4 - 1 Multiplication of Decimal Numbers
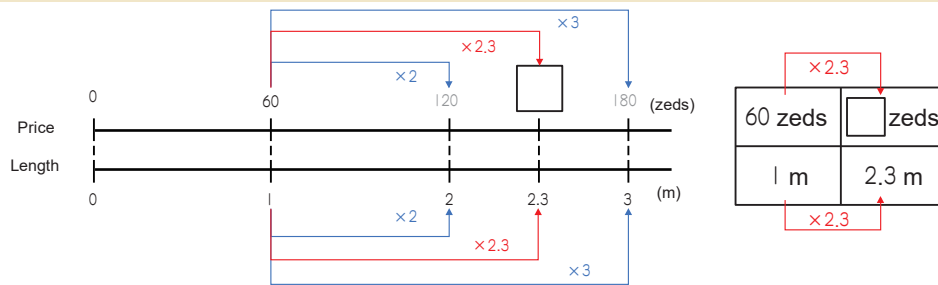# Multiplying with Decimal Numbers (1)

Example A 1 m long ribbon costs 60 zeds*. How much does a 2.3 m ribbon cost? ("zed(s)" is the fictional currency unit.)

×2.3	
60 zeds	☐ zeds
1 m	2.3 m

The math sentence is "60 × 2.3," but how can we calculate it?

According the above diagram, we can make a math sentence of 60 × 2.3.

This multiplication problem can be solved in the following way.

$$60 \times 2.3 = 138$$
$$\downarrow 10 \text{ times}$$
$$60 \times 23 = 1380 \qquad \frac{1}{10}$$

Reference: The multiplication algorithm can be shown as follows;

```
    6 0                      6 0
  ×  2.3    10 times   ×     2 3
  -------   ------->   -----------
                           1 8 0
                           1 2 0
  1 3 8    10 times   -----------
          <-------       1 3 8 0
            1/10
```

The multiplication algorithm with decimal numbers will be explained in detail from the section 4-4.
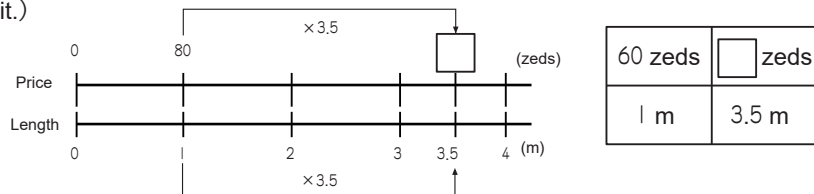
Math sentence

$$60 \times 2.3 = 60 \times 23 \div 10 = 1380 \div 10 = 138$$

Answer **138 zeds**

---

1 A 1 m long hose costs 80 zeds. How much does a 3.5 m hose cost?
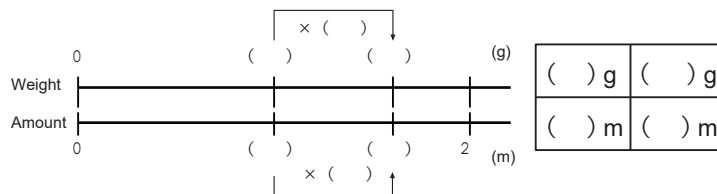("zed(s)" is the fictional currency unit.)

Math sentence

×3.5	
60 zeds	☐ zeds
1 m	3.5 m

Answer _____

---

2 A 1 m long stick weighs 180 g. How much does a 1.6 m stick weigh?

Math sentence

( ) g	( ) g
( ) m	( ) m

Complete the number line diagram and table.

Answer _____

4 - 2 Multiplying with Decimal Numbers (2)

Example A 1 m long pipe weighs 1.8 kg. How much does a 4.2 m pipe weigh?

Weight / Length number line diagram with markings: 0, 1.8, 3.6, 5.4, 7.2, 9 (kg); ×2, ×3, ×4, ×4.2, ×5

	1.8 kg	☐ kg
	1 m	4.2 m

×4.2
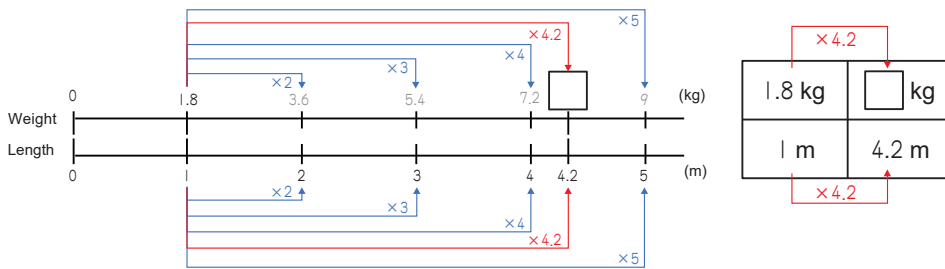
The math sentence is "1.8 × 4.2," but how can we calculate it?

According the above diagram, we can make a math sentence of 1.8 × 4.2.

This multiplication problem can be solved in the following way.

$$1.8 \quad \times \quad 4.2 \quad = \quad \boxed{7.56}$$

↓10 times ↓10 times ⟵ $\frac{1}{100}$

$$18 \quad \times \quad 42 \quad = \quad 756$$

The multiplication algorithm with decimal numbers will be explained in detail from the section 4-4.

Reference: The multiplication algorithm can be shown as follows;

```
    1 . 8      10 times →        1 8
  × 4 . 2      10 times →      × 4 2
                                 3 6
                                 7 2
    7 . 5 6    100 times ←      7 5 6
               $\frac{1}{100}$
```

Math sentence

$$1.8 × 4.2 = 18 × 42 ÷ 100 = 756 ÷ 100 = 7.56$$

Answer **7.56 kg**

---

1 A 1 m long iron pipe weighs 2.1 kg. How much does a 3.5 m pipe weigh?

Math sentence

Weight / Length number line: 0, 2.1 (kg); 0, 1, 3.5 (m); ×3.5

2.1 kg	☐ kg
1 m	3.5 m

Answer _____

2 1 dL of paint was used to paint 1.5 m² of wall. How many m² can be painted with 2.7 dL of paint?

Math sentence

Area / Amount number line: 0, ( ), ( ) (m²); 0, ( ), ( ) (dL); × ( )

( ) m²	( ) m²
( ) dL	( ) dL

Complete the number line diagram and table.

Answer _____

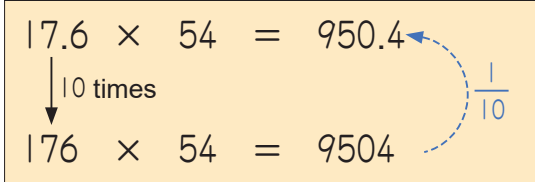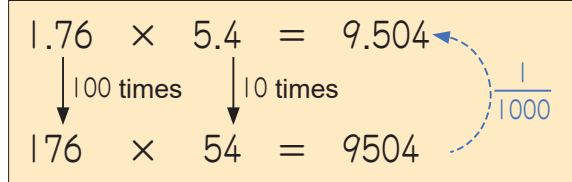## 4 - 3 Multiplication of Decimal Numbers

Multiplication of Decimal Numbers (1)

Example Find the product of each of the following based on $176 \times 54 = 9504$.

① $17.6 \times 54 = \boxed{950.4}$

$17.6 \times 54 = 950.4$
$\downarrow$ 10 times $\qquad \frac{1}{10}$
$176 \times 54 = 9504$

② $1.76 \times 5.4 = \boxed{9.504}$

$1.76 \times 5.4 = 9.504$
$\downarrow$ 100 times $\downarrow$ 10 times $\qquad \frac{1}{1000}$
$176 \times 54 = 9504$

---

1 Find the product of each of the following based on $254 \times 39 = 9906$.

① $25.4 \times 39 = \boxed{}$

② $254 \times 3.9 = \boxed{}$

③ $25.4 \times 3.9 = \boxed{}$

④ $2.54 \times 3.9 = \boxed{}$

⑤ $2.54 \times 39 = \boxed{}$

Pay attention to the location of the decimal point.

2 Find the product of each of the following based on $312 \times 56 = 17472$.

① $31.2 \times 56 = \boxed{}$

② $312 \times 5.6 = \boxed{}$

③ $3.12 \times 5.6 = \boxed{}$

④ $31.2 \times 5.6 = \boxed{}$

⑤ $3.12 \times 56 = \boxed{}$

⑥ $0.312 \times 56 = \boxed{}$

3 Find the product of each of the following based on $47 \times 851 = 39997$.

① $4.7 \times 851 = \boxed{}$

② $47 \times 85.1 = \boxed{}$

③ $4.7 \times 85.1 = \boxed{}$

④ $47 \times 8.51 = \boxed{}$

⑤ $4.7 \times 8.51 = \boxed{}$

⑥ $0.47 \times 851 = \boxed{}$

4 - 4 Multiplication of Decimal Numbers (1)

Example Calculate 1.8×4.2 by using the multiplication algorithm.

```
    1.8      One decimal place   10 times            1 8
  × 4.2      One decimal place   10 times          × 4 2
  ─────                                            ─────
    3 6                                              3 6
  7 2                                              7 2
  ─────                                            ─────
  7.5 6      Two decimal places      1/100         7 5 6
```

```
    1.8  →  1   place to the right
  × 4.2  →  1   place to the right
  ─────
    3 6
  × 7 2 4        1 + 1
  ───────
  7.5 6    ←  2   places to the left
```

Disregard the decimal points for now. Multiply as if they were whole numbers.

To determine the location of the decimal point of the product, add the number of places that are to the right of the decimal points of the multiplicand (**1 place**) and the multiplier (**1 place**). Then move the decimal point of the product from right to left the same number of places (**2 places**).

Even when the multiplier is a decimal number, we can calculate just like we did with whole numbers!

Calculate the following multiplication problems by using the multiplication algorithm.

1 2.1×3.8

```
    2.1
  × 3.8
  ─────
```

2 5.4×1.9

```
    5.4
  × 1.9
  ─────
```

3 1.3×2.7

```
    1.3
  × 2.7
  ─────
```

4 0.8×4.4

```
    0.8
  × 4.4
  ─────
```

5 4.1×3.7

6 2.7×3.4

7 0.7×6.8

8 3.5×2.3

9 2.5×3.3

10 4.4×5.2

11 0.5×7.3

12 7.6×1.2

5		6		7		8	
9		10		11		12	

4 - 5

Multiplication of Decimal Numbers (2)

Example Calculate 2.35×1.7 by using the multiplication algorithm.

$$
\begin{array}{r}
2.3\,5 \\
\times \quad 1.7 \\
\hline
1\ 6\ 4\ 5 \\
2\ 3\ 5 \quad\ \\
\hline
3.9\,9\,5
\end{array}
$$

Two decimal place — 100 times
One decimal place — 10 times
Three decimal places — $\dfrac{1}{1000}$

$$
\begin{array}{r}
2\ 3\ 5 \\
\times \quad 1\ 1 \\
\hline
1\ 6\ 4\ 5 \\
2\ 3\ 5\ \ \\
\hline
3\ 9\ 9\ 5
\end{array}
$$

Multiply as if there were no decimal points.

$$
\begin{array}{r}
2.3\ 5 \\
\times \quad 1.7 \\
\hline
1\ 6\ 4\ 5 \\
\times 2\ 3\ 5 \\
\hline
3.9\ 9\ 5
\end{array}
$$

2 places to the right
1 place to the right
$2 + 1$
3 places to the left

To determine the location of the decimal point of the product, add the number of places that are to the right of the decimal points of the multiplicand (**2 places**) and the multiplier (**1 place**). Then move the decimal point of the product from right to left the same number of places (**3 places**).

Calculate the following multiplication problems by using the multiplication algorithm.

1 1.69×2.5

$$
\begin{array}{r}
1.6\ 9 \\
\times \quad 2.5 \\
\hline
\end{array}
$$

2 2.04×9.2

$$
\begin{array}{r}
2.0\ 4 \\
\times \quad 9.2 \\
\hline
\end{array}
$$

3 0.53×4.3

$$
\begin{array}{r}
0.5\ 3 \\
\times \quad 4.3 \\
\hline
\end{array}
$$

4 0.17×6.2

$$
\begin{array}{r}
0.1\ 7 \\
\times \quad 6.2 \\
\hline
\end{array}
$$

We can cross out "0" at the end of the decimal numbers.

5 2.36×4.6 **6** 3.61×2.5 **7** 4.75×1.5 **8** 9.99×9.9

9 5.03×2.8 **10** 4.18×7.5 **11** 6.54×3.13 **12** 8.25×4.44

Regarding Problems **11** and **12**, the decimal point moves 4 places to the left!

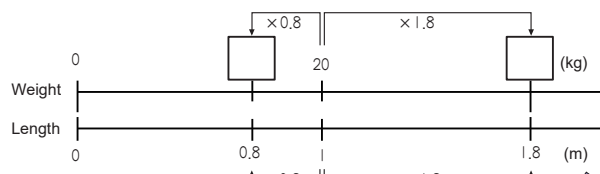| 5		6		7		8	
9		10		11		12	

4 – 6 Multiplication of Decimal Numbers
Multiplier and the Size of the Product

Example A 1 m long iron bar weighs 20 kg. How much does a 1.8 m long iron bar weigh? How much does a 0.8 m long iron bar weigh?

Weight of the 1.8 m iron bar

Math sentence

$$20 \times 1.8 = 36$$

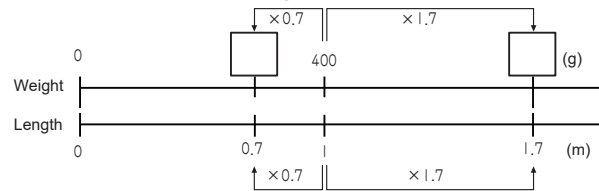Answer ___36 kg___

Weight of the 0.8 m iron bar

Math sentence

$$20 \times 0.8 = 16$$

Answer ___16 kg___

When we multiply by a number less than 1, the product will be less than the multiplicand.

1 There is a 1 m wire that weighs 400 g. How many g does 0.7 m of wire weigh? In addition, how many does 1.7 m of wire weigh?

Weight of 1.7 m wire

Math sentence

Answer _____

Weight of 0.7 m wire

Math sentence

Answer _____

2 Which of the following will have a product that is less than 15?

(a) 15 × 0.9　(b) 15 × 1.4　(c) 15 × 2.08　(d) 15 × 0.76　Answer _____

3 Calculate the following multiplication problems by using the algorithm.

① 4.18 × 0.3
```
    4 . 1 8
×     0 . 3
```

② 1.94 × 0.6
```
    1 . 9 4
×     0 . 6
```

③ 0.56 × 0.49
```
    0 . 5 6
×   0 . 4 9
```

④ 0.37 × 0.85
```
    0 . 3 7
×   0 . 8 5
```

Are all the products smaller than the multiplicand?

33

Calculating Area

Example　To find the area of the following shape, multiply the length times the width.

4.3 cm

2.8 cm

$$
\begin{array}{r}
4.3 \\
\times \quad 2.8 \\
\hline
3\ 4\ 4 \\
8\ 6 \quad\ \\
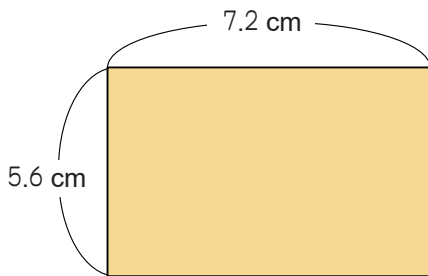\hline
1\ 2.0\ 4
\end{array}
$$
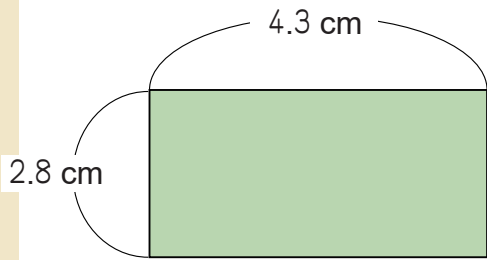
When we try to find the area, we may use decimal numbers.

Area of rectangle = Length × Width

Don't forget to put the decimal point in the correct place in your answer!

Math sentence

$4.3 \times 2.8 = 12.04$

Answer　$12.04\ \text{cm}^2$

To find the area of the following shapes, multiply the length times the width.

1　7.2 cm

5.6 cm

Math sentence

Answer _____

2　4.8 cm

4.8 cm

Math sentence
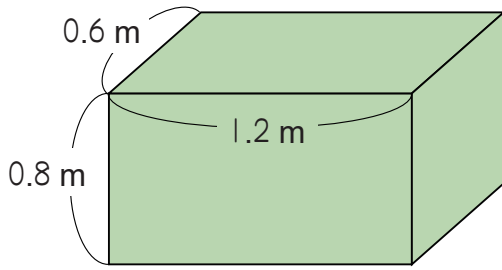
Answer _____

3　12.4 cm

3.5 cm
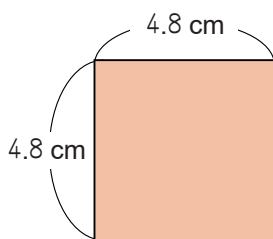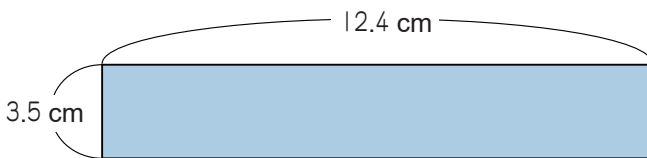
Math sentence

Answer _____

Multiplication of Decimal Numbers

Calculating Volume

Example To find the volume of the following shape, multiply its length, width and height measurements.

0.6 m
1.2 m
0.8 m

$$\begin{array}{r} 1.2 \\ \times\ \ 0.6 \\ \hline 0.72 \end{array}$$

$$\begin{array}{r} 0.72 \\ \times\ \ \ 0.8 \\ \hline 0.576 \end{array}$$

Volume of cuboid = Length × Width × Height.

Don't forget to put the decimal point in the correct place in your answer!
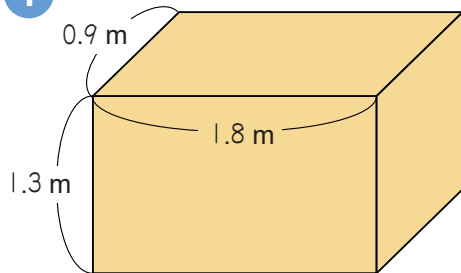
Math sentence

$1.2 × 0.6 × 0.8 = 0.72 × 0.8 = 0.576$

Answer 0.576 m^3

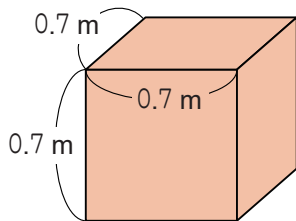To find the volume of the following shapes, multiply its length, width and height measurements.

1

0.9 m
1.8 m
1.3 m

Math sentence

Answer _____

2

0.7 m
0.7 m
0.7 m

Math sentence

Answer _____

3

0.8 m
0.4 m
3.6 m

Math sentence

Answer _____

35
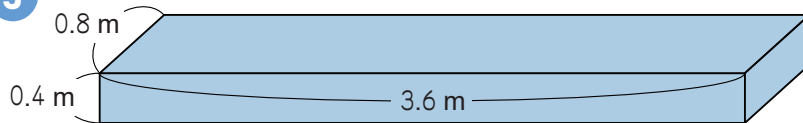
Usage of Properties of Operations

Example The properties of operations for whole numbers also applies to decimal numbers.

The properties of operations are

① ■ × ● = ● × ■
② (■ × ●) × ▲ = ■ × (● × ▲)
③ (■ + ●) × ▲ = ■ × ▲ + ● × ▲
④ (■ − ●) × ▲ = ■ × ▲ − ● × ▲

Example Rewrite the following math sentences using the properties of operations. Then solve the problem.

1 $4.8 \times 4 \times 2.5$ $= 4.8 \times (4 \times 2.5) = 4.8 \times 10 = 48$

2 $2.4 \times 1.8 + 2.6 \times 1.8$ $= (2.4 + 2.6) \times 1.8 = 5 \times 1.8 = 9$

3 $5.7 \times 1.8 - 3.7 \times 1.8$ $= (5.7 - 3.7) \times 1.8 = 2.0 \times 1.8 = 3.6$

Rewrite the following math sentences using the properties of operations. Then solve the problem.

Think about to which example problem the following problems are similar.
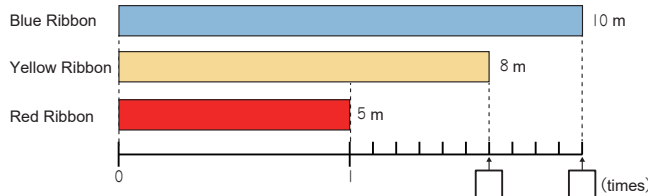
1 $9.2 \times 4 \times 2.5$ $= 9.2 \times (4 \times \boxed{}) =$

2 $2.5 \times 4 \times 8.4$ $= (\boxed{} \times 4) \times 8.4 =$

3 $4 \times 6.8 \times 2.5$ $= (\boxed{} \times \boxed{}) \times \boxed{} =$

4 $0.7 \times 9.8 + 0.3 \times 9.8$ $= (\boxed{} + \boxed{}) \times 9.8 =$

5 $2.7 \times 0.21 + 0.3 \times 0.21$ $= (2.7 + \boxed{}) \times \boxed{} =$

6 $1.3 \times 4.1 + 1.3 \times 5.9$ $= \boxed{} \times (\boxed{} + \boxed{}) =$

7 $3.5 \times 4.3 - 2.5 \times 4.3$ $= (\boxed{} - \boxed{}) \times 4.3 =$

8 $0.3 \times 5.9 - 0.3 \times 2.9$ $= \boxed{} \times (\boxed{} - \boxed{}) =$

9 $25.5 \times 8 = (25 + 0.5) \times 8 = (\boxed{} \times 8) + (\boxed{} \times 8) =$

36

4 – 10 Times as Much with Decimal Numbers (1)

Example I have three ribbons. The red ribbon is 5 m long, the yellow ribbon is 8 m long and the blue ribbon is 10 m long.

Blue Ribbon	10 m
Yellow Ribbon	8 m
Red Ribbon	5 m

0   1   ☐   ☐ (times)

Red Ribbon	Blue Ribbon
5 m	10 m
1	☐ times

Red Ribbon	Yellow Ribbon
5 m	8 m
1	☐ times

1 How many times longer is the white ribbon than the red ribbon?

Math sentence   10 ÷ 5 = 2

Answer   2 times

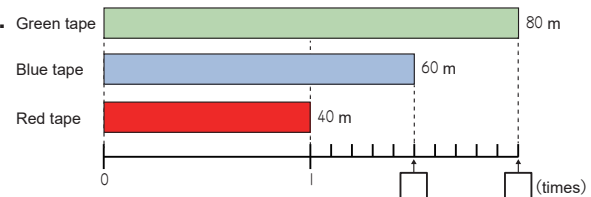**2** How many times longer is the yellow ribbon than red ribbon?

Math sentence   8 ÷ 5 = 1.6

Answer   1.6 times

1 I have three pieces of tape. The red tape is 40 m long, the blue tape is 60 m long, the green tape is 80 m long.

Green tape	80 m
Blue tape	60 m
Red tape	40 m

0   1   ☐   ☐ (times)

Red Tape	Green Tape
40 m	80 m
1	☐ times

Red Tape	Blue Tape
40 m	60 m
1	☐ times

1 How many times longer is the green tape than the red tape?

Math sentence

Answer _____

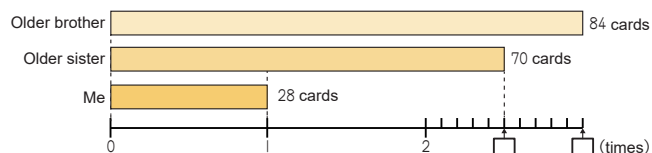**2** How many times longer is the blue tape than the red tape?

Math sentence

Answer _____

2 I have 28 cards, my older brother has 84 cards, and my older sister has 70 cards.

Older brother	84 cards
Older sister	70 cards
Me	28 cards

0   1   2   ☐   ☐ (times)

Me	Older Brother
28 cards	84 cards
1	☐ times

Me	Older sister
28 cards	70 cards
1	☐ times

1 How many times more cards does my older brother have than me?

Math sentence

Answer _____

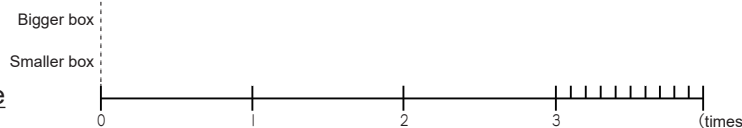**2** How many times more cards does my older sister have than me?

Math sentence

Answer _____

3 I have two boxes. The weight of bigger box is 54 kg and the weight of smaller box is 15 kg. How many times heavier is the bigger box than the smaller box?

Bigger box

Smaller box

Math sentence

0   1   2   3   (times)

( ) kg	( ) kg
( )	( ) times

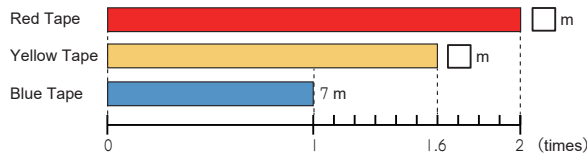Answer _____

Complete the number line diagram and table.

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4 - 11 Multiplication of Decimal Numbers
Times as Much with Decimal Numbers (2)

Example I have three pieces of tape. The blue tape is 7 m long. The yellow tape is 1.6 times longer than the blue tape. The red tape is 2 times longer than the blue tape.

Red Tape ☐ m
Yellow Tape ☐ m
Blue Tape 7 m

0　　　1　　1.6　2 (times)

Blue Tape	Yellow Tape
7 m	☐ m
1 times	2 times

Blue Tape	Red Tape
7 m	☐ m
1 times	1.6 times

1 How long is the red tape?

Math sentence　　7 × 2 = 14

Answer　14 m

2 How long is the yellow tape?

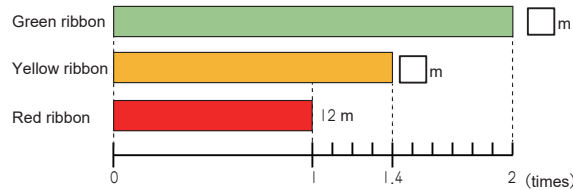Math sentence　　7 × 1.6 = 11.2

Answer　11.2 m

1 I have three ribbons. The red ribbon is 12 m long. The yellow ribbon is 1.4 times longer than the red ribbon. The green ribbon is 2 times longer than the red ribbon.

Red Ribbon	Green Ribbon
12 m	☐ m
1 times	2 times

Red Ribbon	Yellow Ribbon
12 m	☐ m
1 times	1.4 times

Green ribbon ☐ m
Yellow ribbon ☐ m
Red ribbon 12 m

0　　　1　　1.4　2 (times)

1 How long is the green ribbon?

Math sentence

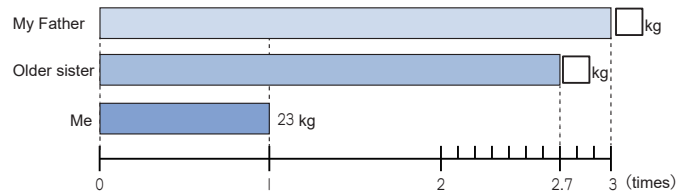Answer _____

2 How long is the yellow ribbon?

Math sentence

Answer _____

2 I weigh 23 kg. My father weighs 3 times more than me. My older brother weighs 2.7 times more than me.

Me	My Father
23 kg	☐ kg
1 times	2 times

Me	Older sister
23 kg	☐ kg
1 times	2.7 times

My Father ☐ kg
Older sister ☐ kg
Me 23 kg

0　　　1　　　　2　2.7　3 (times)

1 What is my father's weight?

Math sentence

Answer _____

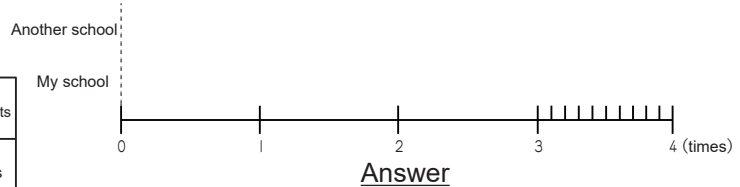**2** What is my older brother's weight?

Math sentence

Answer _____

3 There are 32 grade-5 students at my school. At another school, there are 3.5 times more as grade-5 students than at my school. How many grade-5 students are at the other school?

Math sentence

Complete the number line diagram and table.

( ) students	( ) students
( )	( ) times

Another school
My school

0　　1　　2　　3　4 (times)

Answer _____

38

Review

1 Calculate the following multiplication problems by using the multiplication algorithm.

1 3.9×2.3

```
    3.9
×   2.3
```

2 6.2×4.7

```
    6.2
×   4.7
```

3 3.67×2.4

```
    3.6 7
×     2.4
```

4 7.03×1.9

```
    7.0 3
×     1.9
```

5 5.55×6.4 **6** 9.86×6.5 **7** 8.54×2.5 **8** 0.25×3.2

9 9.16×1.8 **10** 0.75×0.8 **11** 8.37×0.36 **12** 0.35×0.56

5		6		7		8	
9		10		11		12	

2 Which of the following will have a product that is less than 26?

(a) 26×0.95 (b) 26×1.03 (c) 26×2.5 (d) 26×0.9

Answer _____

3 Rewrite the following math sentences using the properties of operations.
Then solve the problem

1 $16.35 \times 2.5 \times 4$ _____

2 $2.4 \times 0.8 + 1.6 \times 0.8$ _____

3 $7.6 \times 2.5 - 6.6 \times 2.5$ _____

4 When a weight was hung on a 7.5 cm long spring, the spring stretched out 2.8 times its original length. How long is the spring with the weight attached?

Math sentence

Spring with a weight

Spring without any weights

```
0            1            2            3 (times)
```

( ) cm	( ) cm
( )	( ) times

Complete the number line diagram and table.

Answer _____