

12-1

Ratio

How to Express a Ratio

Example 1 Your sister and brother practice the shoot.

	Number of shots	Number of scores
(a) Your sister	10	8
(c) Your brother	12	9

1 Let's express the results with fraction

It can be expressed as a fraction as above.

$$\frac{\text{Number of score}}{\text{Number of shots}}$$

Compared quantity

Base quantity

Answer (a) $\frac{8}{10}$ (b) $\frac{9}{12}$



2 Let's compare the results with fraction.

It is easier to compare when the denominators are the same.

(a) $\frac{8}{10} = \frac{\boxed{48}}{60}$ (b) $\frac{9}{12} = \frac{\boxed{45}}{60}$

3 Who has the best shooting record?

Answer (a) your sister

You and your brother practice the shoot.

	Number of shots	Number of scores
(a) You	10	9
(b) Your brother	15	12

1 Let's express the results with fraction

Answer _____

2 Let's compare the results with fraction.

(a) $\frac{9}{10} = \frac{\boxed{\quad}}{30}$ (b) $\frac{12}{15} = \frac{\boxed{\quad}}{30}$

3 Who has the best shooting record?

Answer _____

12-2

Ratio

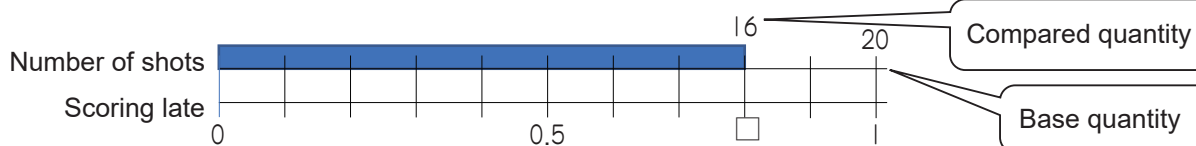
Calculation of Ratio (1)

Example 1 Your sister and brother practice the shoot.

1 Let's calculate (a)'s scoring rate.

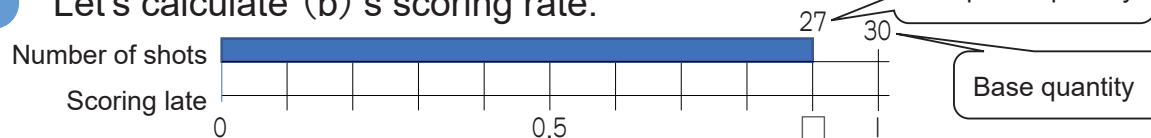
	Number of shots	Number of scores
(a) Your sister	20	16
(b) Your brother	30	27

Let's calculate the number of score per shoot (scoring rate), like amount per unit.



Math sentence $16 \div 20 = 0.8$

1 Let's calculate (b)'s scoring rate.



Math sentence $27 \div 30 = 0.9$

2 Who has the best shooting record?

Answer (a) Your brother

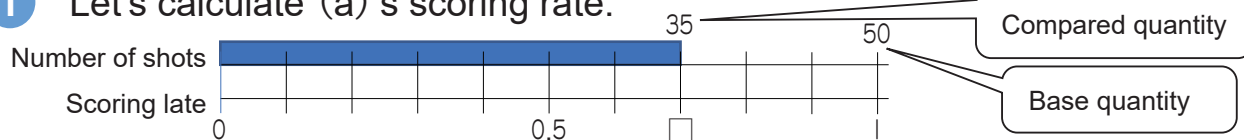
If we consider the performance of a shot as a quantity based on the number of shoot (base quantity) and the number of score of shots as a comparison (compared quantity), it can be expressed by the following math sentence.

Success of record = Number of score \div Number of shots = $\frac{\text{Number of score}}{\text{Number of shots}}$

Your sister and brother practice the shoot.

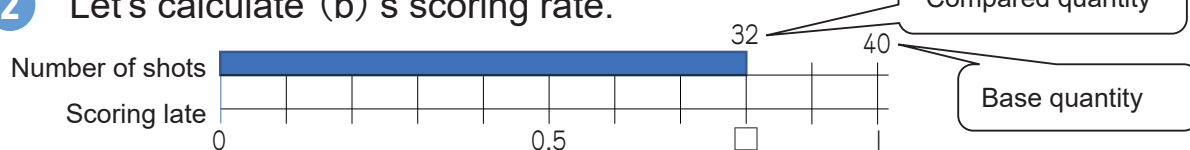
	Number of shots	Number of scores
(a) Your sister	50	35
(b) Your brother	40	32

1 Let's calculate (a)'s scoring rate.



Math sentence

2 Let's calculate (b)'s scoring rate.



Math sentence

3 Who has the best shooting record?

Answer _____

12-3

Ratio

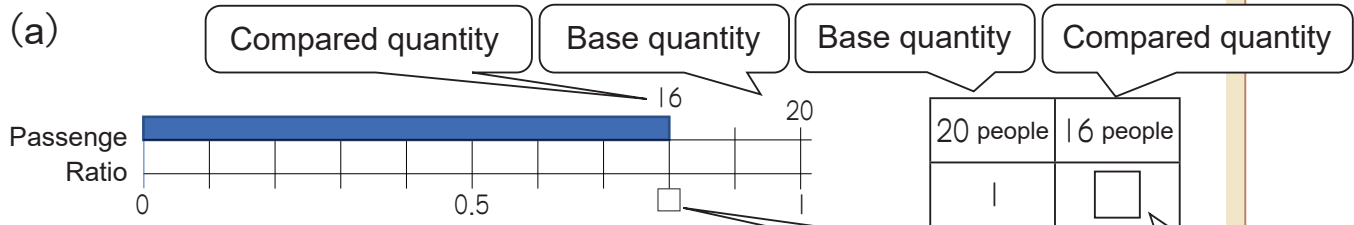
Calculation of Ratio (2)

Example 1 The following table shows the capacity of a bus and the number of passengers on a given day.

	Capacity	Passenger	Ratio
(a) Small bus	20	16	<input type="checkbox"/>
(b) Big bus	80	60	<input type="checkbox"/>

1 Calculate of ratio of congestion of small bus to find the answer.

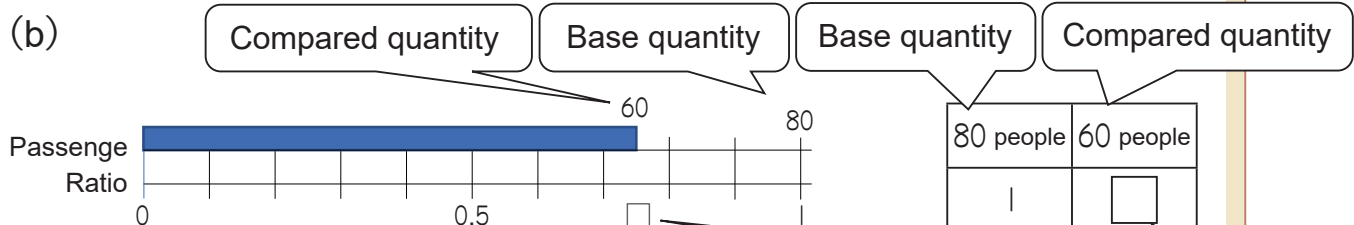
The capacity (ratio of coongestion) can be calculated as the number of passengers compared to the base capacity of 1.



Compared quantity Base quantity Ratio Ratio of congestion Ratio

Math sentence $16 \div 20 = 0.8$

2 Calculate of ratio of congestion of big bus to find the answer.



Compared quantity Base quantity Ratio Ratio of congestion Ratio

Math sentence $60 \div 80 = 0.75$

A congestion ratio of 0.75 means that the ratio of the number of passengers to the capacity of the train is 0.75.



3 Which bus is more crowded?

(a) = 0.8 (b) = 0.75

Answer (a) is more crowded

When the base quantity is considered as the unit, the size of the compared quantity is the **ratio**. And ratio can calculate as following.

$$\text{Ratio} = \text{Compared quantity} \div \text{Base quantity}$$

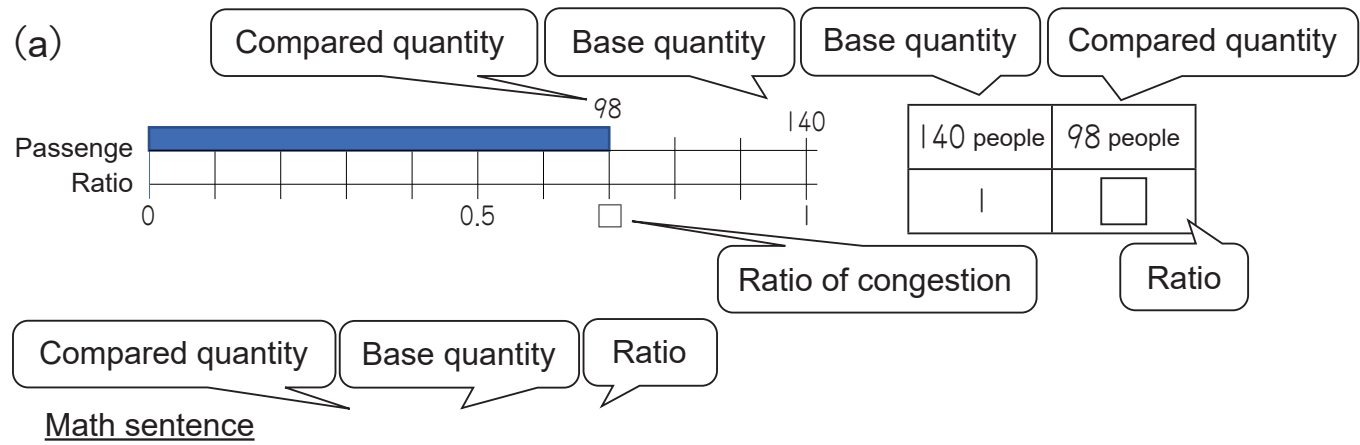
If compared quantity and base quantity are whole number, we can express as following; Ratio =

$$\text{Ratio} = \frac{\text{Compared quantity}}{\text{Base quantity}}$$

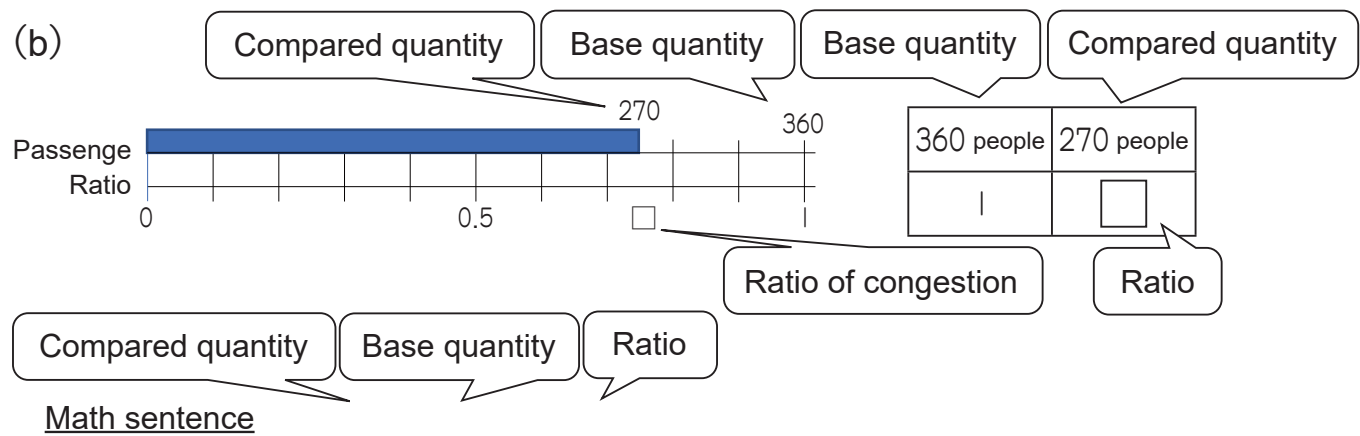
The following table shows the capacity of a bus and the number of passengers on a given day.

	Capacity	Passenger	Ratio
(a) Small airplane	140	98	<input type="checkbox"/>
(b) Big airplane	360	270	<input type="checkbox"/>

1 Calculate of ratio of congestion of small airplane to find the answer.



2 Calculate of ratio of congestion of big bus to find the answer.



3 Which bus is more crowded?

(a) = (b) = Answer _____

12-4

Ratio

Percentage (1)

Example There are 30 passengers in the bus that has 50 seats.

1 Find the ratio of crowdedness of the bus.

Compared quantity: 30
Base quantity: 50
Ratio: $30 \div 50 = 0.6$

Math sentence: $30 \div 50 = 0.6$

Represent this ratio by making the base quantity as 100.

$$30 \div 50 = \square \div 100$$

(Note: Arrows indicate multiplying 30 by 2 and 50 by 2 to reach 100.)

Relative amount 0.01
↕
1%

① % ②
③

We often represent a ratio by making the base quantity as 100.

This representation is called **percentage**.

The ratio 0.01 which is a decimal number, is called one **percent** and is written as 1%. If we represent the ratio 1 as a percentage, it is 100%.

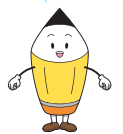
2 Represent the crowdedness of the bus as a percentage.

Compared quantity: 30
Base quantity: 50
Ratio: 0.6

Math sentence: $0.6 \times 100 = 60$

Answer: 60%

Let's multiply by 100 to find the percentage.



There is a train with a capacity of 100 passengers per carriage.

There are 74 passengers in the first car of the train.

1 Find the ratio of crowdedness of the bus.

Compared quantity: 74
Base quantity: 100
Ratio: $74 \div 100 = 0.74$

Math sentence: _____

2 Represent the crowdedness of the bus as a percentage.

Math sentence _____ Answer _____

12-5

Ratio

Percentage (2)

Example 1

1 Express a number expressed as a decimal as a percentage.

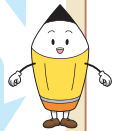
(a) $0.06 \rightarrow$ (b) $0.2 \rightarrow$

Let's multiply by 100 to find the percentage.

2 Express a percentage as a percentage in decimal form.

(a) $7\% \rightarrow$ (b) $35\% \rightarrow$

Let's divide by 100 to find the ratio.



1 Express a number expressed as a decimal as a percentage.

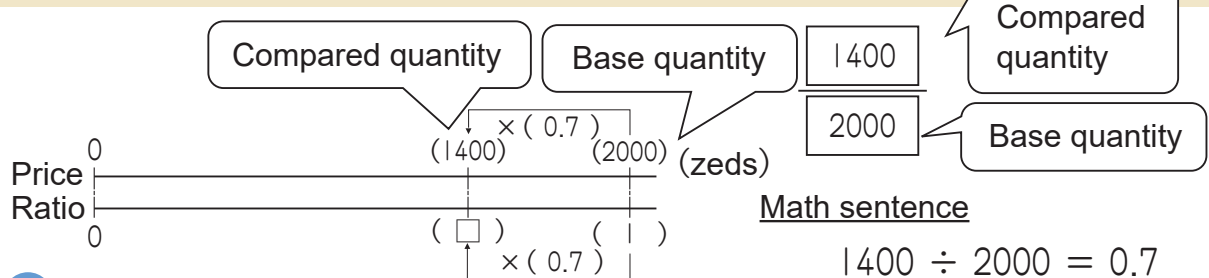
(a) $0.08 \rightarrow$ (b) $0.6 \rightarrow$ (c) $1.25 \rightarrow$

2 Express a percentage as a percentage in decimal form.

(a) $9\% \rightarrow$ (b) $58\% \rightarrow$ (c) $140\% \rightarrow$

Example

A shirts costing 2000 zeds was sold for 1400 zeds. Express the percentage of the price you paid based on the regular price as a percentage.

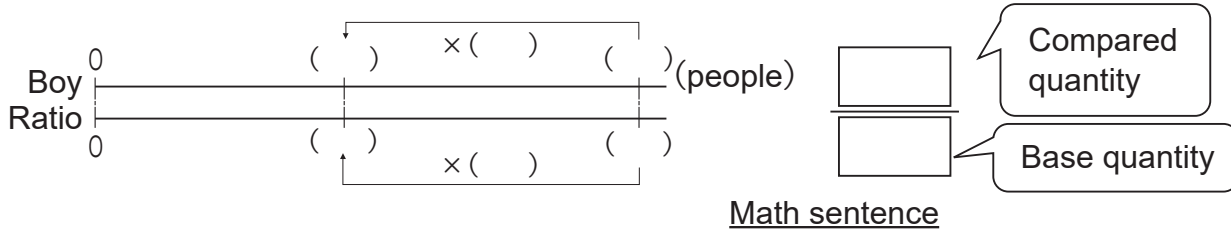


2 Represent the regular price as a percentage.

Math sentence $0.7 \times 100 = 70$

Answer

There are 40 students in the whole class, of which 18 are boys. Express the percentage of boys in the class based on the total number of boys in the class.



2 Represent the total number of boys as a percentage.

Math sentence

Answer

12-6

Ratio

Finding Compared Quality (1)

Example 1 A shirt with a regular price of 2000 zeds is sold for 60% of the regular price. How much will I have to pay to get it. (“zed(s)” is the fictional currency unit.)

Compared quantity Base quantity

Price Ratio: 0 to 2000 (zeds)

Math sentence: $2000 \times 0.6 = 1200$

Answer: 1200 zeds

Example 2 I bought apples for 80 zeds each and decided to sell them for 120% of the purchase price. How much will you sell each apple for? (“zed(s)” is the fictional currency unit.)

Compared quantity Base quantity

Price Ratio: 0 to 80 (zeds)

Math sentence: $80 \times 1.2 = 96$

Answer: 96 zeds

1 A scarf with a regular price of 1200 zeds is sold for 70% of regular price. How much will I have to pay to get it? (“zed(s)” is the fictional currency unit.)

Price Ratio: 0 to () (zeds)

Math sentence: _____

Answer: _____

2 We made a cake which cost 300 zeds per piece. You decide to sell this cake at 150% of the price. How much will you sell one cake for? (“zed(s)” is the fictional currency unit.)

Price Ratio: 0 to 300 (zeds)

Math sentence: _____

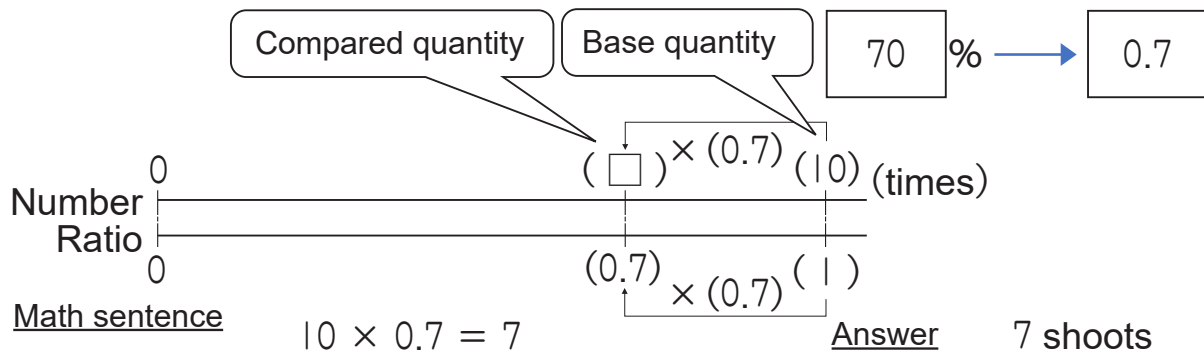
Answer: _____

12-6

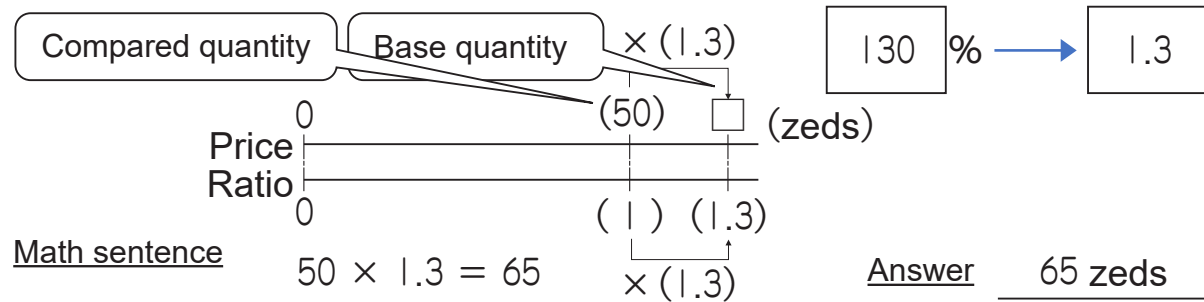
Ratio

Finding Compared Quality (2)

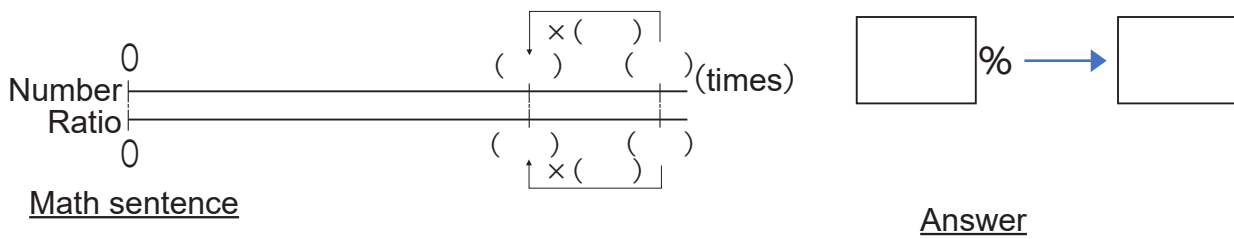
Example 1 In football shooting practice, I shot 10 times and scored 70% of the shoots. How many times did you score?



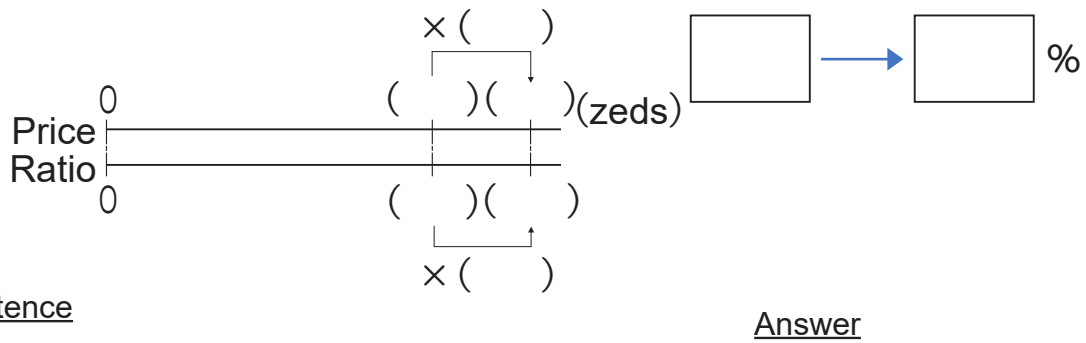
Example 2 I bought onions for 50 zeds each and decided to sell them for 130% of original price. What is the sale price? ("zed(s)" is the fictional currency unit.)



1 In football shooting practice, I shot 12 times and scored 75% of the shoots. How many times did you score?



2 I bought potatoes for 80 zeds each and decided to sell them for 125% of original price. What is the sale price? ("zed(s)" is the fictional currency unit.)

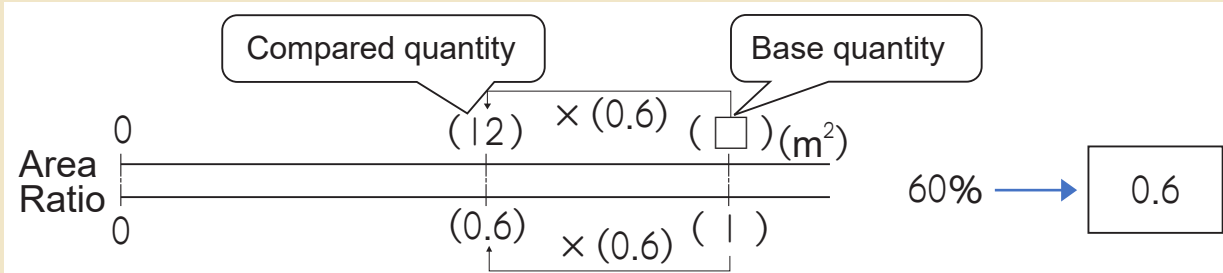


12-7

Ratio

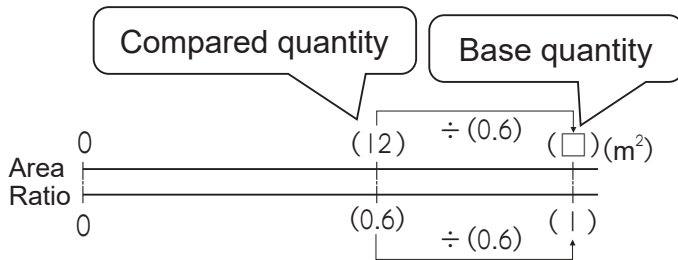
Finding Base Quality (1)

Example We grow maize on our farmland. We grow maize on an area of 12 m^2 , which is 60% of our total area. What is the total area of the farm?



Math sentence $\square \times 0.6 = 12$

To get base quantity, multiply the compared quantity with 0.6

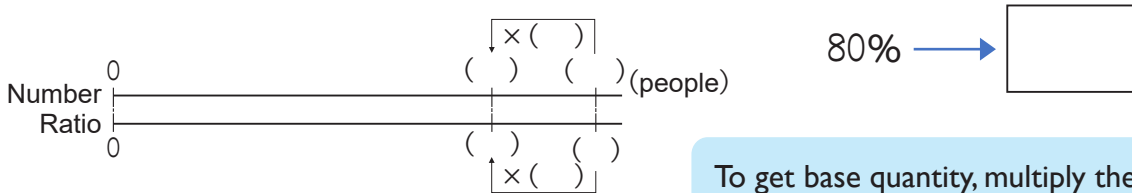


To get base quantity, divide compared quantity into 0.6

Math sentence $12 \div 0.6 = 20$

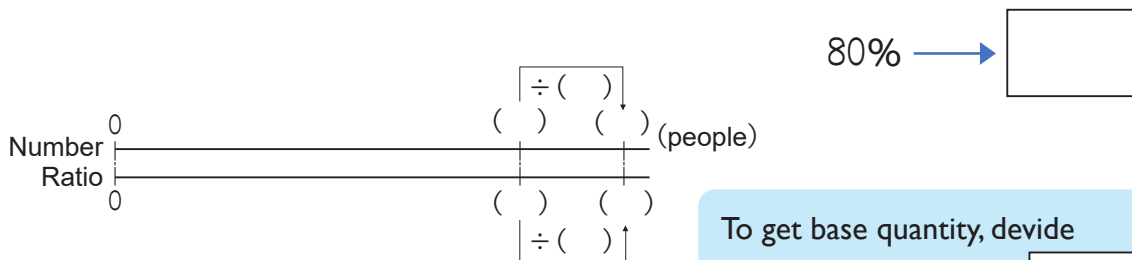
Answer 20 m^2

The track and field club has 24 interested students. This is 80% of available slots. How many slots are available?



To get base quantity, multiply the compared quantity with \square

Math sentence



To get base quantity, divide compared quantity into \square

Math sentence

Answer _____

12-7

Ratio

Finding Base Quality (2)

Example 1 A shirt was sold for 60% of the regular price which is 3000 zeds. How much is the regular price? (“zed(s)” is the fictional currency unit.)

Compared quantity (3000) \div (0.6) Base quantity (□) (zeds) 60% \rightarrow 0.6

Price Ratio 0 (0.6) \div (0.6) (1)

To get base quantity, divide compared quantity into 0.6

Math sentence $\square \times 0.6 = 3000$
Math sentence $3000 \div 0.6 = 5000$

Answer 5000 zeds

Example 2 I bought tomatoes for 60 zeds each and it was for 120% of the original price. How much is the original price? (“zed(s)” is the fictional currency unit.)

Base quantity (□) Compared quantity (60) (zeds) 120% \rightarrow 1.2

Price Ratio 0 (1) \div (1.2) (1.2)

To get base quantity, divide compared quantity into 1.2

Math sentence $\square \times 1.2 = 60$
Math sentence $60 \div 1.2 = 50$

Answer 50 zeds

1 I bought a cake for 80% of regular price which is 1200 zeds. How much is the regular price of the cake? (“zed(s)” is the fictional currency unit.)

Compared quantity (1200) \div (0.8) Base quantity (□) (zeds) 80% \rightarrow 0.8

Price Ratio 0 (0.8) \div (0.8) (1)

To get base quantity, divide compared quantity into 0.8

Math sentence $\square \times 0.8 = 1200$
Math sentence $1200 \div 0.8 = 1500$

Answer 1500 zeds

2 You made a bread and decided to sell it with 120 zeds. This price was 250% of the cost of ingredients. What is the cost of ingredients? (“zed(s)” is the fictional currency unit.)

Base quantity (□) Compared quantity (120) (zeds) 250% \rightarrow 2.5

Price Ratio 0 (1) \div (2.5) (2.5)

To get base quantity, divide compared quantity into 2.5

Math sentence $\square \times 2.5 = 120$
Math sentence $120 \div 2.5 = 48$

Answer 48 zeds

12-8

Ratio

Problems (1)

Example 1 We grow tomatoes on our farmland. We grow tomatoes on an area of 16 m^2 , which is 40% of our total area. What is the total area of the farm?

Area Ratio

Compared quantity: 16 (m²)

Base quantity: (\quad) (m²)

40% \rightarrow 0.4

Math sentence: $16 \div 0.4 = 40$

Answer: 40 m^2

To get base quantity, divide compared quantity into 0.4

Example 2 There were 3000 people who came to the grand park yesterday. That's 120% of yesterday's total. How many people came to the grand park yesterday?

Number Ratio

Base quantity: (\quad) (people)

Compared quantity: 3000 (people)

120% \rightarrow 1.2

Math sentence: $3000 \div 1.2 = 2500$

Answer: 2500 people

To get base quantity, divide compared quantity into 1.2

1 The science club has 42 interested students. This is 20% of total number of students. How many students are there?

Number Ratio

Compared quantity: (\quad)

Base quantity: (\quad) (people)

20% \rightarrow (\quad)

Math sentence: _____

Answer: _____

To get base quantity, divide compared quantity into (\quad)

There were 9000 people who came to the Stadium yesterday. That's 150% of yesterday's total. How many people came to the Stadium yesterday?

Number Ratio

Compared quantity: (\quad)

Base quantity: (\quad) (people)

150% \rightarrow (\quad)

Math sentence: _____

Answer: _____

To get base quantity, divide compared quantity into (\quad)

12-8

Ratio

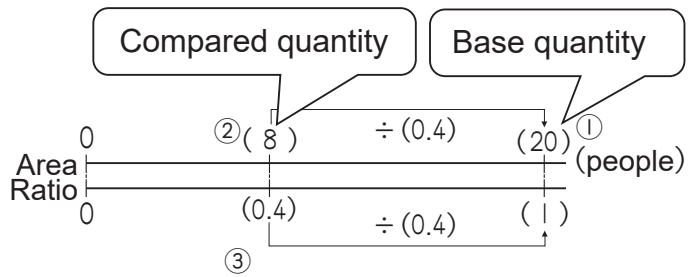
Problems (2)

Example

Let's make math problems.

The sentence below can be expressed using a figure like the one on the right.

20 ^① students gather for the youth group assembly. 8 ^② of them are girls. Girls make up 40 ^③ % of the total number of students.



- 1 Make a math problem for finding ③.

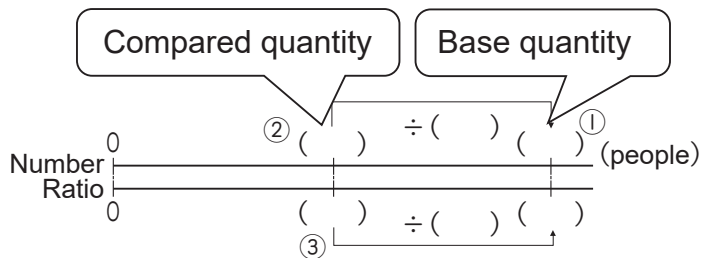
20 students gather for the youth group assembly. 8 of them are girls. What is the percentage of female students?

- 2 Make a math problem for finding ②.

20 students gather for the youth group assembly. Girls make up 40 % of total number of students. What is the number of girls?

The sentence below can be expressed using a figure like the one on the right.

There are a total of 500 ^① children at your school. 260 ^② of them are girls. Girls make up 52 ^③ % of the total number of students.



- 1 Make a math problem for finding ③.

- 2 Make a math problem for finding ②.

12-9

Ratio

Review

1 You shoot 20 times and score 14 times. What is the ratio of the score?

1 Base quantity is and compared quantity is .

2 Express the math sentence by words.

Ratio = ÷

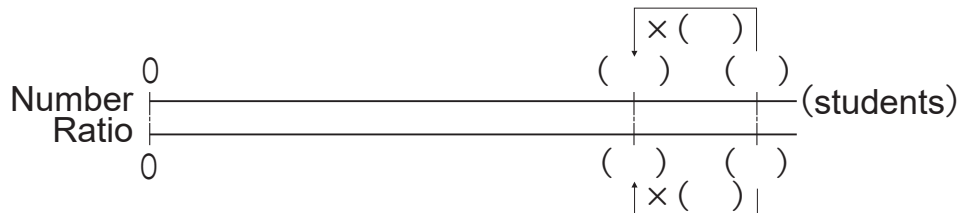
3 Express the math sentence by fraction.

Ratio = $\frac{\text{}}{\text{}}$

Math sentence

Answer _____

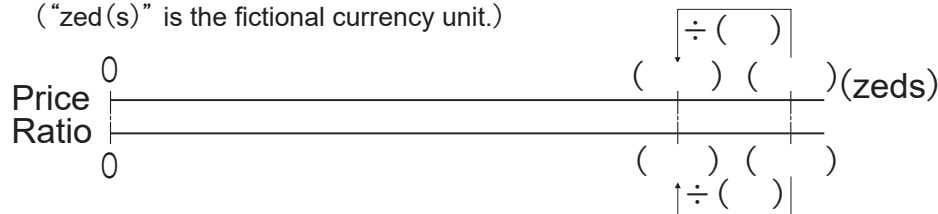
2 The cooking club has 25 available slots. The number of interested students is 0.8 times as many as the number of available slots. How many interested students are there?



Math sentence

Answer _____

3 The reference book for science class costs 1800 zeds. This is 1.2 times as many as the price of a dictionary. What is the price of the dictionary? ("zed(s)" is the fictional currency unit.)



Math sentence

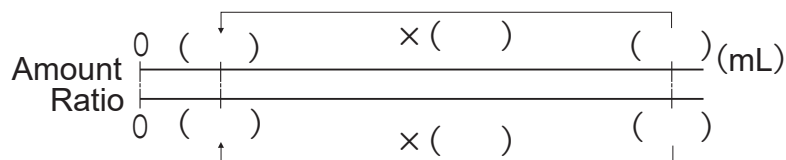
Answer _____

4 The bottle is 300 mL of juice drink. Of the total amount, 15% is fruit juice. How many mL of fruit juice are in the bottle?

1 Express 15% using decimal number.

Answer _____

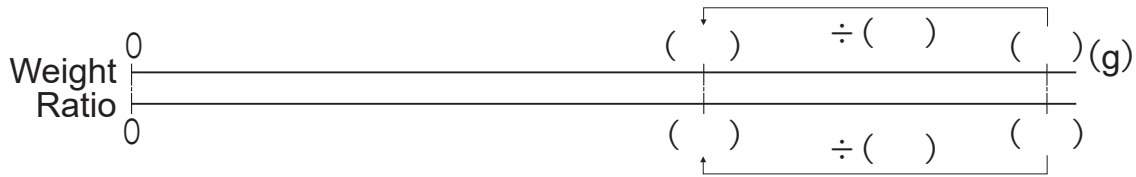
2 Write a math sentence to calculate the compared quantity.



Math sentence

Answer _____

- 5** A dog that was born a week ago. It weighs 184g. The current weight is 160% of its weight a week ago. What was its weight at birth in g?

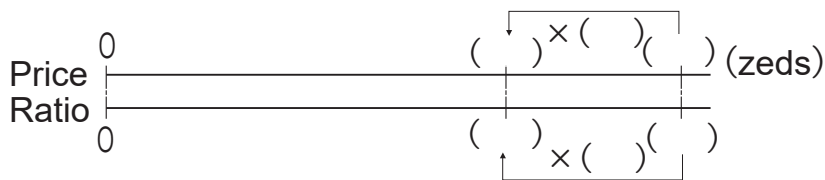


Math sentence _____

Answer _____

- 6** My friend bought a marker for 30% off the regular price of 300 zeds. What is the sale price? (“zed(s)” is the fictional currency unit.)

- 1** 30% off means % of regular price.

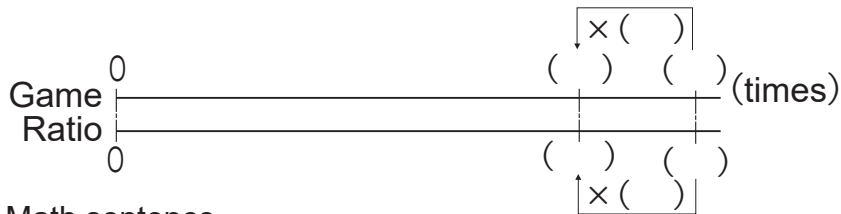


Math sentence _____

Answer _____

- 7** Your football team’s record was 8 wins and 2 losses.

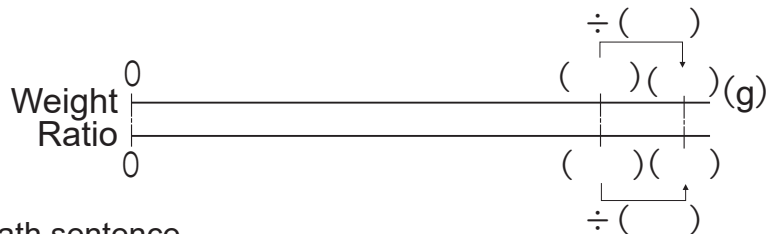
- 1** What is the percentage of games won to games played?



Math sentence _____

Answer _____

- 8** It is said that 86% of the composition of an apple is water. How much does an apple weigh if it contains 258 g of water?

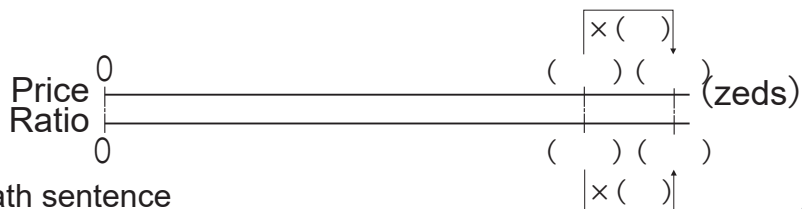


Math sentence _____

Answer _____

- 9** A book is priced at 1200 zeds before tax, what is the price after adding the 10% tax? (“zed(s)” is the fictional currency unit.)

- 1** 10% adding tax means % of regular price.



Math sentence _____

Answer _____