Expressing Fractions (1)

Instruction Four pieces of tape are shown below. How many metres long is each piece?



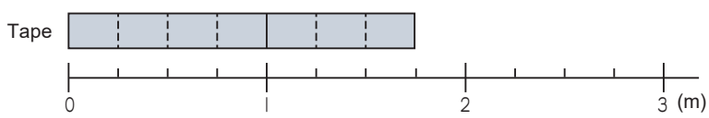
The small tick indicates $\frac{1}{5}$ m.

Fractions where the numerator is less than the denominator, such as $\frac{4}{5}$ called **proper fractions**.

Fractions where the numerator is equal to or greater than the denominator such as $\frac{5}{5}$, $\frac{7}{5}$ and $\frac{13}{5}$ are called **improper fractions**.

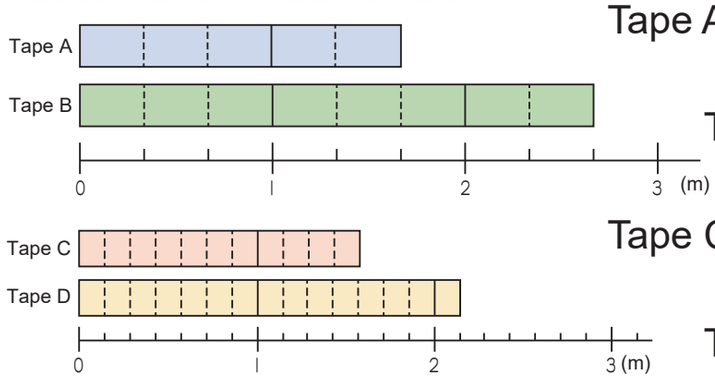
Fractions expressed as a combination of a whole numberand a proper fraction such as $1\frac{2}{5}$ and $2\frac{3}{5}$ are called **mixed numbers**. $1\frac{2}{5}$ is read as "**one and two fifths**."

Example Write down the length of the tape as an improper fraction and as a mixed number.



$\frac{7}{4}$ m or $1\frac{3}{4}$ m

Show the length of the following tapes by using both the improper fraction and mixed number.



Tape A ☐ m or ☐ m

Tape B ☐ m or ☐ m

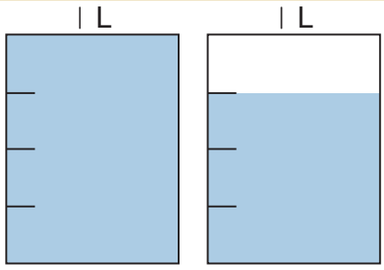
Tape C ☐ m or ☐ m

Tape D ☐ m or ☐ m

15 - 2 Fractions

Expressing Fractions (2)

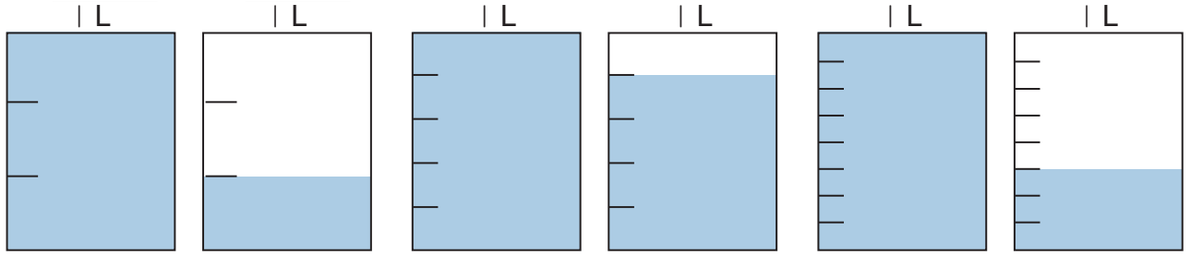
Example 1 Write the total amount of water as an improper fraction and as a mixed number.



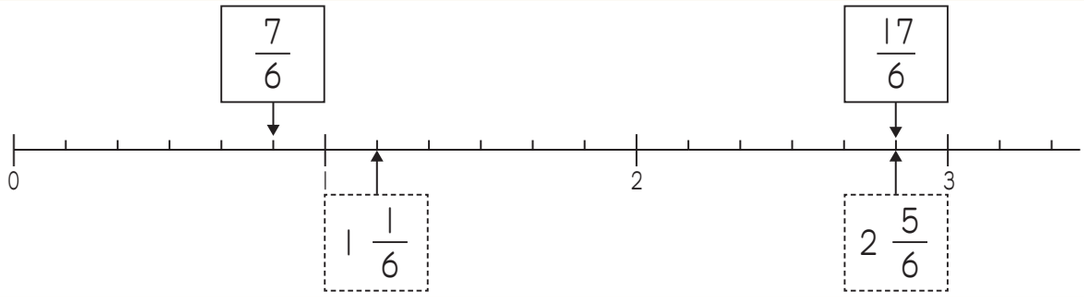
$\frac{7}{4}$ L or $1\frac{3}{4}$ L

1 Write the total amount of water as an improper fraction and as a mixed number.

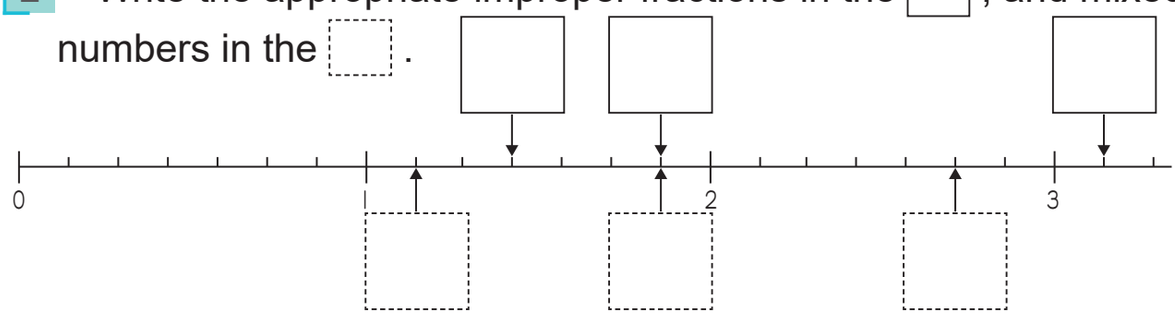
① ☐ L or ☐ L ② ☐ L or ☐ L ③ ☐ L or ☐ L



Example 2 Write the appropriate improper fraction in the ☐ and mixed number in the ⬚.



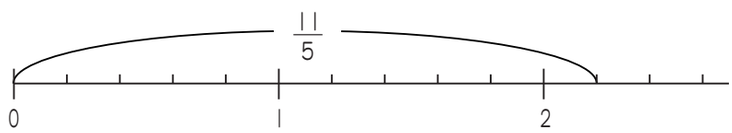
2 Write the appropriate improper fractions in the ☐, and mixed numbers in the ⬚.



15 - 3 Fractions

Expressing Fractions (3)

Example Change $\frac{11}{5}$ into a mixed number.



$\frac{11}{5} = 2\frac{1}{5}$

$11 \div 5 = 2$ R 1

$\frac{5}{5} = 1$

So $\frac{10}{5} = 2$

$\frac{11}{5} = 2\frac{1}{5}$

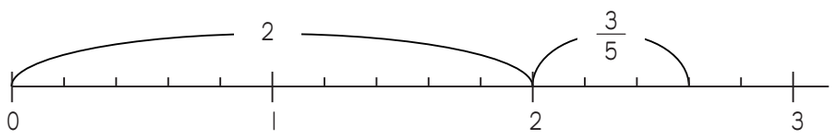
Change the following improper fractions into mixed numbers or whole numbers.

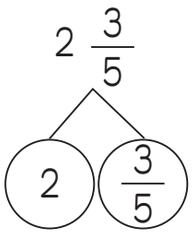
1. $\frac{12}{5}$ ☐
2. $\frac{9}{4}$ ☐
3. $\frac{13}{3}$ ☐
4. $\frac{7}{2}$ ☐
5. $\frac{30}{7}$ ☐
6. $\frac{56}{9}$ ☐
7. $\frac{31}{6}$ ☐
8. $\frac{40}{8}$ ☐
9. $\frac{35}{4}$ ☐
10. $\frac{28}{5}$ ☐

15 - 4 Fractions

Expressing Fractions (4)

Example Change $2\frac{3}{5}$ into an improper fraction.





$2\frac{3}{5} = \frac{13}{5}$

$5 \times 2 + 3 = 13$

Because 1 is the same as five $\frac{1}{5}$'s, 2 must be 10 $\frac{1}{5}$'s.

$2\frac{3}{5} = \frac{13}{5}$

Change the following mixed numbers into improper fractions.

1. $1\frac{1}{3}$ ☐
2. $1\frac{1}{4}$ ☐
3. $4\frac{2}{5}$ ☐
4. $3\frac{5}{6}$ ☐
5. $2\frac{4}{7}$ ☐
6. $4\frac{3}{8}$ ☐
7. $5\frac{1}{2}$ ☐
8. $3\frac{5}{9}$ ☐
9. $4\frac{1}{6}$ ☐
10. $6\frac{2}{3}$ ☐

15 - 5 Fractions

Comparing the Size of Numbers (1)

Example Use an inequality symbol (< or >) to express the relationship between the two numbers.

$$\frac{25}{7} \boxed{<} 3\frac{5}{7}$$

Compare them as mixed numbers:

$$\frac{25}{7} = 3\frac{4}{7}$$

$$25 \div 7 = 3 \text{ R } 4$$

Therefore, this problem is...

$$3\frac{4}{7} \boxed{<} 3\frac{5}{7}$$

Compare them as improper fractions:

$$3\frac{5}{7} = \frac{26}{7}$$

$$7 \times 3 + 5 = 26$$

Therefore, this problem is...

$$\frac{25}{7} \boxed{<} \frac{26}{7}$$

It is easier to compare the two numbers if they are both mixed numbers or both improper fractions.

Use an inequality symbol (< or >) to express the relationship between the two numbers.

1. $\frac{8}{3} \square 2\frac{1}{3}$
2. $\frac{23}{4} \square 5\frac{1}{4}$
3. $\frac{23}{7} \square 3\frac{3}{7}$
4. $\frac{30}{6} \square 4\frac{5}{6}$
4. $\frac{16}{5} \square 3\frac{2}{5}$
6. $\frac{22}{9} \square 2\frac{2}{9}$
7. $\frac{27}{8} \square 3\frac{1}{8}$
8. $\frac{26}{3} \square 9\frac{1}{3}$
9. $\frac{15}{2} \square 6\frac{1}{2}$
10. $\frac{45}{7} \square 6\frac{4}{7}$

15 - 6 Fractions

Comparing the Size of Numbers (2)

Example Put the numbers in the ( ) in order from the largest to smallest.

$$\left(\frac{7}{8}, \quad 2\frac{3}{8}, \quad \frac{15}{8} \right)$$

Compare them as improper fractions:

$$2\frac{3}{8} = \frac{19}{8}$$

$$8 \times 2 + 3 = 19$$

It is easier to compare the three numbers if they are all mixed numbers or all improper fractions.

Answer $2\frac{3}{8}$ ➡ $\frac{15}{8}$ ➡ $\frac{7}{8}$

Put the numbers in the ( ) in order from the largest to smallest.

1. $\left(\frac{12}{5}, \quad 2\frac{1}{5}, \quad \frac{9}{5} \right)$ Answer ☐ ➡ ☐ ➡ ☐

2. $\left(\frac{20}{7}, \quad 3, \quad 2\frac{5}{7} \right)$ Answer ☐ ➡ ☐ ➡ ☐

3. $\left(\frac{25}{6}, \quad 3\frac{5}{6}, \quad 4 \right)$ Answer ☐ ➡ ☐ ➡ ☐

4. $\left(7\frac{1}{4}, \quad \frac{31}{4}, \quad 7, \quad 6\frac{3}{4} \right)$ Answer ☐ ➡ ☐ ➡ ☐ ➡ ☐

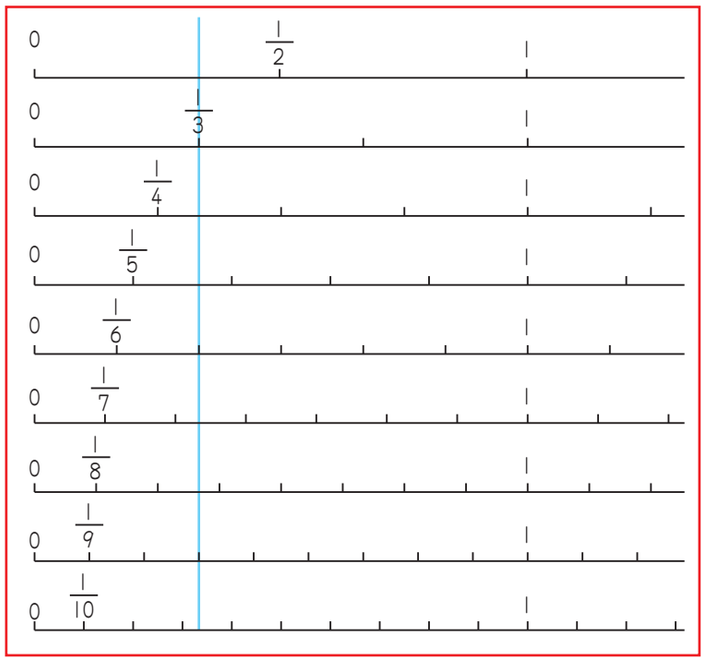
5. $\left(\frac{70}{9}, \quad 8\frac{8}{9}, \quad \frac{68}{9}, \quad 8 \right)$ Answer ☐ ➡ ☐ ➡ ☐ ➡ ☐

15 - 7 Fractions

Fractions of Equal Size

Example Look at the number line below and find two fractions that are exactly equal in size to $\frac{1}{3}$.

In which two number lines does the blue line land on a tick mark? Now what is the fraction for that tick mark?



Answer

$\frac{2}{6}$ and $\frac{3}{9}$

Look at the number line above and answer the following questions.

1. Find the fractions that are equal in size to $\frac{1}{4}$. ☐

2. Find four fractions that are equal in size to $\frac{1}{2}$. ☐

3. Find the fractions that are equal in size to $\frac{2}{3}$. ☐

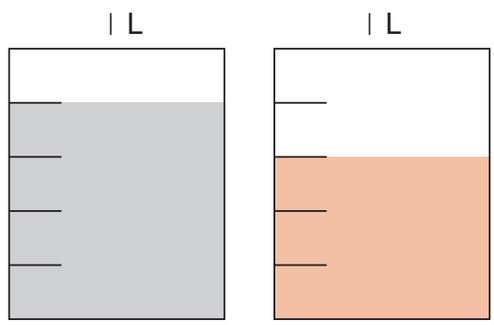
4. Which is greater, $\frac{1}{6}$ or $\frac{1}{7}$? ☐

5. Which is greater, $\frac{3}{5}$ or $\frac{5}{9}$? ☐

15 - 8 Fractions

Addition of Fractions (1)

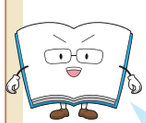
Example My mother made milk tea by mixing $\frac{4}{5}$ L of milk and $\frac{3}{5}$ L of black tea. How many litres of milk tea did she make?



Milk: Four $\frac{1}{5}$'s

Black Tea: Three $\frac{1}{5}$'s

Altogether: Seven $\frac{1}{5}$'s



When proper fractions have the same denominator, add the numerators only.

Math sentence $\frac{4}{5} + \frac{3}{5} = \frac{7}{5}$

Answer $\frac{7}{5}$ L or 1 $\frac{2}{5}$ L

It is fine that the answer will be either improper fraction or mixed number.

1 I have two pieces of tape. One is $\frac{4}{7}$ m long and the other is $\frac{6}{7}$ m long. How long is the total length of both pieces together?

Math sentence

Answer

2 Calculate the following addition problems.

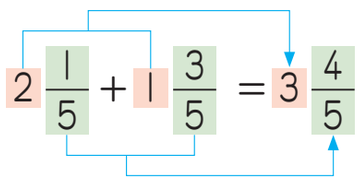
1. $\frac{2}{3} + \frac{2}{3} =$ ☐
2. $\frac{5}{7} + \frac{6}{7} =$ ☐
3. $\frac{11}{5} + \frac{3}{5} =$ ☐
4. $\frac{7}{4} + \frac{5}{4} =$ ☐
5. $\frac{14}{9} + \frac{3}{9} =$ ☐
6. $\frac{15}{6} + \frac{5}{6} =$ ☐

15 - 9 Fractions

Addition of Fractions (2)

Example 1 Calculate $2\frac{1}{5}+1\frac{3}{5}$

Calculate the whole number parts and the fraction parts separately.



It is also fine to change the mixed number to an improper fraction and to calculate.

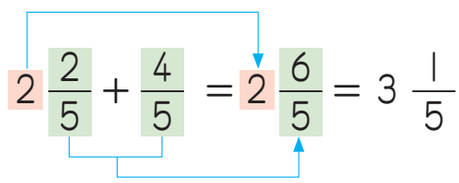
$$2\frac{1}{5}+1\frac{3}{5}=\frac{11}{5}+\frac{8}{5}=\frac{19}{5}=3\frac{4}{5}$$

1 Calculate the following addition problems.

① $1\frac{2}{5}+4\frac{1}{5}=$ ☐ ② $1\frac{1}{7}+2\frac{4}{7}=$ ☐ ③ $4\frac{1}{9}+2\frac{7}{9}=$ ☐

Example 2 Calculate $2\frac{2}{5}+\frac{4}{5}$

Calculate the whole number parts and then write the fraction part.



As same as Example 1, it is also fine to change the mixed number to an improper fraction and to calculate.

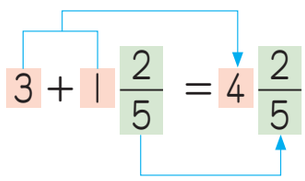
$$2\frac{2}{5}+\frac{4}{5}=\frac{12}{5}+\frac{4}{5}=\frac{16}{5}=3\frac{1}{5}$$

2 Calculate the following addition problems.

① $3\frac{4}{5}+\frac{3}{5}=$ ☐ ② $1\frac{3}{7}+\frac{5}{7}=$ ☐ ③ $\frac{7}{9}+2\frac{4}{9}=$ ☐

Example 3 Calculate $3+1\frac{2}{5}$

Calculate the whole number parts and the fraction parts separately.



It is also fine to change both whole number and mixed number to improper fractions and to calculate.

$$3+1\frac{2}{5}=\frac{15}{5}+\frac{7}{5}=\frac{22}{5}=4\frac{2}{5}$$

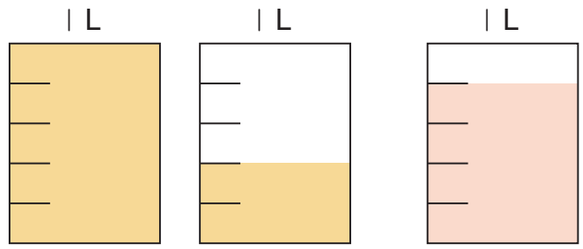
3 Calculate the following addition problems.

① $2+3\frac{1}{2}=$ ☐ ② $1+2\frac{3}{5}=$ ☐ ③ $1\frac{5}{8}+3=$ ☐

15 - 10 Fractions

Subtraction of Fractions (1)

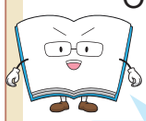
Example My mother made $\frac{7}{5}$ L of orange juice and $\frac{4}{5}$ L of apple juice. How many more L of orange juice did she make?



Orange Juice: Seven $\frac{1}{5}$'s

Apple Juice: Four $\frac{1}{5}$'s

The difference: Three $\frac{1}{5}$'s



When the denominators of both fractions have the same number, subtract the numerators.

Math sentence $\frac{7}{5} - \frac{3}{5} = \frac{4}{5}$

Answer $\frac{4}{5}$ L

1 I have a $\frac{11}{7}$ m red tape and $\frac{6}{7}$ m white tape. How many more metres of red tape do I have?

Math sentence

Answer

2 Calculate the following subtraction problems.

1. $\frac{10}{7} - \frac{4}{7} =$ ☐
2. $\frac{11}{3} - \frac{4}{3} =$ ☐
3. $\frac{13}{9} - \frac{8}{9} =$ ☐
4. $\frac{9}{7} - \frac{5}{7} =$ ☐
5. $\frac{14}{5} - \frac{7}{5} =$ ☐
6. $\frac{10}{9} - \frac{5}{9} =$ ☐
7. $\frac{8}{3} - \frac{4}{3} =$ ☐
8. $\frac{7}{5} - \frac{4}{5} =$ ☐

15 - 11 Fractions

Subtraction of Fractions (2)

Example 1 Calculate $3\frac{4}{5} - 1\frac{3}{5}$

Calculate the whole number parts and the fraction parts separately.

$3\frac{4}{5} - 1\frac{3}{5} = 2\frac{1}{5}$

It is also correct to change the mixed number to an improper fraction and to calculate.

$3\frac{4}{5} - 1\frac{3}{5} = \frac{19}{5} - \frac{8}{5} = \frac{11}{5} = 1\frac{1}{5}$

1 Calculate the following subtraction problems.

① $4\frac{4}{7} - 1\frac{1}{7} =$ ☐ ② $6\frac{3}{5} - 4\frac{2}{5} =$ ☐ ③ $5\frac{2}{3} - 4\frac{1}{3} =$ ☐

Example 2 Calculate $2\frac{1}{5} - \frac{4}{5}$

Make the fractional part of the mixed number an improper fraction by re-grouping and then calculate.

$2\frac{1}{5} - \frac{4}{5} = 1\frac{6}{5} - \frac{4}{5} = 1\frac{2}{5}$

It is also fine to change the mixed number to an improper fraction and to calculate. $2\frac{1}{5} - \frac{4}{5} = \frac{11}{5} - \frac{4}{5} = \frac{7}{5}$

2 Calculate the following subtraction problems.

① $2\frac{2}{5} - \frac{4}{5} =$ ☐ ② $2\frac{1}{3} - \frac{2}{3} =$ ☐ ③ $3\frac{2}{7} - 1\frac{6}{7} =$ ☐

Example 3 Calculate $2 - \frac{1}{3}$

Change the whole number to the fraction and then calculate it.

$2 - \frac{1}{3} = 1\frac{3}{3} - \frac{1}{3} = 1\frac{2}{3}$

It is also fine to change the mixed number to an improper fraction completely and to calculate.

$2 - \frac{1}{3} = \frac{6}{3} - \frac{1}{3} = \frac{5}{3} = 1\frac{2}{3}$

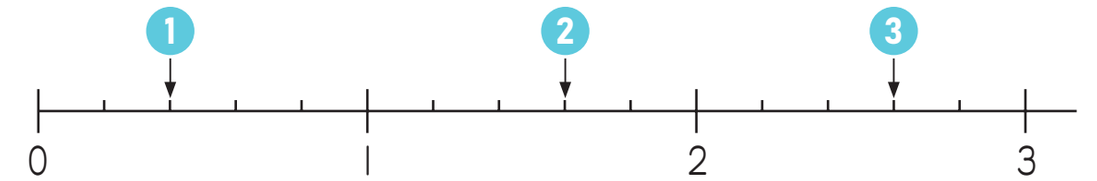
3 Calculate the following subtraction problems.

① $2 - \frac{4}{5} =$ ☐ ② $3 - \frac{2}{7} =$ ☐ ③ $5 - 2\frac{3}{10} =$ ☐

15 - 12 Fractions

Review

1 What fractions are represented by the tick marks labeled ①, ② and ③ ? If the fraction is greater than 1, express it as an improper fraction and as a mixed number.



① ＿＿＿＿　② ＿＿＿＿　③ ＿＿＿＿

2 Write the fractions in the ( ) in order from the largest to smallest.

① $\left(\frac{13}{5},\ 3\frac{2}{5},\ \frac{11}{5}\right)$ Answer ＿＿＿＿ → ＿＿＿＿ → ＿＿＿＿

② $\left(2\frac{5}{9},\ \frac{28}{9},\ 3\frac{3}{9}\right)$ Answer ＿＿＿＿ → ＿＿＿＿ → ＿＿＿＿

③ $\left(\frac{22}{7},\ 3\frac{3}{7},\ 3\right)$ Answer ＿＿＿＿ → ＿＿＿＿ → ＿＿＿＿

It is fine that the answer will be either improper fraction or mixed number.

3 Calculate the following problems.

① $\frac{2}{5}+\frac{4}{5}$　② $1\frac{2}{9}+3\frac{5}{9}$　③ $1\frac{7}{9}+3\frac{4}{9}$　④ $1\frac{2}{3}+\frac{2}{3}$

⑤ $2+\frac{5}{6}$　⑥ $\frac{6}{7}-\frac{5}{7}$　⑦ $5\frac{4}{5}-3\frac{3}{5}$　⑧ $7\frac{1}{3}-5\frac{2}{3}$

⑨ $4\frac{3}{5}-\frac{4}{5}$　⑩ $3\frac{5}{9}-\frac{7}{9}$　⑪ $2-\frac{7}{8}$　⑫ $3-1\frac{4}{9}$

①		②		③		④	
⑤		⑥		⑦		⑧	
⑨		⑩		⑪		⑫	