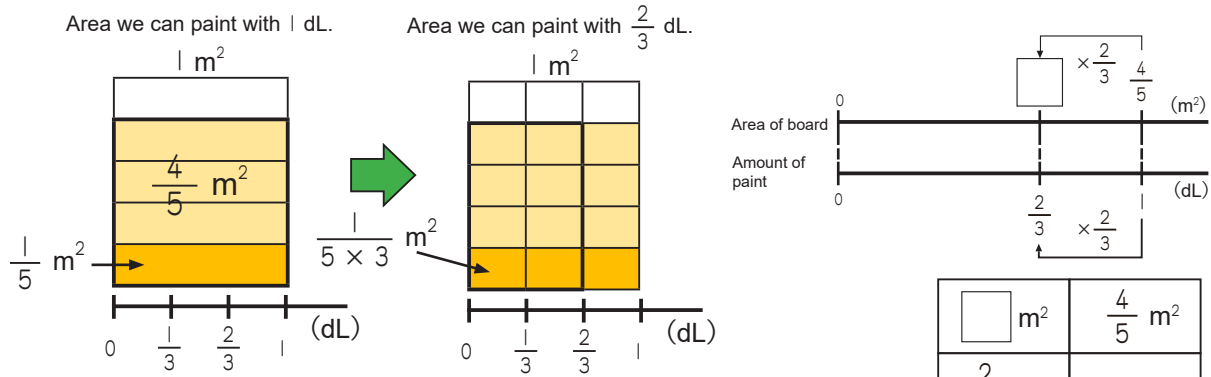


# 4 - 1

## How to Multiply by Fractions

### Multiplying by Fractions (1)

**Example** | dL of paint can cover  $\frac{4}{5} \text{ m}^2$ . How much area can  $\frac{2}{3}$  dL of paint cover?



Math sentence

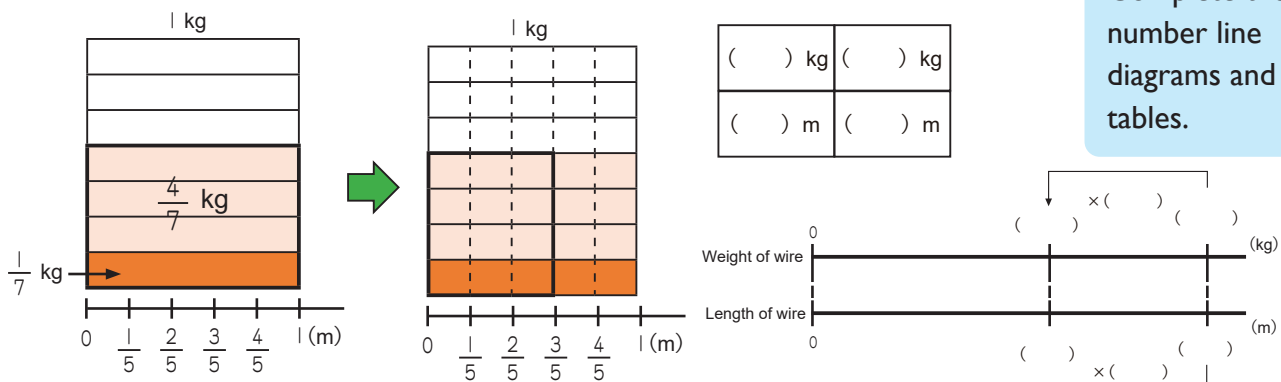
$$\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$$

Answer  $\frac{8}{15} \text{ m}^2$

Multiply the numerators to equal the answer's numerator. Multiply the denominators to equal the answer's denominator.

$$\frac{b}{a} \times \frac{d}{c} = \frac{b \times d}{a \times c}$$

**1** A 1 m piece of wire weighs  $\frac{4}{7}$  kg. How much will a  $\frac{3}{5}$  m piece of wire weigh?



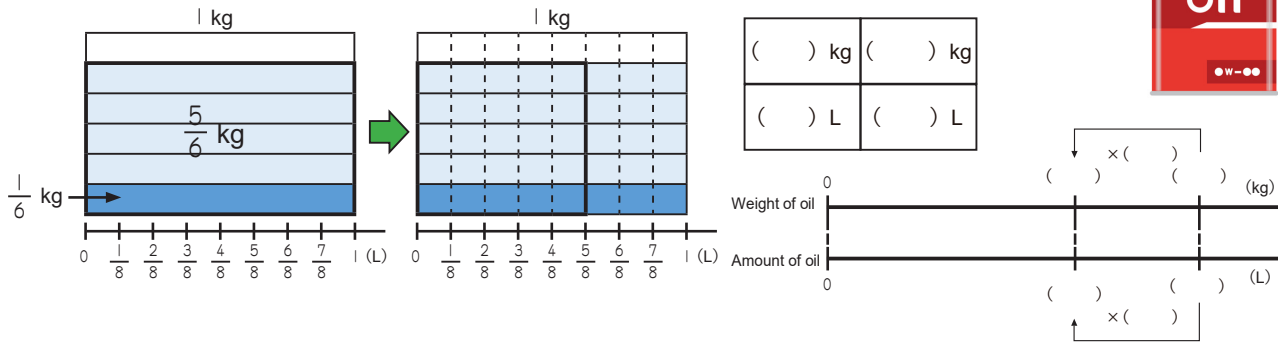
Complete the number line diagrams and tables.

Math sentence

Answer



**2** A 1 L oil weighs  $\frac{5}{6}$  kg. How much does a  $\frac{5}{8}$  L of this oil weigh?



Math sentence

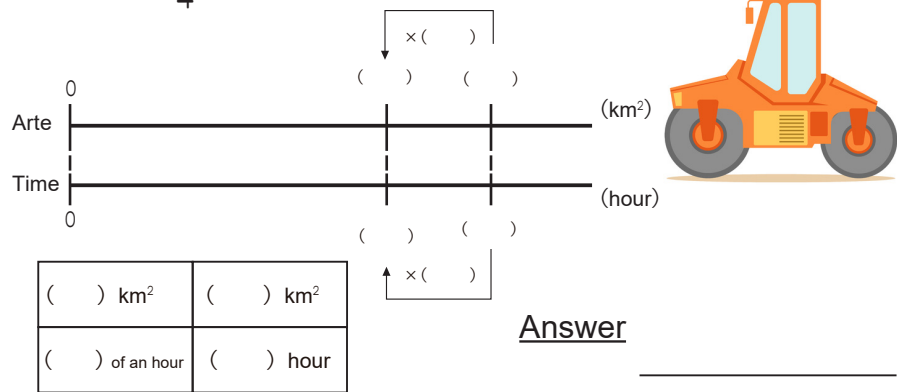
Answer \_\_\_\_\_

**3** A paver is a machine that can lay  $\frac{5}{7}$  km<sup>2</sup> of pavement in 1 hour. How much pavement can it lay in  $\frac{3}{4}$  hour?



Math sentence

Complete the number line diagram and table.



Answer \_\_\_\_\_

**4** Calculate the following multiplication problems. Simplify the answers when possible. Leave the answers as improper fractions.

**1**  $\frac{1}{5} \times \frac{4}{9} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$

**2**  $\frac{7}{9} \times \frac{5}{6} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$

**3**  $\frac{3}{2} \times \frac{5}{4}$

**4**  $\frac{4}{9} \times \frac{2}{3}$

**5**  $\frac{2}{3} \times \frac{7}{5}$

**6**  $\frac{5}{4} \times \frac{5}{2}$

**7**  $\frac{9}{5} \times \frac{7}{2}$

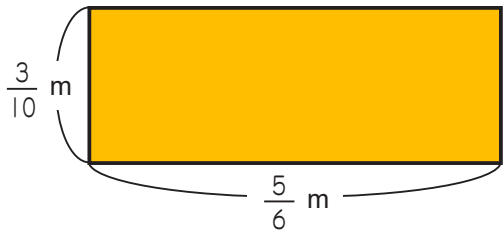
**8**  $\frac{3}{8} \times \frac{3}{4}$

# 4 - 2

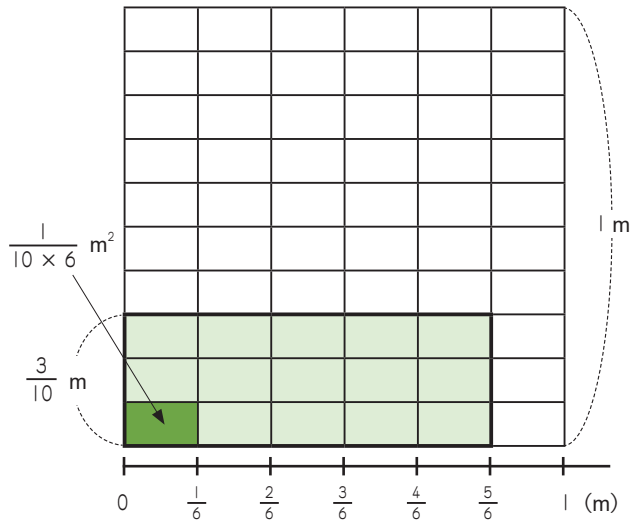
## How to Multiply by Fractions

### Multiplying by Fractions (2)

**Example 1** There is a rectangle with a length of  $\frac{5}{6}$  m and a width of  $\frac{3}{10}$  m. What is the area of this rectangle?



This rectangle has 15 small rectangles with an area of  $\frac{1}{10 \times 6}$  m<sup>2</sup>. Therefore, the area of this rectangle is  $\frac{15}{60}$  m<sup>2</sup> (or  $\frac{1}{4}$  m<sup>2</sup>). This area can be calculated by the formula of the area of rectangle.



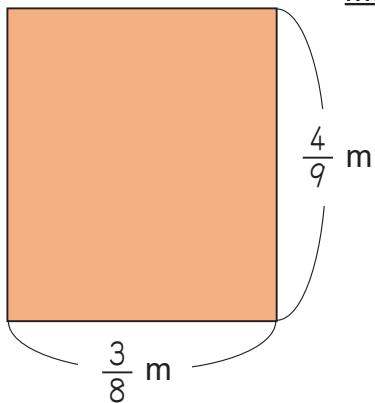
Math sentence

$$\frac{5}{6} \times \frac{3}{10} = \frac{\cancel{5} \times \cancel{3}}{\cancel{6} \times \cancel{10}} = \frac{1}{4}$$

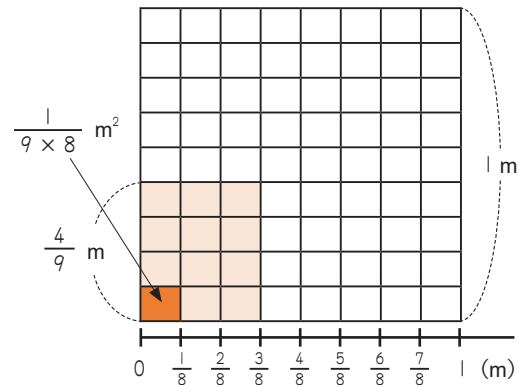
Answer  $\frac{1}{4}$  m<sup>2</sup>

Simplifying fractions by dividing a numerator and denominator by their greatest common factors is easier to be done during the calculation. It is also fine to simplify it after calculation.

**1** There is a rectangle with a length of  $\frac{4}{9}$  m and a width of  $\frac{3}{8}$  m. What is the area of this rectangle? Calculate it by using the formula of the area of rectangle.



Math sentence



Answer \_\_\_\_\_

How many rectangles with the area of  $\frac{1}{9 \times 8}$  m<sup>2</sup> are there?



**Example 2** 1 L of water contains  $\frac{3}{8}$  g of minerals. How many g of minerals contained in  $\frac{4}{9}$  L of water?

<input type="text"/> g	$\frac{3}{8}$ g
$\frac{4}{9}$ L	1 L

**Math sentence**

$$\frac{3}{8} \times \frac{4}{9} = \frac{\cancel{3} \times \cancel{4}}{\cancel{8} \times \cancel{9}} = \frac{1}{6}$$

**Answer**  $\frac{1}{6}$  g

**2** 1 kg of potato contains  $\frac{5}{6}$  L of water. How many L of water is contained in  $\frac{3}{10}$  kg of potatoes?

( ) g	( ) g
( ) L	( ) L

**Math sentence**

**Answer** \_\_\_\_\_

**3** A machine can run for  $\frac{14}{15}$  of an hour on 1 L of oil. How long can the machine run on  $\frac{5}{7}$  L of oil?

**Math sentence**

( ) hours	( ) of an hour
( ) L	( ) L

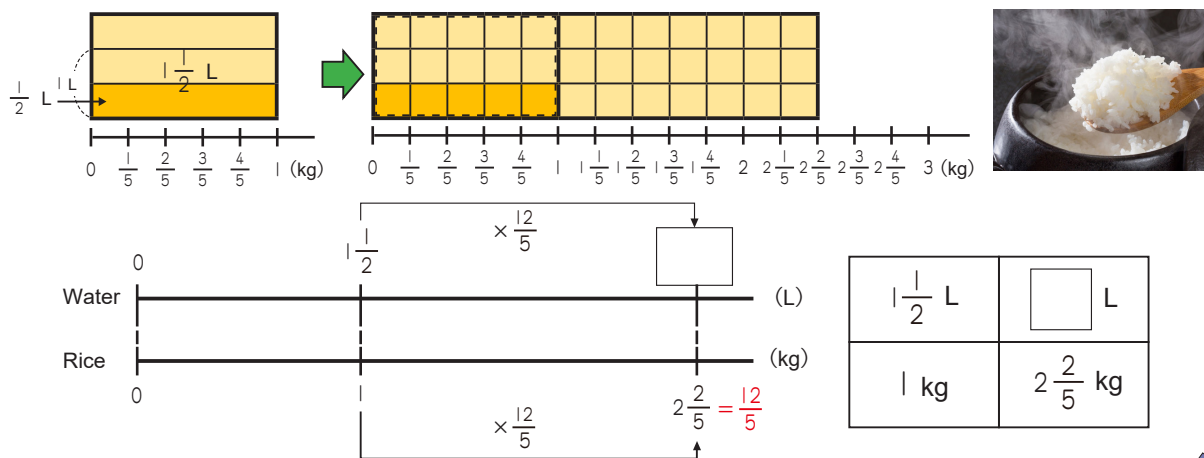
**Answer** \_\_\_\_\_

# 4 - 3

## How to Multiply by Fractions

### Multiplying Two Mixed Numbers

**Example**  $1\frac{1}{2}$  L of water is needed to cook 1 kg of rice. How much water is needed to cook  $2\frac{2}{5}$  kg of rice?



**Math sentence**

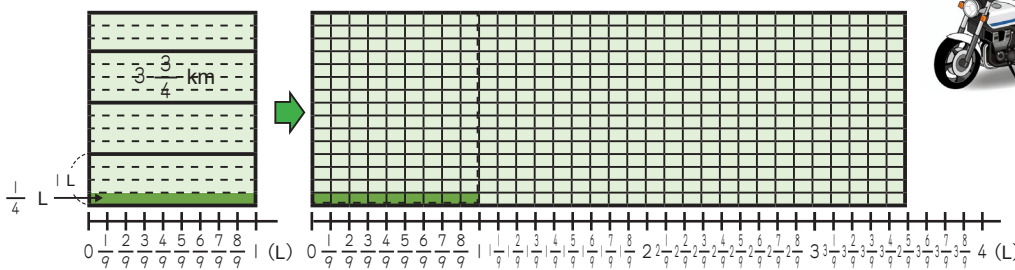
$$1\frac{1}{2} \times 2\frac{2}{5} = \frac{3}{2} \times \frac{12}{5} = \frac{3 \times \cancel{12}^6}{2 \times 5} = \frac{18}{5} \text{ or } 3\frac{3}{5} \text{ Answer } \frac{18}{5} \text{ L or } 3\frac{3}{5} \text{ L}$$



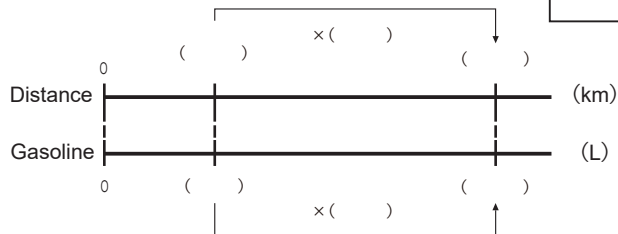
**Multiply two mixed numbers: (a mixed number) × (a mixed number)**

Change both mixed numbers to improper fractions. Multiply both numerators to get the answer's numerator. Multiply both denominators to get the answer's denominator.

**1** My motorcycle can run  $3\frac{3}{4}$  km on 1 L of gasoline. How many km can it go when it has  $3\frac{5}{9}$  L of gasoline?



**Math sentence**



**Answer**

Complete the number line diagrams and tables.



- 2** I have  $2\frac{11}{12}$  a of farmland. I need  $3\frac{3}{5}$  kg of fertilizer per a of the farmland. How many kg of fertilizer do I need to fertilize all of my farmland?

Math sentence

( ) kg	( ) kg
( ) a	( ) a

Answer \_\_\_\_\_

- 3** A  $1\frac{1}{8}$  m hose weighs  $1\frac{7}{8}$  kg. How much does a  $10\frac{2}{3}$  m hose weigh?



Math sentence

Complete the number line diagram and table.

( ) kg	( ) kg
( ) m	( ) m

Answer \_\_\_\_\_

- 4** Calculate the following multiplication problems. Simplify the answers when possible. Convert improper fractions to mixed numbers.

**1**  $1\frac{3}{4} \times 1\frac{1}{5} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$  or  $\square\frac{\square}{\square}$

**2**  $1\frac{5}{6} \times 2\frac{1}{4} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$  or  $\square\frac{\square}{\square}$

**3**  $4\frac{2}{3} \times 1\frac{6}{7}$

**4**  $2\frac{1}{3} \times 1\frac{2}{5}$

**5**  $2\frac{1}{4} \times 3\frac{1}{6}$

**6**  $2\frac{5}{8} \times 2\frac{2}{9}$

**7**  $1\frac{3}{7} \times 1\frac{13}{15}$

Improper fractions are convenient for calculations. On the other hand, mixed numbers make it easy to imagine the size of numbers.

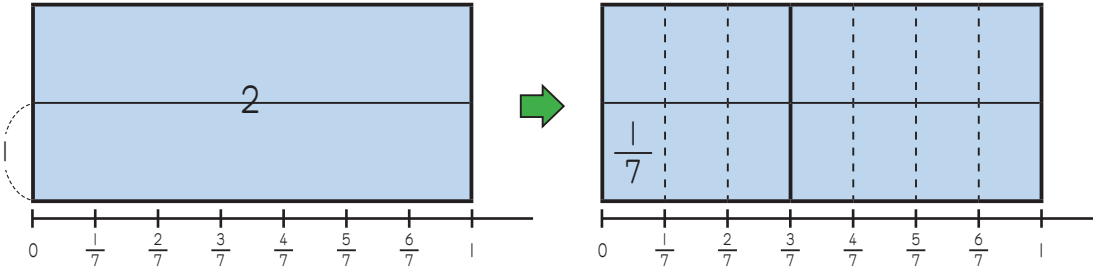


# 4 - 4

## How to Multiply by Fractions

### Multiplying Whole Numbers by Fractions

**Example** Calculate  $2 \times \frac{3}{7}$



$$2 \times \frac{3}{7} = \frac{2}{1} \times \frac{3}{7} = \frac{2 \times 3}{1 \times 7} = \frac{6}{7}$$

When we consider 2 as  $\frac{2}{1}$ , we can use the formula of

$$\frac{b}{a} \times \frac{d}{c} = \frac{b \times d}{a \times c}$$

If we consider it as  $\frac{3}{7} \times 2$ , we can use the following rule, too.

$$\frac{\bullet}{\blacksquare} \times \blacktriangle = \frac{\bullet \times \blacktriangle}{\blacksquare}$$

Calculate the following multiplication problems. Simplify the answers when possible. Leave the answers as improper fractions.

1  $5 \times \frac{3}{8} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$       2  $3 \times \frac{3}{4} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$

3  $8 \times \frac{5}{7}$

4  $6 \times \frac{2}{9}$

5  $3 \times \frac{7}{6}$

6  $12 \times \frac{4}{3}$

7  $\frac{5}{4} \times 6$

8  $\frac{2}{9} \times 15$

9  $3 \times 1\frac{5}{6}$

10  $4 \times 1\frac{1}{12}$

# 4 - 5

## How to Multiply by Fractions

### Multiplying Decimal Numbers and Fractions

**Example** Calculate  $0.3 \times \frac{3}{7}$



$$0.3 \times \frac{3}{7} = \frac{3}{10} \times \frac{3}{7} = \frac{3 \times 3}{10 \times 7} = \frac{9}{70}$$

We have already learned that decimal numbers can be changed into fractions by using denominators of 10, 100, and so on.

When we change 0.3 into  $\frac{3}{10}$ , we can use the formula of  $\frac{b}{a} \times \frac{d}{c} = \frac{b \times d}{a \times c}$

**1** Calculate the following multiplication problems. Simplify the answers when possible. Leave the answers as improper fractions.

**1**  $0.3 \times \frac{1}{7} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$       **2**  $0.9 \times \frac{5}{6} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$

**3**  $0.8 \times \frac{2}{3}$

**4**  $1.6 \times \frac{1}{2}$

**5**  $0.9 \times 2\frac{2}{3}$

**6**  $1.2 \times 1\frac{1}{6}$

**7**  $1\frac{1}{5} \times 1.5$

**8**  $3\frac{1}{8} \times 0.8$

**2** Explain the following calculation errors and calculate them correctly.

**1**  $2.5 \times \frac{3}{5} = \frac{25}{1} \times \frac{3}{5} = \frac{25 \times 3}{1 \times 5} = \frac{15}{1} = 15$

**2**  $0.8 \times \frac{3}{4} = \frac{8}{10} \times \frac{3}{4} = \frac{32 \times 30}{40} = \frac{24}{1} = 24$



# 4 - 6

## How to Multiply by Fractions

### Multiplying More than Two Numbers

**Example 1** Calculate  $\frac{3}{4} \times \frac{2}{5} \times \frac{1}{3}$



$$\frac{3}{4} \times \frac{2}{5} \times \frac{1}{3} = \frac{\cancel{3} \times \cancel{2} \times 1}{\cancel{4}_2 \times 5 \times \cancel{3}_1} = \frac{1}{10}$$

This type of multiplication is an application of the following formula:

$$\frac{b}{a} \times \frac{d}{c} = \frac{b \times d}{a \times c}$$

When we multiply several fractions, multiply all numerators together to equal answer's numerator. Multiply all denominators together to equal the answer's denominators.

**1** Calculate the following multiplication problems. Simplify the answers when possible. Leave the answers as improper fractions.

**1**  $\frac{4}{5} \times \frac{5}{6} \times \frac{2}{3}$

**2**  $\frac{3}{8} \times \frac{3}{4} \times \frac{4}{9}$

**3**  $\frac{7}{8} \times \frac{3}{14} \times \frac{1}{3}$

**4**  $\frac{6}{7} \times \frac{5}{8} \times \frac{7}{15}$

**Example 2** Calculate  $0.2 \times \frac{1}{4} \times 3$



Decimal numbers and whole numbers can be changed into fractions. Then we can calculate this problem.

$$0.2 \times \frac{1}{4} \times 3 = \frac{\cancel{2} \times 1 \times 3}{10 \times 4 \times 1} = \frac{3}{20}$$

Convert decimal numbers and whole numbers into fractions and multiply. Remember, 3 can be written as  $\frac{3}{1}$ .

**2** Calculate the following multiplication problems. Simplify the answers. Leave the answers as improper fractions.

**1**  $1.2 \times \frac{2}{9} \times 6$

**2**  $0.4 \times \frac{1}{6} \times 3$

**3**  $0.3 \times \frac{2}{9} \times 4$

**4**  $8 \times 2 \frac{3}{16} \times 0.6$

# 4 - 7

## How to Multiply by Fractions

### Using the Properties of Operations (1)

**Instruction** For multiplication, there are four important properties of operations:

- ①  $a \times b = b \times a$
- ②  $(a \times b) \times c = a \times (b \times c)$
- ③  $(a + b) \times c = a \times c + b \times c$
- ④  $(a - b) \times c = a \times c - b \times c$

**Example** By using properties of operations, simplify the math sentence and calculate.

$$\left(\frac{7}{8} \times \frac{5}{6}\right) \times \frac{6}{5}$$



Student A

$$\begin{aligned} \left(\frac{7}{8} \times \frac{5}{6}\right) \times \frac{6}{5} &= \frac{7 \times 5}{8 \times 6} \times \frac{6}{5} \\ &= \frac{35}{48} \times \frac{6}{5} = \frac{\overset{7}{\cancel{35}} \times \cancel{6}}{\underset{8}{\cancel{48}} \times \underset{1}{\cancel{5}}} = \boxed{\frac{7}{8}} \end{aligned}$$

He calculated it normally in order from the beginning of the math sentence.



Student B

$$\begin{aligned} \left(\frac{7}{8} \times \frac{5}{6}\right) \times \frac{6}{5} &= \frac{7}{8} \times \left(\frac{5}{6} \times \frac{6}{5}\right) \\ &= \frac{7}{8} \times \frac{\cancel{5} \times \cancel{6}}{\cancel{6} \times \cancel{5}} = \boxed{\frac{7}{8}} \end{aligned}$$

She calculated it by using the properties of operations, especially the above ②.



The method that Student B used is easier to calculate because there are not large numbers like “35” and “45” in the math sentence.



Simplify the following calculations by using the properties of operations and calculate. Simplify the answer.

Think about which fractions should be multiplied first.

1  $\left(\frac{1}{3} \times \frac{5}{7}\right) \times \frac{7}{5}$

2  $\left(\frac{7}{10} \times \frac{3}{4}\right) \times \frac{4}{3}$

3  $\frac{11}{12} \times \left(\frac{1}{5} \times \frac{12}{11}\right)$

4  $\frac{10}{13} \times \left(\frac{2}{7} \times \frac{13}{10}\right)$

4  $\frac{7}{9} \times \left(\frac{3}{4} \times \frac{9}{14}\right)$

5  $\frac{5}{7} \times \left(\frac{9}{11} \times \frac{7}{10}\right)$

# 4 - 8

## How to Multiply by Fractions

### Using the Properties of Operations (2)


**Example** By using the properties of operations, simplify the math sentence and calculate.

$$\frac{3}{4} \times 5 + \frac{3}{4} \times 7$$

 Student A

$$\begin{aligned} \frac{3}{4} \times 5 + \frac{3}{4} \times 7 &= \frac{3}{4} \times (5 + 7) \\ &= \frac{3}{4} \times 12 = \frac{3 \times \overset{3}{\cancel{12}}}{\cancel{4}} = \frac{9}{1} = \boxed{9} \end{aligned}$$

He calculated it by **using the properties of operations, especially ③ in the previous page.**

 Student B

$$\begin{aligned} \frac{3}{4} \times 5 + \frac{3}{4} \times 7 &= \frac{3 \times 5}{4} + \frac{3 \times 7}{4} \\ &= \frac{15}{4} + \frac{21}{4} = \frac{\overset{9}{\cancel{36}}}{\cancel{4}} = \frac{9}{1} = \boxed{9} \end{aligned}$$

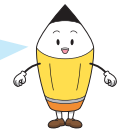
She calculated it normally according to the given math sentence.



The method that Student A used is easier to calculate because there are not large numbers like “21” and “39” in the math sentence.

**1** Simplify the following calculations by using the properties of operations and calculate. Simplify the answer and leave them as improper fractions.

To solve Problems (3), (4), (7) and (8), ④ of the properties of operations can be used.



**1**  $\frac{7}{6} \times 7 + \frac{7}{6} \times 11$

**2**  $\frac{3}{5} \times 7 + \frac{3}{5} \times 8$

**3**  $\frac{4}{9} \times 16 - \frac{4}{9} \times 7$

**4**  $\frac{5}{7} \times 29 - \frac{5}{7} \times 15$

**5**  $\frac{10}{9} \times \frac{2}{5} + \frac{10}{9} \times \frac{1}{5}$

**6**  $\frac{15}{4} \times \frac{5}{7} + \frac{15}{4} \times \frac{3}{7}$

**7**  $\frac{18}{13} \times \frac{20}{23} - \frac{18}{13} \times \frac{7}{23}$

**8**  $\frac{24}{5} \times \frac{9}{11} - \frac{24}{5} \times \frac{4}{11}$

**2** What properties of operations ( from ① to ④ on page 33 ) can be used in order to make the following problems easier to do? Write the numbers ( ① to ④ ) in the  and calculate them by using that property of operations. However, there are some in which the properties of operations cannot be used. In this case, write X in the .

(a)  $\frac{10}{11} \times \frac{7}{8} \times \frac{8}{7}$

(b)  $\frac{12}{13} \times \frac{15}{7} - \frac{5}{13} \times \frac{15}{7}$

(c)  $13 \times \frac{9}{4} + 11 \times \frac{9}{4}$

(d)  $\frac{2}{3} \times \frac{4}{5} - \frac{1}{2} \times \frac{3}{5}$

(e)  $\frac{2}{3} \times \frac{7}{12} \times \frac{12}{7}$

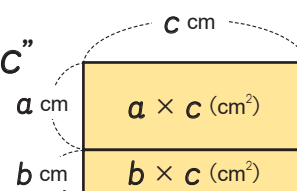
(f)  $\frac{9}{5} \times \frac{9}{8} + \frac{7}{5} \times \frac{9}{8}$

(g)  $\frac{11}{13} \times 25 - \frac{11}{13} \times 12$

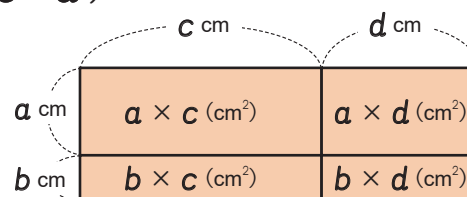
(h)  $\frac{1}{6} \times \frac{5}{7} \times \frac{5}{8}$

**Let's Try!**

This property of operations: “ $(a + b) \times c = a \times c + b \times c$ ” is easy to understand if we think about finding the area of a rectangle showed on the right;



In the future, we will also learn “ $(a + b) \times (c + d)$ .” Let's think about what the math sentence looks like when we expand this formula.



# 4 - 9

## How to Multiply by Fractions

### Reciprocal

**Instruction** The reciprocal of a number is 1 divided by the number.

Think of a reciprocal as the numerator and denominator switching numbers.

$$\frac{b}{a} \rightarrow \frac{a}{b}$$

For example,

The reciprocal of  $\frac{5}{6}$  is  $\frac{6}{5}$ .

The reciprocal of 4 which can be written as  $\frac{4}{1}$  is  $\frac{1}{4}$ .

$$\frac{5}{6} \times \frac{6}{5} = 1$$

When multiplying a number by its reciprocal, the product is always 1.

$$\frac{1}{4} \times 4 = 1$$

**Example** Find the reciprocals of each of the following numbers.

1  $\frac{5}{7}$

2  $0.7 = \frac{7}{10}$



To find the reciprocal of a decimal, convert it to a fraction first. In the above case, 0.7 is converted to  $\frac{7}{10}$  first.

Find the reciprocals of each of the following numbers.

1  $\frac{2}{3}$

2  $\frac{3}{5}$

3  $\frac{1}{7}$

4  $\frac{3}{10}$

5 8

6 12

7 56

8  $0.3 = \frac{3}{10}$

9  $0.57 = \frac{57}{100}$

10  $2.11 = \frac{211}{100}$

Remember, to what fractions are 0.3, 0.57 and 2.11 converted?

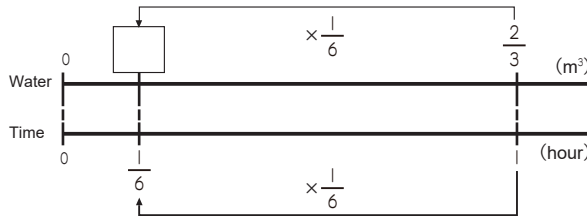
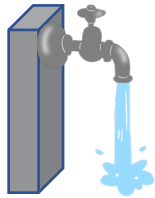


# 4 - 10

## How to Multiply by Fractions

### Various Multiplication Problems

**Example** A faucet gives  $\frac{2}{3}$  m<sup>3</sup> of water in 1 hour. How much water will the faucet give in  $\frac{1}{6}$  of an hour?



<input type="text"/> m <sup>3</sup>	$\frac{2}{3}$ m <sup>3</sup>
$\frac{1}{6}$ of an hour	1 hour

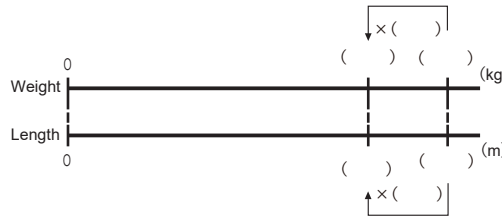
Math sentence

$$\frac{2}{3} \times \frac{1}{6} = \frac{\cancel{2}^1 \times 1}{3 \times \cancel{6}_3} = \frac{1}{9}$$

Answer  $\frac{1}{9}$  m<sup>3</sup>

**1** 1 m of metal pipe weighs  $\frac{3}{5}$  kg. How much does a  $\frac{5}{6}$  m pipe weigh?

Math sentence



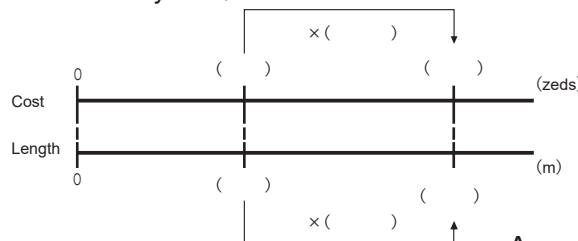
( ) kg	( ) kg
( ) m	( ) m

Complete the number line diagrams and tables.

Answer \_\_\_\_\_

**2** One m of cloth costs 240 zeds\*. How much does  $2\frac{1}{4}$  m of this cloth cost? (\* "zed(s)" is the fictional currency unit.)

Math sentence



( ) zeds	( ) zeds
( ) m	( ) m

Answer \_\_\_\_\_

**3** An outdoor pump gives  $\frac{17}{4}$  L of water per minute. A house faucet gives  $\frac{11}{4}$  L of water per minute. If the pump and faucet are running for 10 minutes, how many L of water will we have in total?

Math sentence



Answer \_\_\_\_\_

# 4 - 11

## How to Multiply by Fractions

### Review

**1** Calculate the following multiplication problems. Leave the answers as improper fractions.

**1**  $\frac{2}{3} \times \frac{2}{5}$

**2**  $\frac{14}{9} \times \frac{12}{7}$

**3**  $1\frac{1}{8} \times 1\frac{1}{6}$

**4**  $2\frac{3}{4} \times 1\frac{3}{11}$

**5**  $0.3 \times 2\frac{2}{9}$

**6**  $1\frac{3}{7} \times 2.1$

**7**  $\frac{5}{8} \times \frac{7}{10} \times \frac{3}{14}$

**8**  $\frac{7}{12} \times \frac{16}{21} \times \frac{9}{10}$

**9**  $6 \times 1\frac{1}{8} \times 1.4$

**10**  $0.4 \times 2\frac{1}{2} \times 1\frac{1}{3}$

**2** Simplify the following calculations by using the properties of operations and calculate. Leave the answer as improper fractions.

**1**  $(\frac{11}{5} \times \frac{7}{9}) \times \frac{12}{7}$

**2**  $\frac{9}{7} \times (\frac{14}{27} \times \frac{4}{5})$

**3**  $\frac{3}{4} \times 5 + \frac{3}{4} \times 7$

**4**  $\frac{13}{8} \times \frac{3}{7} + \frac{13}{8} \times \frac{5}{7}$

**5**  $\frac{3}{7} \times 25 - \frac{3}{7} \times 11$

**6**  $\frac{5}{13} \times \frac{17}{9} - \frac{5}{13} \times \frac{4}{9}$

**3** Find the reciprocals of each of the following numbers.

**1**  $\frac{3}{4}$

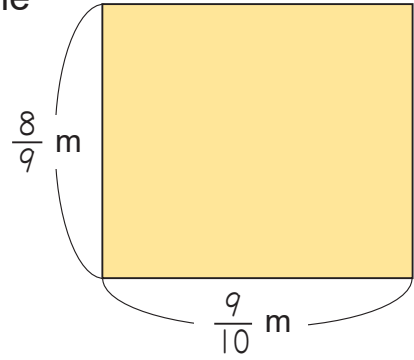
**2** 7

**3** 5.3

**4** 0.61

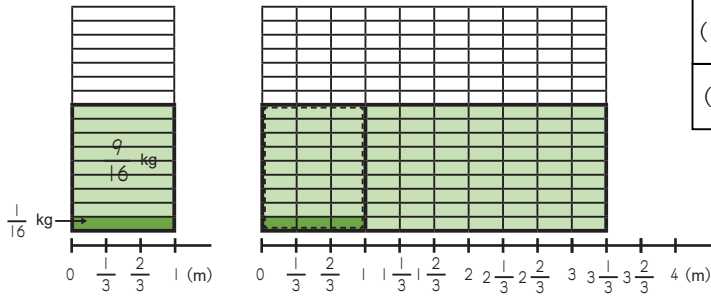
- 4** Find the following area of a rectangle. Write the math sentence and calculate it.

Math sentence

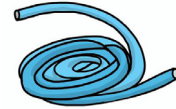


Answer \_\_\_\_\_

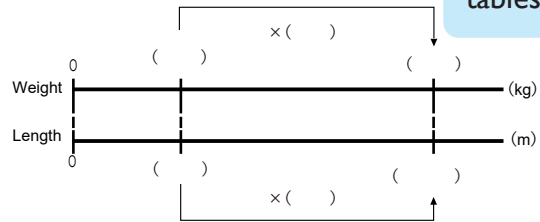
- 5** There is a hose that weighs  $\frac{9}{16}$  kg per metre. How many kg does  $3\frac{1}{3}$  m of this hose weigh?



( ) kg	( ) kg
( ) m	( ) m



Complete the number line diagrams and tables.



Math sentence

Answer \_\_\_\_\_

- 6** A weaving machine can produce  $1\frac{1}{5}$  m of the cloth per minute. Another machine can produce  $\frac{2}{5}$  m of the cloth per minute. If we use both machines, how many m of cloth can we make in 60 minutes?

Math sentence

Answer \_\_\_\_\_

- 7** Calculate every math sentence when the  has the number from 2 to 20. Then answer the following questions.

$$\frac{11}{3} \times \square$$

- 1** List the numbers whose product was a whole number.

Answer \_\_\_\_\_

- 2** What do the numbers in your answer **1** have in common?

Answer \_\_\_\_\_