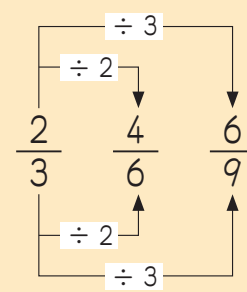
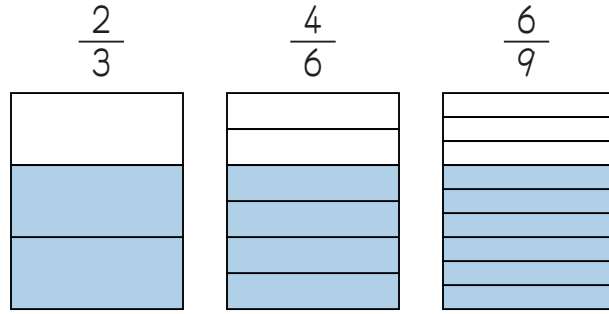
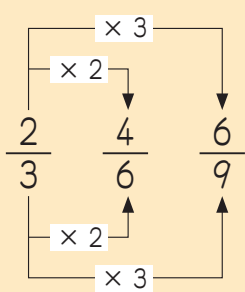


8 - 1

Addition and Subtraction of Fractions

Fractions of the Same Size

Instruction $\frac{2}{3}$, $\frac{4}{6}$ and $\frac{6}{9}$ are the same size.



The size of a fraction does not change if we multiply or divide both the denominator and the numerator by the same number.

$$\frac{\bullet}{\blacksquare} = \frac{\bullet \times \blacktriangle}{\blacksquare \times \blacktriangle}$$

$$\frac{\bullet}{\blacksquare} = \frac{\bullet \div \blacktriangle}{\blacksquare \div \blacktriangle}$$

Example Write the appropriate numbers in the .



1 $\frac{3}{4} = \frac{6}{8}$

2 $\frac{1}{5} = \frac{3}{15}$

Regarding (1), we can multiply both the denominator and the numerator by 2.

Write the appropriate numbers in the .

1 $\frac{1}{3} = \frac{\square}{6}$

2 $\frac{5}{6} = \frac{\square}{12}$

3 $\frac{6}{7} = \frac{\square}{21}$

4 $\frac{3}{8} = \frac{\square}{24}$

5 $\frac{2}{9} = \frac{\square}{27}$

6 $\frac{3}{10} = \frac{\square}{40}$

7 $\frac{1}{4} = \frac{\square}{12}$

8 $\frac{4}{5} = \frac{\square}{30}$

9 $\frac{1}{2} = \frac{\square}{18}$

10 $\frac{10}{16} = \frac{5}{\square}$

11 $\frac{2}{24} = \frac{1}{\square}$

12 $\frac{27}{30} = \frac{9}{\square}$

13 $\frac{32}{18} = \frac{16}{\square}$

14 $\frac{15}{21} = \frac{5}{\square}$

15 $\frac{20}{24} = \frac{5}{\square}$

8 - 2

Addition and Subtraction of Fractions

Simplifying the Fraction

Instruction Multiples of the numerator and denominator by the same number are all equal to each other.

For example $\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}, \frac{7}{28}, \frac{8}{32}, \dots$



To **simplify the fraction**, divide the numerator and denominator by the greatest common factor.

$$\frac{\cancel{9}}{\cancel{36}} = \frac{1}{4}$$

When we divide the numerator and denominator by common factor repeatedly such as “ $\div 2$ ”, and “ $\div 3$ ”, we can also simplify the fraction.

$$\frac{\overset{3}{\cancel{9}}}{\cancel{36}} = \frac{\overset{1}{\cancel{3}}}{\cancel{12}} = \frac{1}{4}$$

Example Simplify the following fractions.

1 $\frac{7}{21} = \frac{1}{3}$

2 $\frac{4}{10} = \frac{2}{5}$

We can divide both the denominator and the numerator by the greatest common factor, cannot we?



Simplify the following fractions.

1 $\frac{3}{9} = \square$

2 $\frac{12}{15} = \square$

3 $\frac{24}{16} = \square$

4 $\frac{15}{25} = \square$

5 $\frac{8}{10} = \square$

6 $\frac{18}{24} = \square$

7 $\frac{14}{35} = \square$

8 $\frac{16}{36} = \square$

9 $\frac{56}{49} = \square$

10 $2 \frac{14}{18} = \square$

11 $3 \frac{9}{24} = \square$

12 $1 \frac{3}{18} = \square$

13 $4 \frac{9}{30} = \square$

14 $1 \frac{21}{36} = \square$

15 $2 \frac{30}{48} = \square$

You can also simplify the fraction part of a mixed number.



8 - 3

Addition and Subtraction of Fractions

Finding a Common Denominator

Instruction When comparing two fractions, they must have the same denominator. If they do not, a common denominator must be found. The numerator must be recalculated so that it is the equivalent of the original fraction.

When comparing the size of fractions between $\frac{3}{4}$ and $\frac{4}{5}$, the least common denominator is 20.

$\frac{3}{4}$	$\frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}, \frac{24}{32}, \frac{27}{36}, \frac{30}{40}, \dots$
$\frac{4}{5}$	$\frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \frac{20}{25}, \frac{24}{30}, \frac{28}{35}, \frac{32}{40}, \frac{36}{45}, \frac{40}{50}, \dots$

Example 1 Compare the following fractions by finding a common denominator.

- Find the **least common denominator** by thinking about multiples of 3 and 5.
- Convert to fractions** with the least common denominator.
- Compare two fractions with the same denominator.

$\frac{2}{3}, \frac{3}{5}$

Multiples of 3: 3, 6, 9, 12, **15**, 18, 21, ...

Multiples of 5: 5, 10, **15**, 20, 25, 30, ...

$\frac{2}{3}$	$\frac{10}{15}$	$\frac{3}{5}$	$\frac{9}{15}$
$\times 5$		$\times 3$	
	$\times 5$		$\times 3$

$\frac{10}{15} > \frac{9}{15}$

The least common denominator can be found by multiplying the numbers of both denominators (3×5).

1 Compare the following fractions by finding a common denominator.

1 $\frac{3}{4}, \frac{5}{7} \rightarrow$

2 $\frac{9}{5}, \frac{11}{6} \rightarrow$

3 $\frac{2}{3}, \frac{3}{4} \rightarrow$

4 $\frac{11}{7}, \frac{11}{8} \rightarrow$

5 $\frac{3}{5}, \frac{4}{7} \rightarrow$

6 $\frac{3}{8}, \frac{4}{9} \rightarrow$

Example 2 Compare the following fractions by finding a common denominator.

1. Find the **least common denominator** by thinking about multiples of 3 and 9.

2. **Convert to one fraction** with the least common denominator.

3. Compare two fractions with the same denominator.

$$\frac{2}{3}, \frac{7}{9}$$

Multiples of 3: 3, 6, **9**, 12, 15, 18, 21, ...

Multiples of 9: **9**, 18, 27, 36, 45, 54, ...

$$\begin{array}{c} \times 3 \downarrow \\ \frac{2}{3} \quad \frac{6}{9} \\ \times 3 \uparrow \end{array}$$

$$\frac{7}{9}$$

$$\frac{6}{9} < \frac{7}{9}$$

The least common denominator is the denominator of the larger fraction. In this case, it is 9.

2 Compare the following fractions by finding a common denominator.

1 $\frac{1}{2}, \frac{5}{6} \rightarrow$

2 $\frac{5}{6}, \frac{13}{18} \rightarrow$

3 $\frac{3}{4}, \frac{7}{12} \rightarrow$

4 $\frac{11}{15}, \frac{4}{5} \rightarrow$

Example 3 Compare the following fractions by finding a common denominator.

1. Find the **least common denominator** by thinking about multiples of 6 and 8.

2. **Convert to one fraction** with the least common denominator.

3. Compare two fractions with the same denominator.

$$\frac{5}{6}, \frac{7}{8}$$

Multiples of 6: 6, 12, 18, **24**, 30, 36, ...

Multiples of 8: 8, 16, **24**, 32, 40, 48, ...

$$\begin{array}{c} \times 4 \downarrow \\ \frac{5}{6} \quad \frac{20}{24} \\ \times 4 \uparrow \end{array}$$

$$\begin{array}{c} \times 3 \downarrow \\ \frac{7}{8} \quad \frac{21}{24} \\ \times 3 \uparrow \end{array}$$

$$\frac{20}{24} < \frac{21}{24}$$

The least common denominator is between the larger denominator of the fraction (8) and the number multiplying both denominators (6 × 8).

3 Compare the following fractions by finding a common denominator.

1 $\frac{3}{4}, \frac{5}{6} \rightarrow$

2 $\frac{5}{10}, \frac{13}{25} \rightarrow$

3 $\frac{11}{12}, \frac{5}{9} \rightarrow$

4 $\frac{7}{9}, \frac{13}{15} \rightarrow$

8 - 4

Addition and Subtraction of Fractions

Addition of Fractions (1)

Example My mother made $\frac{1}{2}$ L and $\frac{1}{3}$ L of orange juice. How much orange juice is there altogether?



The math sentence is $\frac{1}{2} + \frac{1}{3}$.

To calculate this, we must find a common denominator of 2 and 3. The common denominator is 6. Therefore, the math

sentence can be changed to $\frac{3}{6} + \frac{2}{6}$.

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

$$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

Math sentence $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

Answer $\frac{5}{6}$ L

1 A small carton of milk contains $\frac{1}{5}$ L of milk and a large carton of milk contains $\frac{1}{2}$ L of milk. How much milk is there altogether?

Math sentence

Answer _____

2 Calculate the following by finding a common denominator.

1 $\frac{1}{4} + \frac{1}{3} = \frac{\square}{12} + \frac{\square}{12} = \frac{\square}{\square}$

2 $\frac{2}{3} + \frac{3}{7} = \frac{\square}{21} + \frac{\square}{21} = \frac{\square}{\square}$

3 $\frac{2}{5} + \frac{1}{6} = \frac{\square}{30} + \frac{\square}{30} = \frac{\square}{\square}$

4 $\frac{1}{4} + \frac{2}{5} = \frac{\square}{20} + \frac{\square}{20} = \frac{\square}{\square}$

5 $\frac{4}{7} + \frac{1}{2}$

6 $\frac{4}{3} + \frac{6}{5}$

7 $\frac{3}{8} + \frac{2}{3}$

8 $\frac{2}{5} + \frac{1}{4}$

9 $\frac{1}{2} + \frac{2}{9}$

10 $\frac{2}{9} + \frac{3}{5}$

11 $\frac{1}{6} + \frac{1}{7}$

12 $\frac{3}{8} + \frac{3}{5}$

8 - 5

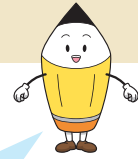
Addition and Subtraction of Fractions

Addition of Fractions (2)

Example Calculate $\frac{5}{6} + \frac{2}{3}$.

To calculate this, we must find a common denominator of 6 and 3. The common denominator is 6. Therefore, the math sentence can be changed to $\frac{5}{6} + \frac{4}{6}$.

$$\frac{5}{6} + \frac{2}{3} = \frac{5}{6} + \frac{4}{6} = \frac{\cancel{9}^3}{\cancel{6}_2} = \frac{3}{2}$$



$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

It is fine to show the answer as a mixed number.



If the answer can be simplified, we should do it. In the above case, we simplify $\frac{9}{6}$ as $\frac{3}{2}$.

Calculate the following by finding a common denominator.

1 $\frac{1}{6} + \frac{1}{12} = \frac{\square}{12} + \frac{\square}{12}$

2 $\frac{4}{15} + \frac{2}{5} = \frac{\square}{15} + \frac{\square}{15}$

3 $\frac{3}{4} + \frac{5}{12} = \frac{\square}{12} + \frac{\square}{12}$

4 $\frac{5}{6} + \frac{3}{2} = \frac{\square}{6} + \frac{\square}{6}$

5 $\frac{11}{36} + \frac{1}{9}$

6 $\frac{1}{13} + \frac{5}{26}$

7 $\frac{3}{8} + \frac{1}{24}$

8 $\frac{1}{9} + \frac{1}{18}$

9 $\frac{5}{12} + \frac{13}{30} = \frac{\square}{60} + \frac{\square}{60}$

10 $\frac{11}{30} + \frac{41}{45} = \frac{\square}{90} + \frac{\square}{90}$

11 $\frac{1}{15} + \frac{5}{6}$

12 $\frac{13}{10} + \frac{13}{15}$

8 - 6

Addition and Subtraction of Fractions

Addition of Fractions (3)

Example 1 Calculate $1\frac{2}{3} + \frac{5}{6}$.



It is better to simplify the answer and change it to an appropriate mixed number.

$$1\frac{2}{3} + \frac{5}{6} = 1\frac{4}{6} + \frac{5}{6} = 1\frac{\cancel{9}^3}{\cancel{6}_2} = 2\frac{1}{2}$$

A common denominator must be found in order to calculate this problem. We do not touch the whole number and move to $\frac{1}{6}$ to the answer. After adding two fractions, we must simplify the answer if possible.

In addition, when the part of the fraction of the answer is an improper fraction, we must change it to an appropriate mixed number.

1 Calculate the following by finding a common denominator.

1 $1\frac{1}{5} + \frac{1}{3}$

2 $1\frac{1}{6} + \frac{1}{12}$

3 $2\frac{1}{2} + \frac{1}{6}$

4 $2\frac{5}{6} + \frac{7}{15}$

Example 2 Calculate $1\frac{2}{3} + 2\frac{5}{6}$.

$$1\frac{2}{3} + 2\frac{5}{6} = 1\frac{4}{6} + 2\frac{5}{6} = 3\frac{\cancel{9}^3}{\cancel{6}_2} = 4\frac{1}{2}$$



It is necessary to simplify the answer and change it to an appropriate mixed number.

Use the common denominator of 6 to add the fractions of both $\frac{2}{3}$ and $\frac{5}{6}$. Adding the two fractions $\frac{4}{6}$ and $\frac{5}{6}$ together equals to $\frac{9}{6}$.

Add the whole numbers together to get the answer $3\frac{9}{6}$.

By using the greatest common factor, $\frac{9}{6}$ is simplified to $\frac{3}{2}$.

Finally simplify into the appropriate mixed number of $4\frac{1}{2}$.

2 Calculate the following by finding a common denominator.

1 $1\frac{3}{5} + 2\frac{2}{3}$

2 $2\frac{5}{6} + 1\frac{1}{2}$

3 $2\frac{3}{4} + 3\frac{5}{12}$

4 $1\frac{7}{15} + 2\frac{5}{6}$



Don't forget to make the most appropriate mixed numbers.

8 - 7

Addition and Subtraction of Fractions

Subtraction of Fractions (1)

Example A container has $\frac{3}{4}$ L of tea in it. $\frac{2}{3}$ L is poured out.
How much tea is left?



The math sentence is $\frac{3}{4} - \frac{2}{3}$.

To calculate this, we must find a common denominator of 4 and 3. The common denominator is 12.

Therefore, the math sentence can be changed to

$$\frac{9}{12} - \frac{8}{12}$$

Which fraction is greater?

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Math sentence

$$\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$$

Answer

$$\frac{1}{12} \text{ L}$$

1 The red ribbon is $\frac{1}{2}$ m long. The blue ribbon is $\frac{3}{5}$ m long. Which ribbon is longer? How much longer is one ribbon than the other?

Math sentence

Answer _____

2 Calculate the following by finding a common denominator.

1 $\frac{3}{4} - \frac{1}{3} = \frac{\square}{12} - \frac{\square}{12} = \frac{\square}{\square}$

2 $\frac{5}{7} - \frac{2}{5} = \frac{\square}{35} - \frac{\square}{35} = \frac{\square}{\square}$

3 $\frac{1}{3} - \frac{1}{8} = \frac{\square}{24} - \frac{\square}{24} = \frac{\square}{\square}$

4 $\frac{4}{7} - \frac{1}{3} = \frac{\square}{21} - \frac{\square}{21} = \frac{\square}{\square}$

5 $\frac{5}{8} - \frac{2}{5}$

6 $\frac{5}{6} - \frac{3}{7}$

7 $\frac{5}{9} - \frac{1}{4}$

8 $\frac{4}{5} - \frac{2}{3}$

9 $\frac{7}{10} - \frac{5}{9}$

10 $\frac{5}{8} - \frac{2}{7}$

8 - 8

Addition and Subtraction of Fractions

Subtraction of Fractions (2)

Example Calculate $\frac{2}{3} - \frac{1}{6}$.

To calculate this, we must find a common denominator of 3 and 6. The common denominator is 6. So, the math sentence will be $\frac{4}{6} - \frac{1}{6}$.



$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

$$\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6} = \frac{\cancel{3}}{\cancel{6}_2} = \frac{1}{2}$$

Simplify the fraction by finding the greatest common factor. In this case it is 3.

We can simplify $\frac{3}{6}$ to $\frac{1}{2}$.

Calculate the following by finding a common denominator and simplify the answer by finding the greatest common factor.

1 $\frac{2}{3} - \frac{5}{12} = \frac{\square}{12} - \frac{\square}{12}$

2 $\frac{2}{3} - \frac{7}{15} = \frac{\square}{15} - \frac{\square}{15}$

3 $\frac{5}{6} - \frac{5}{24} = \frac{\square}{24} - \frac{\square}{24}$

4 $\frac{3}{2} - \frac{7}{10} = \frac{\square}{10} - \frac{\square}{10}$

5 $\frac{25}{24} - \frac{5}{12}$

6 $\frac{11}{13} - \frac{7}{39}$

7 $\frac{15}{28} - \frac{1}{4}$

8 $\frac{3}{5} - \frac{7}{20}$

9 $\frac{27}{30} - \frac{3}{4} = \frac{\square}{60} - \frac{\square}{60}$

10 $\frac{7}{12} - \frac{5}{15} = \frac{\square}{60} - \frac{\square}{60}$

11 $\frac{11}{10} - \frac{14}{15}$

12 $\frac{5}{6} - \frac{3}{10}$

Subtraction of Fractions (3)

ExampleCalculate $1\frac{1}{4} - \frac{1}{2}$.

$$1\frac{1}{4} - \frac{1}{2} = 1\frac{1}{4} - \frac{2}{4}$$

$$= \frac{5}{4} - \frac{2}{4}$$

$$= \frac{3}{4}$$



Step 1: Find common denominators.

Step 2: Convert to improper fractions if needed.

In order to subtract fractions, both fractions must have the same denominator.

4 is the common denominator.

However, we cannot subtract $\frac{2}{4}$ from $\frac{1}{4}$. Therefore, the mixed number $1\frac{1}{4}$ must be converted into an improper fraction: $\frac{5}{4}$.

Then we can do $\frac{5}{4} - \frac{2}{4}$.

Calculate the following.

1 $1\frac{1}{4} - \frac{2}{3}$

2 $1\frac{1}{3} - \frac{3}{5}$

3 $1\frac{1}{7} - \frac{3}{5}$

4 $1\frac{1}{6} - \frac{3}{5}$

5 $1\frac{2}{5} - \frac{3}{4}$

6 $3\frac{1}{6} - \frac{5}{7}$

7 $2\frac{2}{9} - \frac{2}{3}$

8 $2\frac{1}{12} - \frac{1}{6}$

9 $3\frac{2}{3} - \frac{7}{9}$

10 $2\frac{3}{8} - \frac{9}{10}$

11 $1\frac{1}{4} - \frac{5}{6}$

12 $3\frac{3}{10} - \frac{13}{15}$

8 - 10

Addition and Subtraction of Fractions

Subtraction of Fractions (4)

Example Calculate $2\frac{1}{4} - 1\frac{2}{3}$.



It is important to the whole number can be changed into fraction if necessary, and the whole numbers can be calculate by themselves.

$$\begin{aligned} 2\frac{1}{4} - 1\frac{2}{3} &= 2\frac{3}{12} - 1\frac{8}{12} \\ &= 1\frac{15}{12} - 1\frac{8}{12} \\ &= \frac{7}{12} \end{aligned}$$

To calculate this, we must find the common denominator of 4 and 3.

However, it is impossible to subtract $\frac{8}{12}$ from $\frac{3}{12}$. Both mixed numbers must be converted to improper fractions in order to subtract them.

$2\frac{3}{12}$ becomes $1\frac{15}{12}$.

The whole numbers, 1 and 1 can be calculate $1 - 1$.

Calculate the following.

1 $2\frac{1}{5} - 1\frac{1}{3}$

2 $2\frac{2}{3} - 1\frac{3}{4}$

3 $3\frac{1}{7} - 1\frac{3}{4}$

4 $3\frac{1}{4} - 1\frac{4}{5}$

5 $5\frac{2}{5} - 3\frac{2}{3}$

6 $4\frac{1}{6} - 3\frac{2}{5}$

7 $3\frac{2}{15} - 1\frac{2}{3}$

8 $4\frac{1}{8} - 1\frac{1}{2}$

9 $2\frac{1}{6} - 1\frac{7}{12}$

10 $3\frac{1}{6} - 1\frac{8}{15}$

11 $4\frac{1}{6} - 2\frac{7}{8}$

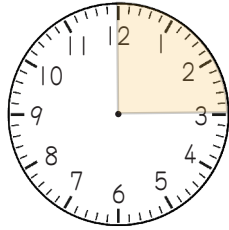
12 $4\frac{7}{10} - 2\frac{3}{4}$

8 - 11

Addition and Subtraction of Fractions

Time and Fractions

Instruction We can express time by using fractions as follows:

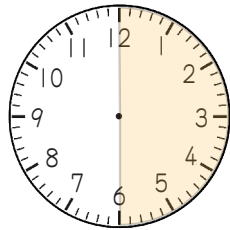


15 minutes



$$\frac{15}{60} = \frac{1}{4}$$

of an hour

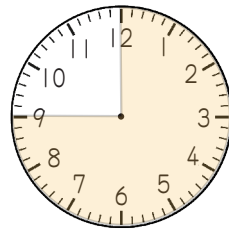


30 minutes



$$\frac{30}{60} = \frac{1}{2}$$

of an hour

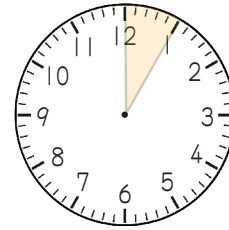


45 minutes



$$\frac{45}{60} = \frac{3}{4}$$

of an hour



5 minutes



$$\frac{5}{60} = \frac{1}{12}$$

of an hour

Because 1 hour is 60 minutes, we can express time by using a fraction whose denominator is 60.



Example Write the appropriate fractions in the .

1 10 minutes = $\frac{10}{60} = \frac{1}{6}$ of an hour

2 30 seconds = $\frac{30}{60} = \frac{1}{2}$ of a minute

Because 1 minute is 60 seconds, we can express time by using a fraction whose denominator is 60. We can do the same with minutes to hours.

1 minute is 60 seconds. Therefore, we can use the same way as the converting minutes to hours and seconds to minutes.



Write the appropriate fractions in the .

1 20 minutes = $\frac{20}{60} = \frac{\quad}{\quad}$ of an hour 2 50 minutes = $\frac{50}{60} = \frac{\quad}{\quad}$ of an hour

3 90 minutes = $\frac{90}{60} = \frac{\quad}{\quad}$ of an hour 4 100 minutes = $\frac{100}{60} = \frac{\quad}{\quad}$ of an hour

5 1 minutes = $\frac{\quad}{\quad}$ of an hour 6 45 seconds = $\frac{\quad}{\quad}$ of a minute

7 25 seconds = $\frac{\quad}{\quad}$ of a minute 8 1 seconds = $\frac{\quad}{\quad}$ of a minute

8 - 12

Addition and Subtraction of Fractions

Review

1 Compare the following fractions and write the appropriate inequal sign in the .

1 $\frac{5}{21}$ $\frac{2}{7}$ **2** $\frac{5}{6}$ $\frac{7}{9}$ **3** $\frac{8}{9}$ $\frac{13}{15}$ **4** $\frac{3}{8}$ $\frac{5}{12}$

2 Calculate the following and simplify the answer.

1 $\frac{1}{6} + \frac{2}{7}$

2 $\frac{5}{9} + \frac{1}{5}$

3 $\frac{3}{5} + \frac{1}{15}$

4 $\frac{1}{6} + \frac{3}{10}$

5 $\frac{2}{3} - \frac{1}{5}$

6 $\frac{3}{4} - \frac{2}{5}$

7 $\frac{19}{28} - \frac{1}{4}$

8 $\frac{7}{9} - \frac{1}{6}$

9 $3\frac{4}{7} + 2\frac{2}{3}$

10 $1\frac{5}{6} + \frac{11}{18}$

11 $1\frac{2}{15} - \frac{3}{10}$

12 $3\frac{7}{8} - 1\frac{9}{10}$

3 Write the appropriate fractions in the .

1 10 minutes = of an hour **2** 40 minutes = of an hour

3 30 seconds = of a minute **4** 5 seconds = of a minute

4 Write the appropriate numbers from 1 to 9 in the . The same number cannot be used twice.

1 $\frac{\square}{6} + \frac{\square}{24} = \frac{5}{24}$ **2** $\frac{32}{45} - \frac{\square}{15} = \frac{8}{45}$ **3** $\square \frac{\square}{13} + 1\frac{7}{26} = 4\frac{\square}{26}$