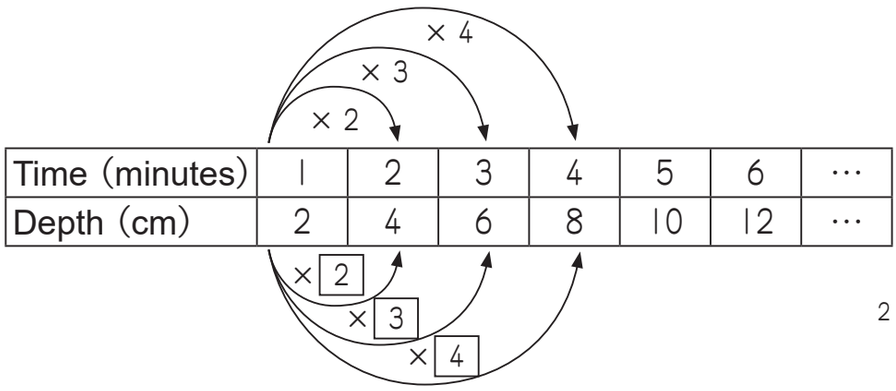
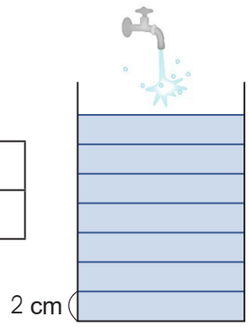
8 - 1 Proportion and Inverse Proportion

Meaning of Proportion

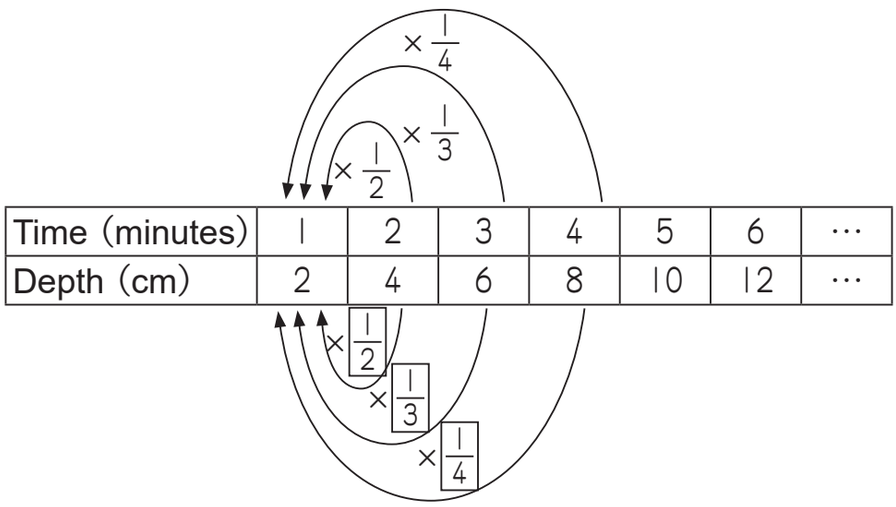
Example 1 The diagram below shows the depth of water in a rectangular shaped tank at one minute intervals.
Let's think about the relationship between the time it takes to fill the tank with water and the depth of the water.

1 If you double, triple, or quadruple the time to fill the water in the tank, how does the depth of the water change accordingly?





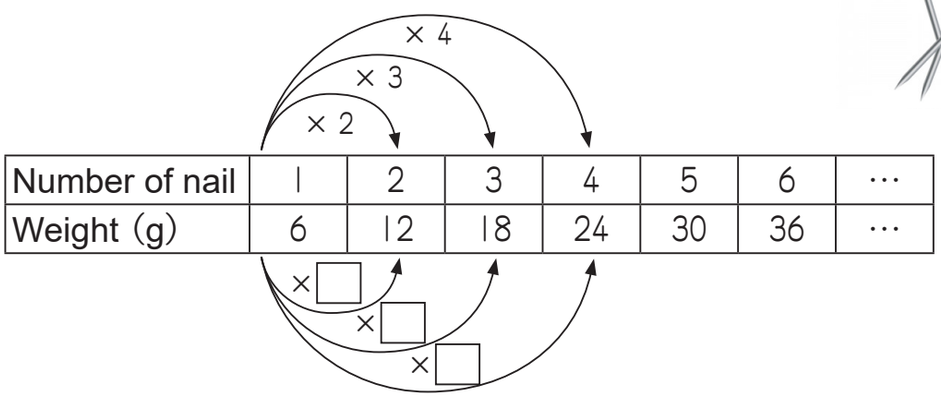
2 If you one-half, one-third, or one-fourth the time to fill the water in the tank, how does the depth of the water change accordingly?



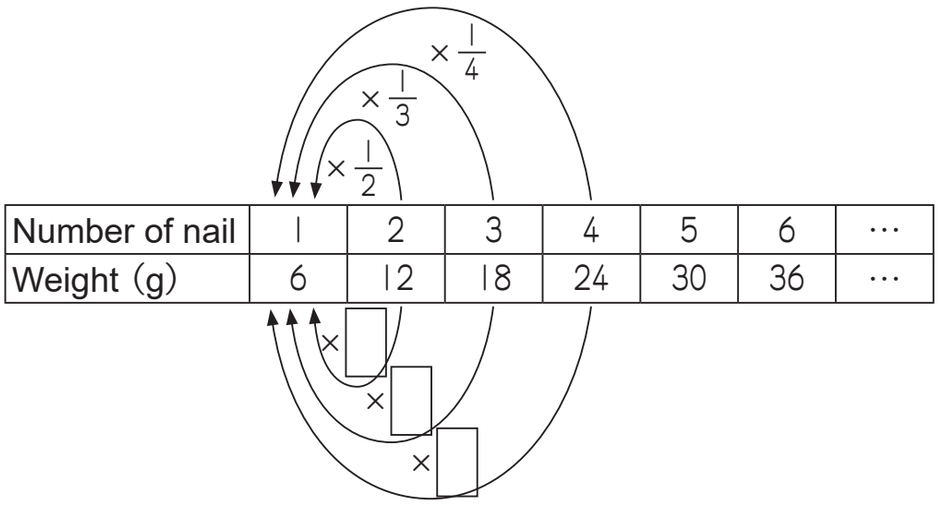
If we have two numbers x and y, and If x is doubled, tripled or quadrupled, or 1/2, 1/3 or 1/4 times as many, and y is also doubled, tripled or quadrupled, or 1/2, 1/3 or 1/4 as many, we say that x and y are proportional. In other words, we say that "**y is proportional to x**".

The table below shows the number of nails and their weights. Each nail weighs 6 g.

1 If the number of nails doubles, triples or quadruples, how does the weight change? Fill in the □ .



2 If you make the number of nails one-half, one-third, or one-fourth, how does the weight change? Fill in the □ .



3 What is the relationship between the number of nails and their weight?

Answer _____

8 - 2 Proportion and Inverse Proportion

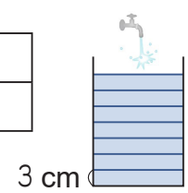
Expressing Proportion with a Math Sentence

Example The table below shows the depth of water in a rectangular shaped tank at one minute intervals.

Time x (minutes)	1	2	3	4	5	6	…
Depth y (cm)	3	6	9	12	15	18	…

1 Find out how many values of y increase as the value of x increases by one.

		+1		+1		+1		
x	1	2	3	4	5	6	…	
y	3	6	9	12	15	18	…	
		+3		+3		+3		



2 What is the relationship between the value for time x and the corresponding value for the depth y of the water?

Answer y is proportional to x

3 Using the fact that the depth of water per 1 minute is 3 cm, investigate the relationship between the amount of water and its depth, and express the relationship between x and y in an math sentence.

Depth of water y (cm)		Depth of water per minute (cm)		time x (minute)
3	←	3	×	1
6	←	3	×	2
9	←	3	×	3
12	←	3	×	4
y	←	3	×	x

Answer $y = 3 \times x$

If we have two quantities x and y, and y is proportional to x, we can express this relationship by the following math sentence

y = **Fix number** × x

4 What is the depth of the water if it is filled for 15 minutes?

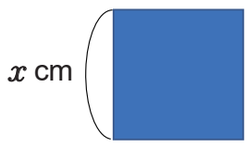
Math sentence $y = 3 \times 15 = 45$ Answer 45 cm

5 how many minutes will it take for the depth of the water to reach 60 cm?

Math sentence $60 = 3 \times x$

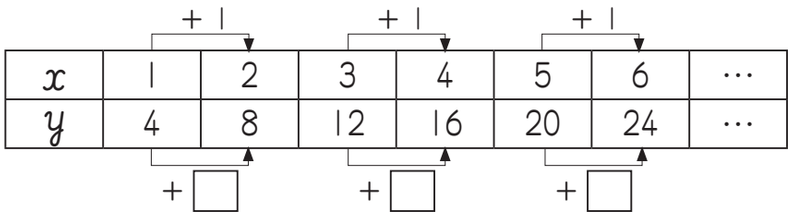
$x = 60 \div 3 = 20$ Answer 20 minutes

For the square on the right, here is a table showing the relationship between the length of one side, x cm, and the length of the perimeter.



Length of 1 side x (cm)	1	2	3	4	5	6	…
Perimeter y (cm)	4	8	12	16	20	24	…

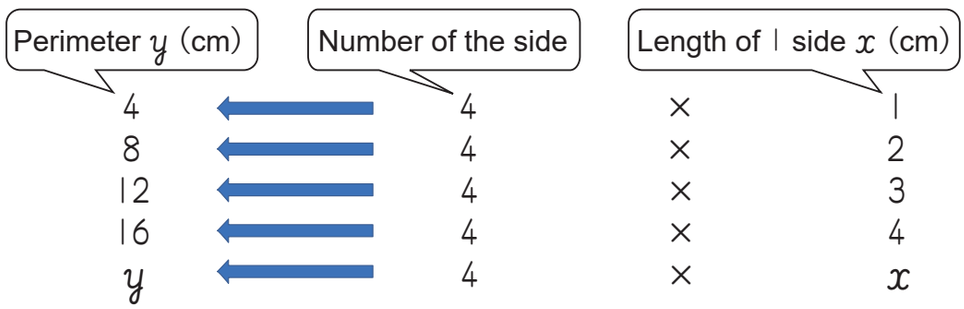
1 Find out how many values of y increase as the value of x increases by one.



2 What is the relationship between the value for the length of 1 side x and the corresponding value for the perimeter y?

Answer _____

3 Using the fact that number of the side is 4 cm, investigate the relationship between the length of 1 side and its perimeter, and express the relationship between x and y in an math sentence.



Answer _____

4 If the value of x is 8, what is the value of y?

Math sentence

Answer _____

5 What is the perimeter if the length of 1 side is 12 cm?

Math sentence

Answer _____

6 How long the length of 1 side is if the perimeter is 60 cm?

Math sentence

Answer _____

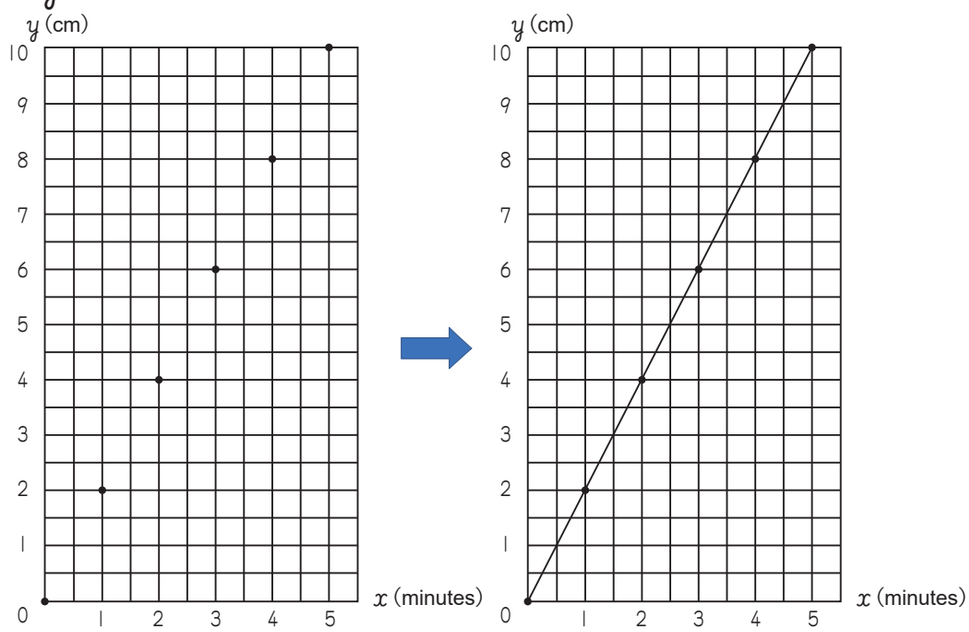
8 – 3 Proportion and Inverse Proportion

Drawing a Graph of Proportion

Example The table below shows the depth of water in a rectangular shaped tank at one minute intervals.

Time x (minutes)	0	1	2	3	4	5	…
Depth y (cm)	0	2	4	6	8	10	…

1 For a time of x minutes and a corresponding depth of water of y cm, draw a point on the graph below that represents a pair of values of x and y.



2 What is the relationship of x (minutes) and y (depth)?

Answer y is proportional to x

3 Using the fact that depth of water per minute is 2 cm, investigate the relationship between time and depth of water, and express the relationship between x and y in an math sentence.

Depth of water y (cm)	Depth of water per minute (cm)		Time x (minutes)
2	2	×	1
4	2	×	2
8	2	×	3
y	2	×	x

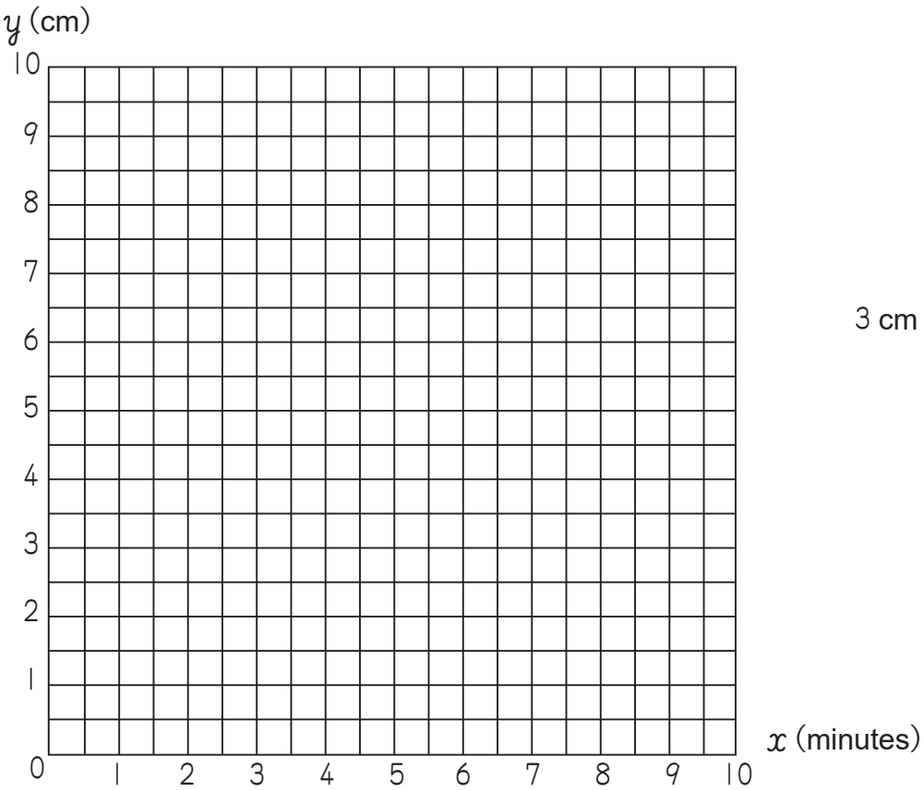
Answer $y = 2 \times x$

If we connect the points representing the corresponding pairs of x and y values with a line, we get a line like the one above. This line is the graph of $y = 2 \times x$.

The table below shows the depth of water in a rectangular shaped tank at one minute intervals.

Time x (minutes)	0	1	2	3	4	5	⋯
Depth y (cm)	0	3	6	9	12	15	⋯

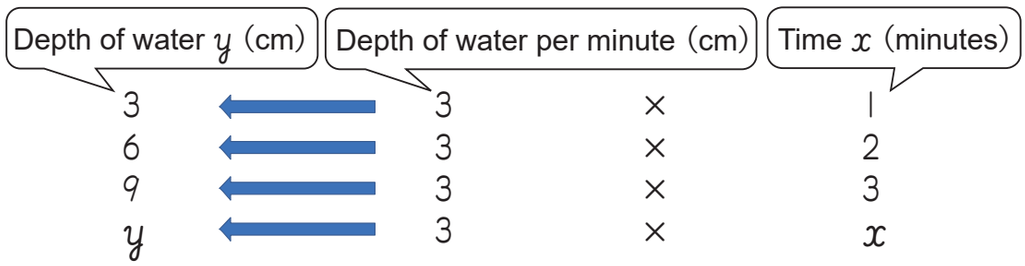
1 For a time of x minutes and a corresponding depth of water of y cm, draw a point on the graph below that represents a pair of values of x and y.



2 What is the relationship between x (minutes) and y (depth)?

Answer _____

3 Using the fact that depth of water per minute is 3 cm, investigate the relationship between time and depth of water, and express the relationship between x and y in a math sentence.



Answer _____

4 Write the graph represent the relationship between x and y.

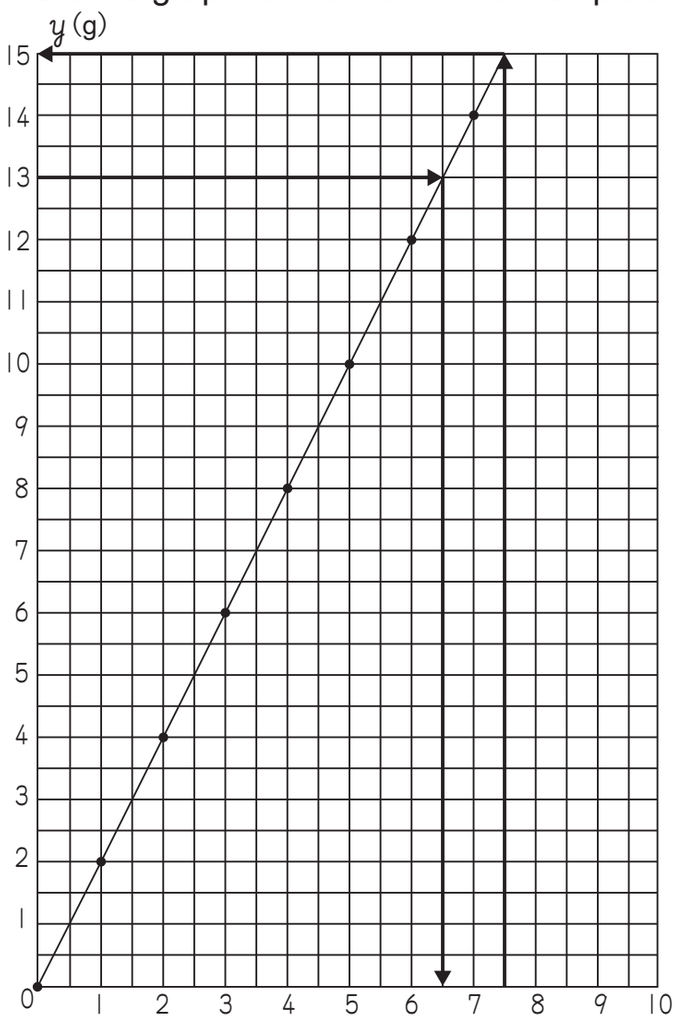
8 - 4 Proportion and Inverse Proportion

Reading a Graph of Proportion (1)

Example The table below shows the relationship between the length of the wire x cm and the weight y g.

Length x (cm)	0	1	2	3	4	5	6	7	…
Weight y (g)	0	2	4	6	8	10	12	14	…

1 Draw a graph to show the relationship between x and y.



You can find the answer of ③, ④ by using the graph!

2 Using the fact that weight of wire per cm is 2 g, investigate the relationship between length of wire and its weight, and express the relationship between x and y in an math sentence

Answer $y = 2 \times x$

3 When a wire is 7.5 cm long, how many g does it weigh?

Answer 15 g

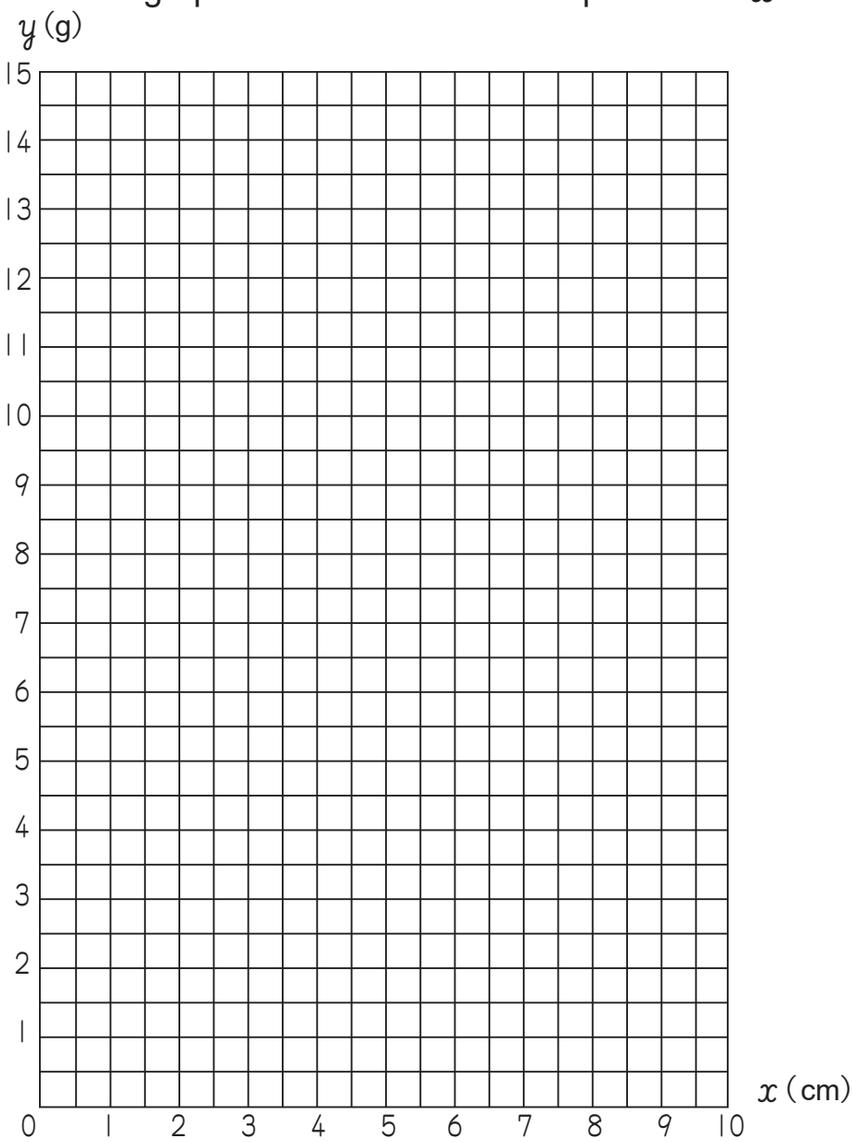
4 How many cm of wire will it take if it weighs 13 g?

Answer 6.5 cm

The table below shows the relationship between the length of the wire x cm and the weight y g.

Length x (cm)	0	1	2	3	4	5	6	⋯
Weight y (g)	0	2	4	6	8	10	12	⋯

1 Draw a graph to show the relationship between x and y.



2 Using the fact that weight of wire per cm is 2 g, investigate the investigate the relationship between length of the wire and weight and express relationship between x and y in an math sentence.

Answer _____

3 When a wire is 5.5 cm long, how many g does it weigh?

Answer _____

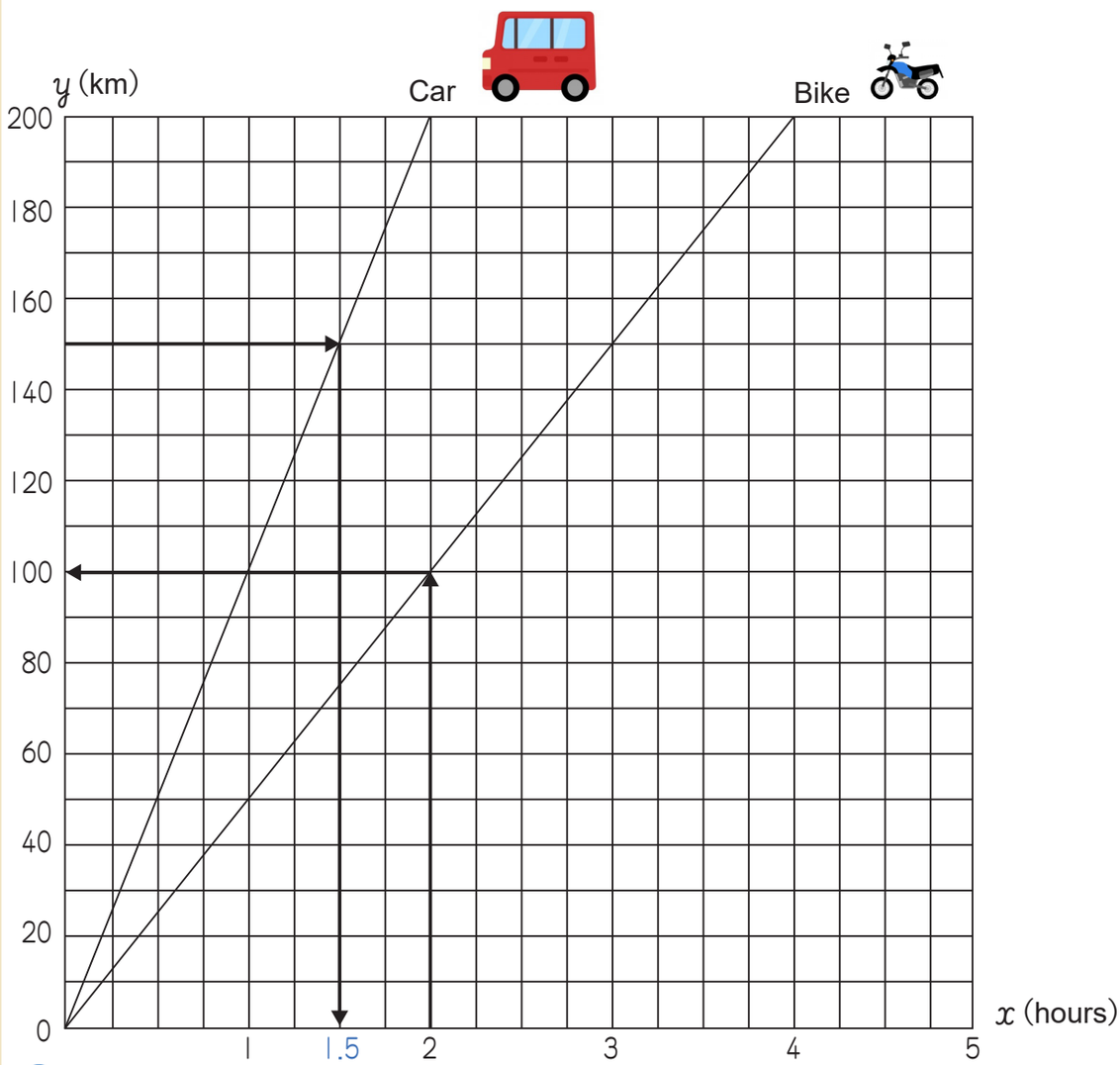
4 How many cm of wire weighs 14 g?

Answer _____

8 - 5 Proportion and Inverse Proportion

Reading a Graphs of Proportion (2)

Example The graph below shows the relationship between the time (x hours) and the journey (y km) since the bike and the car set off at the same time. Read the graph.



1. How many km does a bike go in two hours?

Answer 100 km

2. How many hours does it take for a car to travel 150 km?

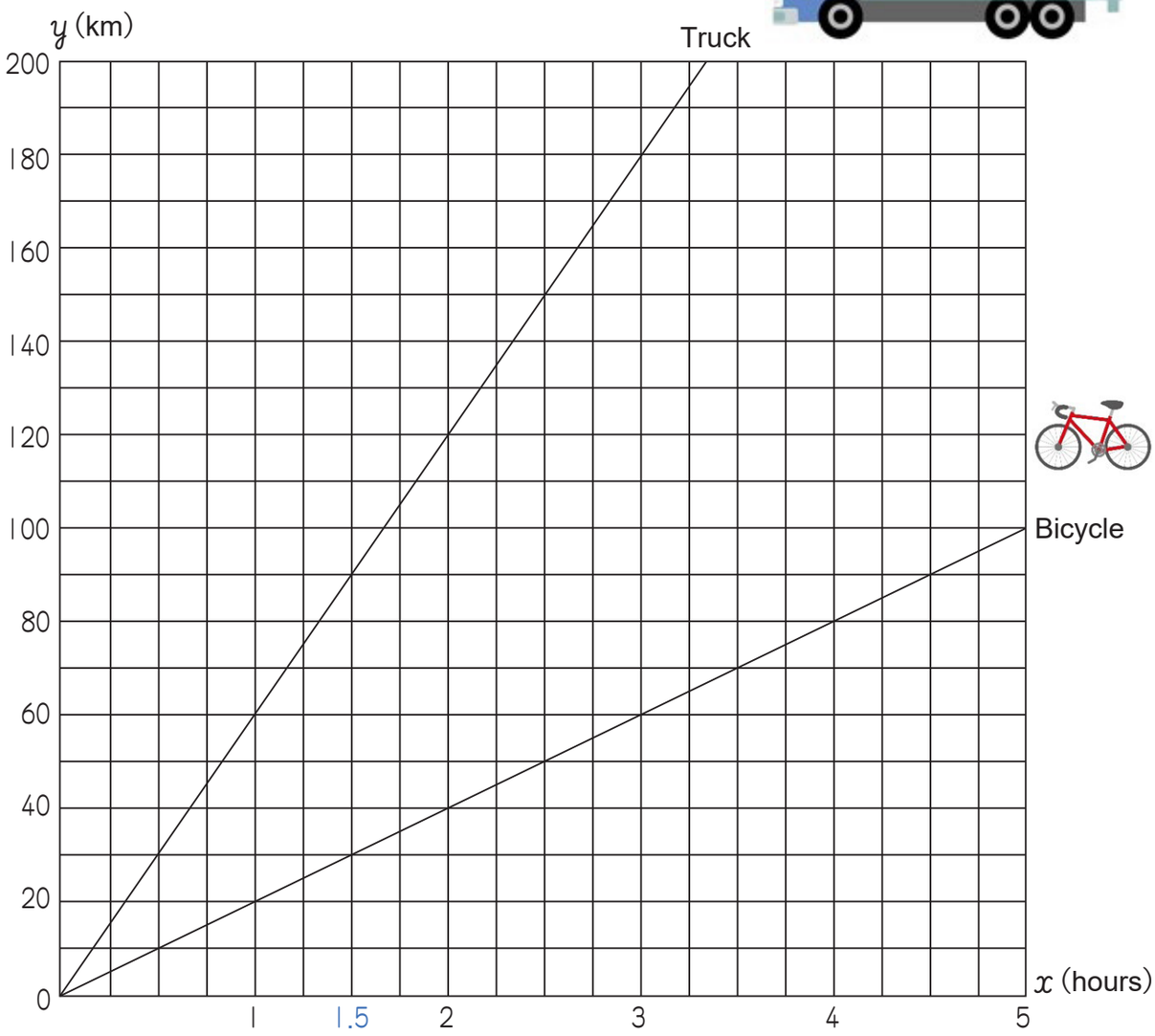
Answer 1.5 hours

3. Looking at the graph, what are the speeds of the bike and the car?

Speed is km per one hour, so…

Answer Car 100 km/hours Bike 50 km/hours

The graph below shows the relationship between the time (x hours) and the journey (y km) since the bicycle and the truck set off at the same time. Read the graph.



1 How many km does a bicycle go in two hours?

Answer _____

2 How many hours does it take for a truck to travel 150 km?

Answer _____

3 Looking at the graph, what are the speeds of the truck and the car?

Answer Truck _____ Bicycle _____

8 - 6 Proportion and Inverse Proportion

Review (1)

1 The table below shows the relationship between the price of the wire x cm and the price y zeds.

Length x (x cm)	0	1	2	3	4	5	6	…
Price y (y zeds)	0	30	60	90	120	150	180	…

1 What is the relationship between the length (x cm) and the price (y zeds)?

Answer _____

2 How can you express the relationship between x and y in the math sentence if the length of wire is x cm and the price of wire is y zeds?

Answer _____

3 When the value of x is 8, what is the value of y?

Math sentence

Answer _____

4 What is the price of an 8 cm wire?

Math sentence

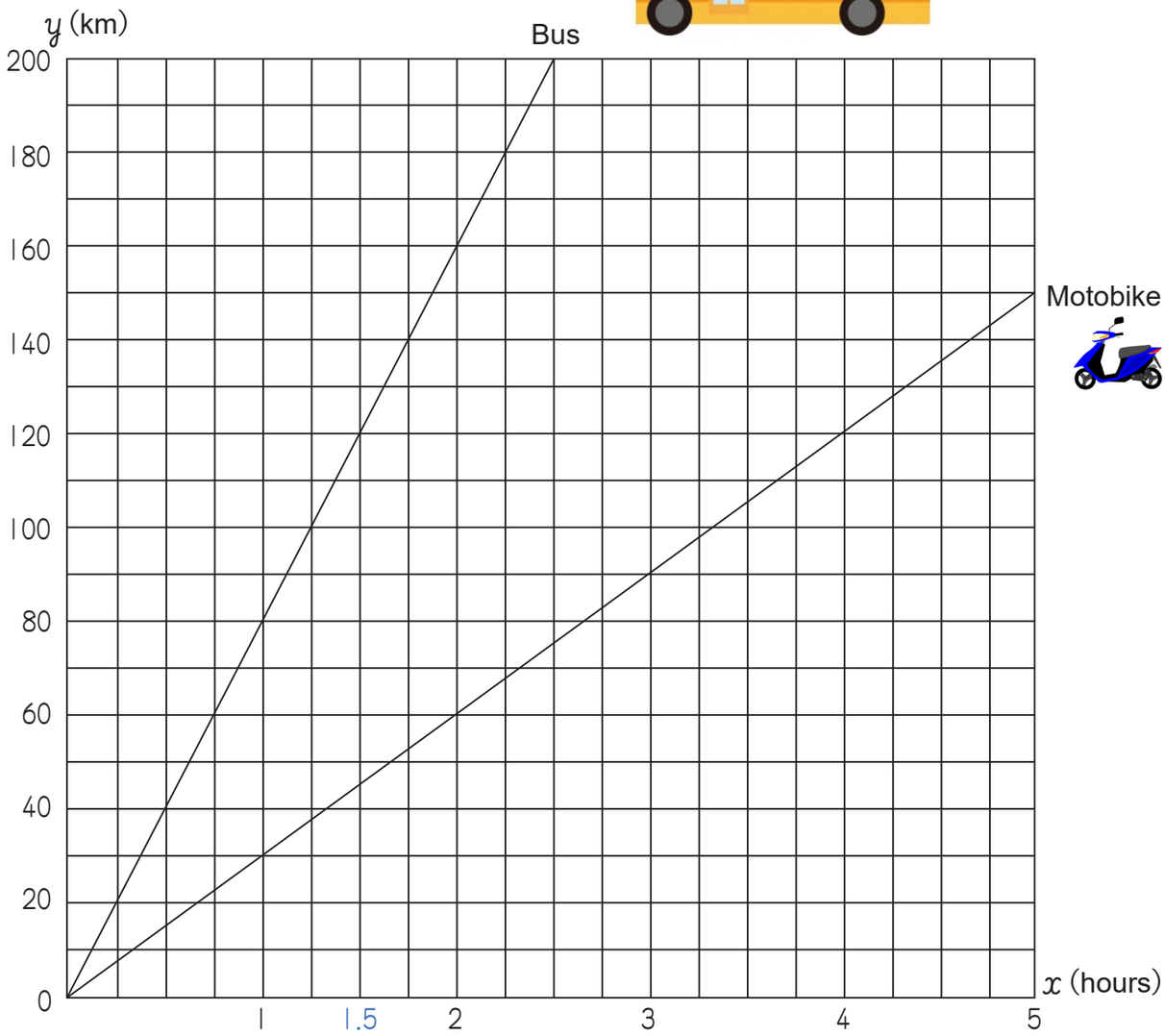
Answer _____

5 How long is a wire whose price is 360 zeds?

Math sentence

Answer _____

2 The graph below shows the relationship between the time (x hours) and the journey (y km) since the bus and the motorbike set off at the same time. Read the graph.



1 How many km does a motorbike go in three hours?

Answer _____

2 How many hours does it take for a bus to travel 120 km?

Answer _____

3 Looking at the graph, what are the speeds of the bus and the motorbike?

Answer

Bus _____

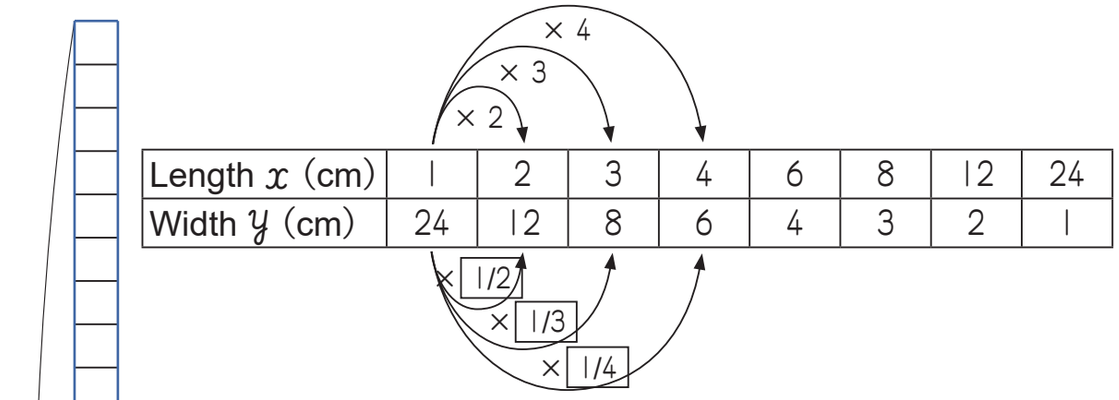
Motorbike _____

8 – 7 Proportion and Inverse Proportion

Expressing Inverse Proportion with a Math Sentence

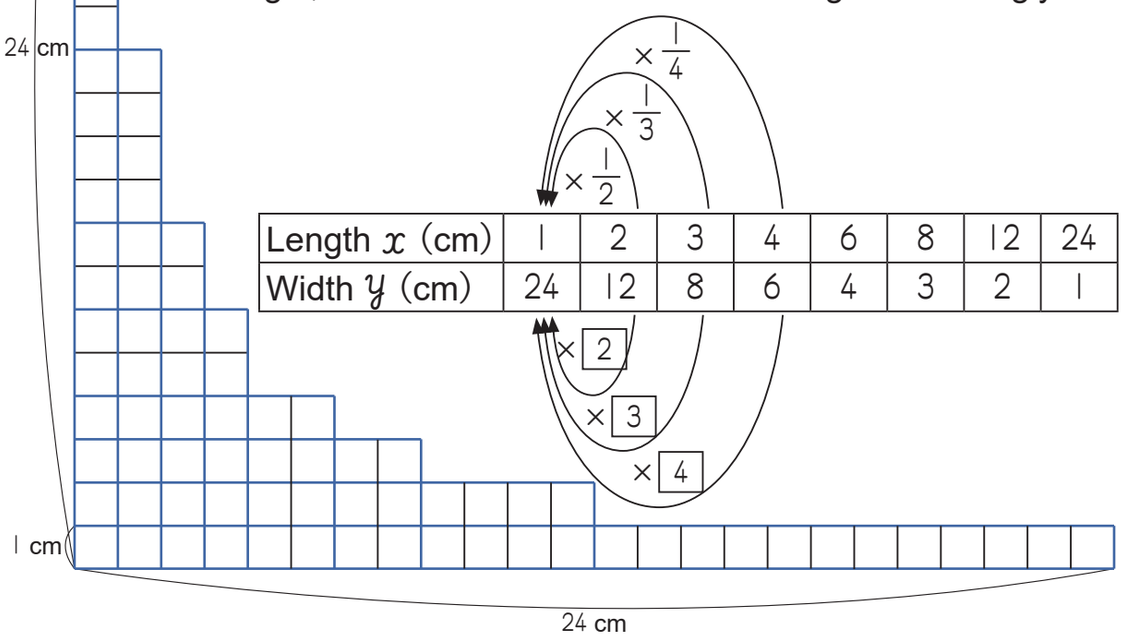
Example The table summarizes the relationship between the length and width of a rectangle with an area of 24 cm^2.
Let's think about the relationship between the length and the width of the rectangle.

1 If you double, triple, or quadruple the length of the rectangle, how does the width of the rectangle change accordingly?



Length x (cm)	1	2	3	4	6	8	12	24
Width y (cm)	24	12	8	6	4	3	2	1

2 If you one-half, one-third, or one-fourth the length of the rectangle, how does the width of the rectangle accordingly?

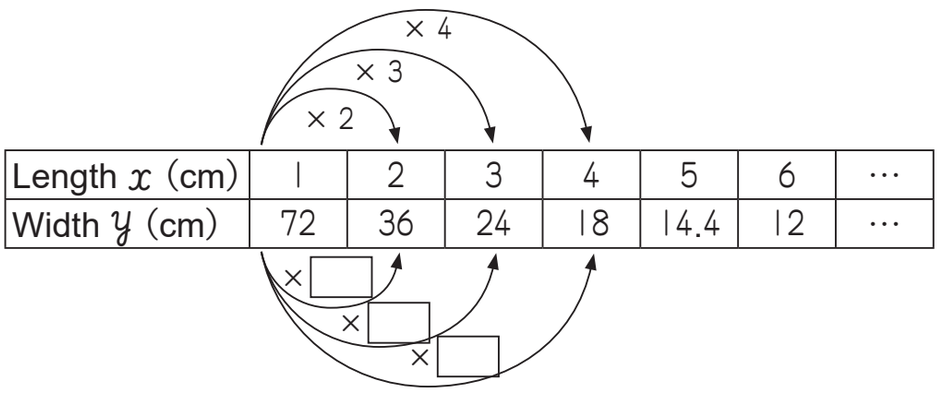


Length x (cm)	1	2	3	4	6	8	12	24
Width y (cm)	24	12	8	6	4	3	2	1

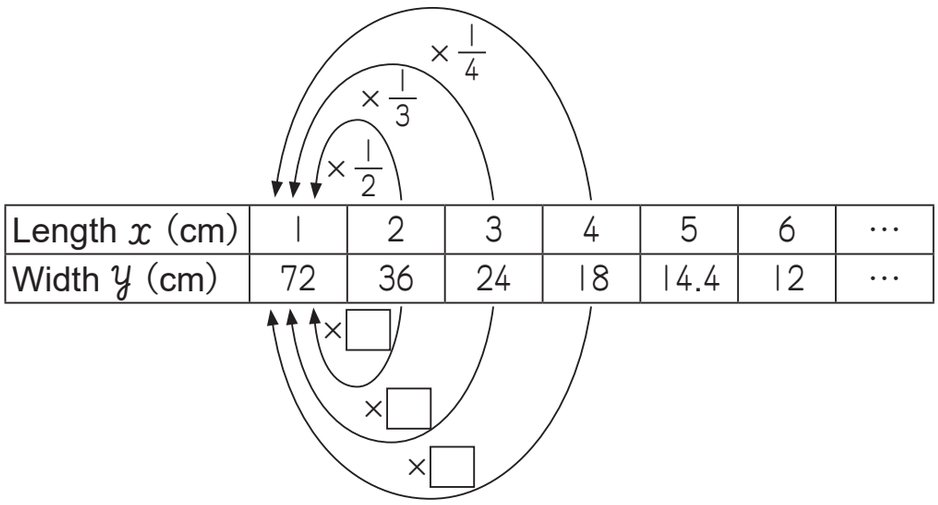
If we have two numbers x and y, and If x is doubled, tripled or quadrupled, y is 1/2, 1/3 or 1/4 as many, we say that x and y are inversely proportional. In other words, "**y is inversely proportional to x**"

The table summarizes the relationship between length and width of the rectangle with 72 cm^2.
Let's think about the relationship between the length and the width of the rectangle.

1 If you double, triple, or quadruple the length of the rectangle, how does the width of the rectangle change accordingly?



2 If you make the number of nails one-half, one-third, or one-fourth, how does the weight change? Fill in the □ .



3 What is the relationship between x and y?

Answer _____

8 – 8 Proportion and Inverse Proportion

Math Sentence of Inverse Proportion

Example The table summarizes the relationship between length and width of the rectangle with 24 cm^2.

Length x (cm)	1	2	3	4	5	6	…
Width y (cm)	24	12	8	6	4.8	4	…

1 Find out how many values of y increase as the value of x increases by one.

	× 2		× 3				
Length x (cm)	1	2	3	4	5	6	…
Width y (cm)	24	12	8	6	4.8	4	…
	÷ 2		÷ 3				

2 What is the relationship between x and y is?

Answer y is inversely proportional to x

3 Using the fact that the area of rectangle is 24 m^2, investigate the relationship between the length and width, and express the relationship between x and y in an math sentence.

Length x (cm)		Width y (cm)		Area (m^2)
1	×	24	=	24
2	×	12	=	24
3	×	8	=	24
x	×	y	=	24

If there are two quantities x and y, and y is inversely proportional to x, then this relationship can be expressed by the math sentence.

"$x \times y$ = Fixed number"

4 If the value of x is 8, what is the value for y?

Math sentence $8 \times y = 24$

$y = 24 \div 8$

Answer $y = 6$

5 If the width of rectangle is 12 cm, what is the length of the rectangle?

Math sentence $12 \times y = 24$

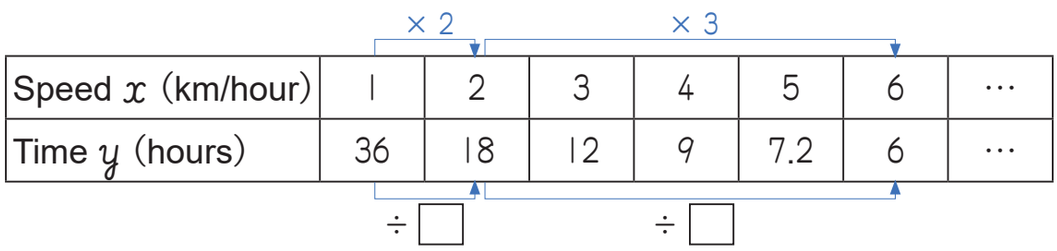
$y = 24 \div 12$

Answer 2 cm

The table summarizes the relationship between speed of walking and time for a 36 km walk.

Speed x (km/hour)	1	2	3	4	5	6	…
Time y (hours)	36	18	12	9	7.2	6	…

1 Find out how many values of y increase as the value for x increases by one.



2 What is the relationship between x and y is?

Answer _____

3 Using the fact that the distance is 36 m, investigate the relationship between the speed and time, and express the relationship between x and y in a math sentence?

Speed x (km/hour)		Time y (hours)		distance (km)
1	×	36	=	36
2	×	18	=	36
3	×	12	=	36
x	×	y	=	36

Answer _____

4 If the value of x is 9, what is the value for y?

Math sentence

Answer _____

5 If the time is 8 hours, what is the speed?

Math sentence

Answer _____

8 – 9 Proportion and Inverse Proportion

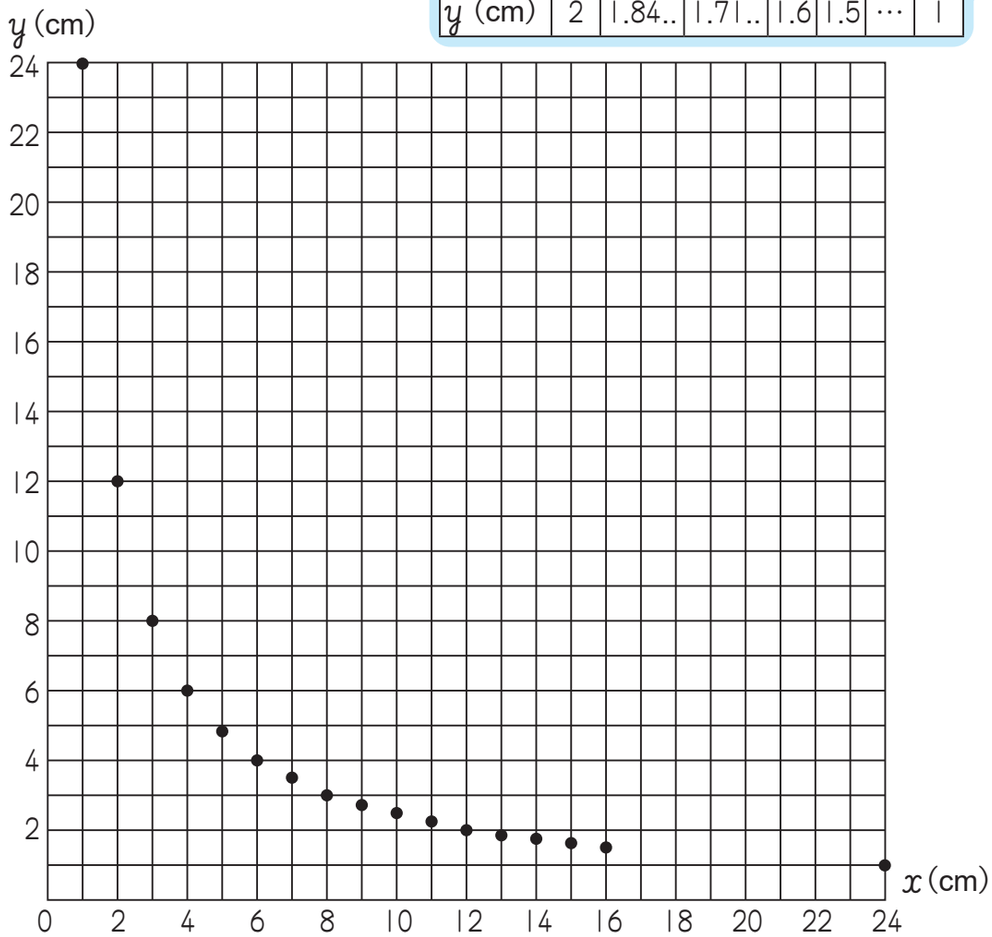
Drawing a Graph of Inverse Proportion (1)

Example The table summarizes the relationship between length and width of the rectangle with 24 cm^2.

Length x (cm)	1	2	3	4	5	6	7	8	9	10	11
Width y (cm)	24	12	8	6	4.8	4	3.42..	3	2.66..	2.4	2.18..

1 For a length of x cm and a corresponding width of y cm, find and draw a point on the graph below that represents a pair of values for x and y.

x (cm)	12	13	14	15	16	⋯	24
y (cm)	2	1.84..	1.71..	1.6	1.5	⋯	1



2 What is the relationship the length (x cm) and the width (y cm)?

Answer y is inversely proportional to x

3 How can you express the relationship between x and y in the math sentence if the length is x cm and the width is y cm?

$x \times y$ = fix number. So y = fix number $\div x$

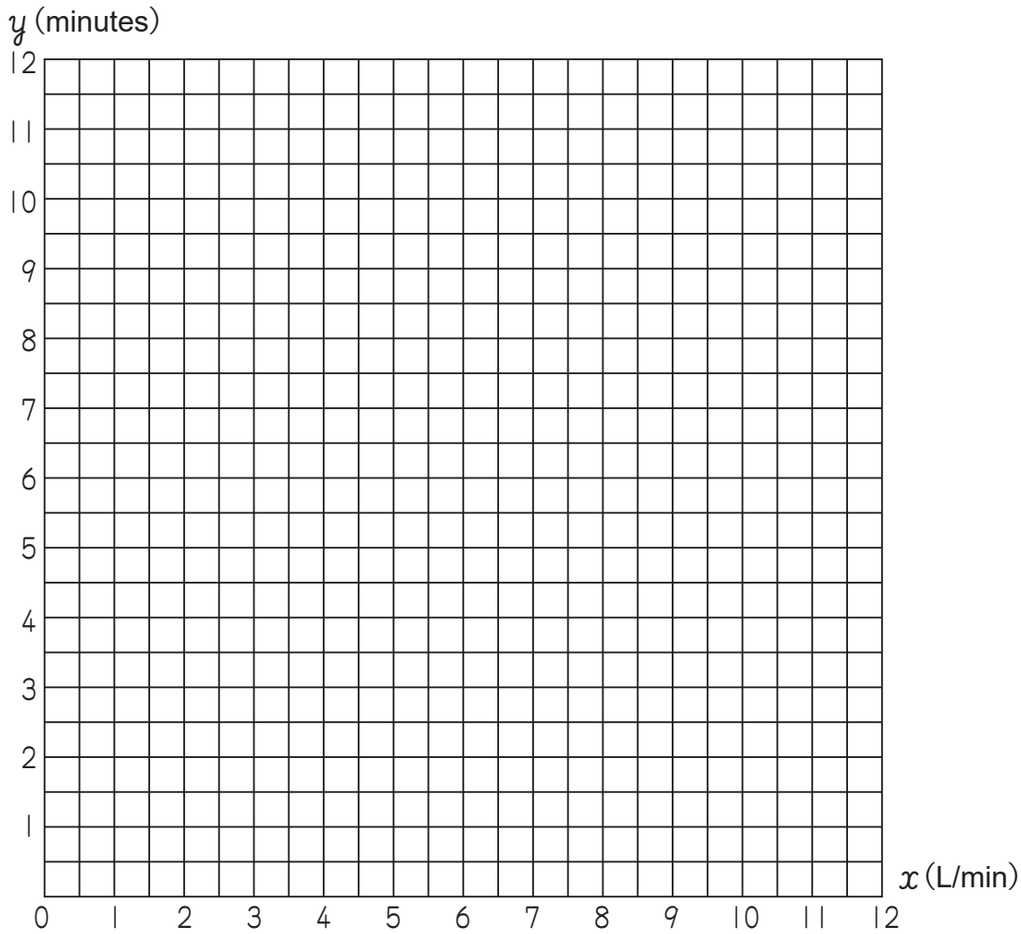
Answer $x \times y = 24$

There is a tank that holds 12 L of water. Find the relationship between the volume of water you fill the tank per minute and the time it takes for the tank to fill up in the table below.

Water x (L/min)	1	2	3	4	5	6	7	8	9
Time y (minutes)	12	6	4	3	2.4	2	1.71..	1.5	1.33..

x (L/min)	10	11	12
y (minutes)	1.2	1.09..	1

1 For a water of x L/min and a corresponding time of y minutes, draw a point on the graph below that represents a pair of values for x and y.



2 What the relationship between water (x L/min) and time (y minutes)?

Answer _____

3 How can you express the relationship between x and y in the math sentence if the time to fill the water is x L/min and the time is y minutes?

Answer _____

8 – 10 Proportion and Inverse Proportion

Drawing a Graph of Inverse Proportion (2)

Example The table summarizes the relationship between speed and the time it takes a car to travel between villages A and B.

1 How can you express the relationship between x and y in the math sentence if the speed is x km/h and the time is y hours?

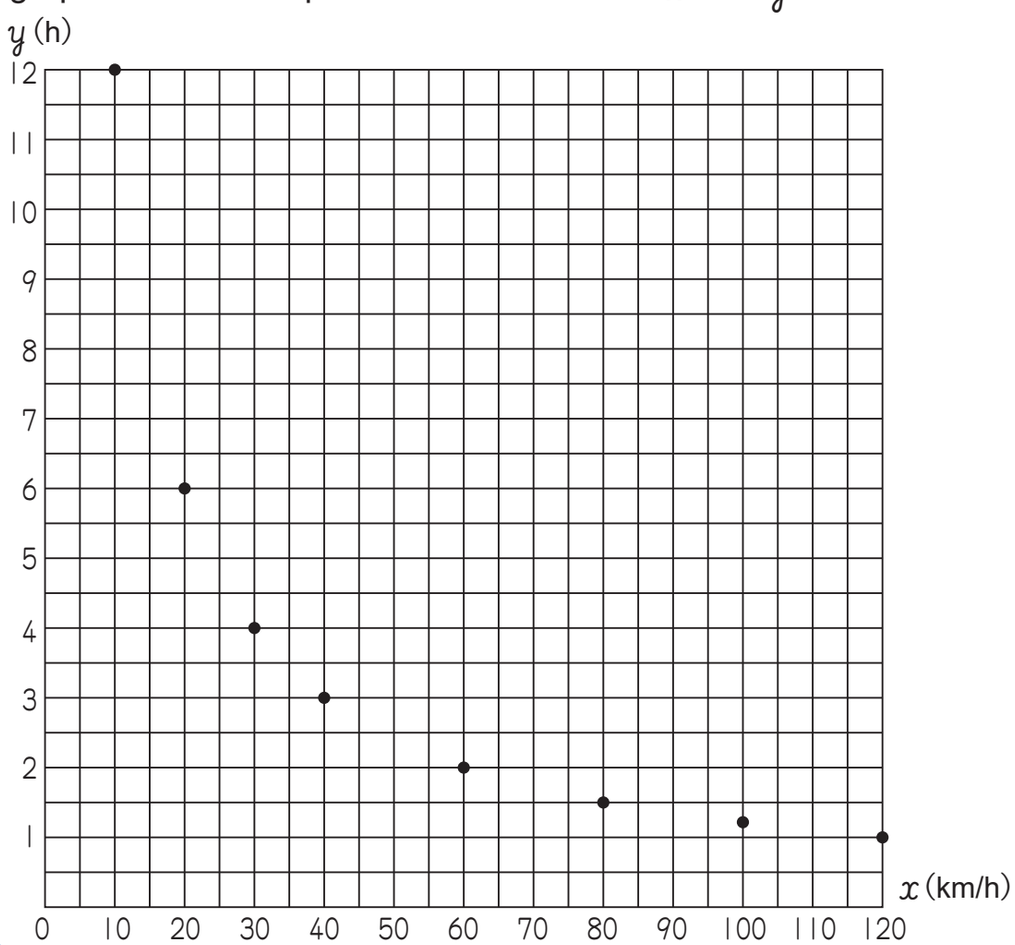
Answer $x \times y = 120$

$x \times y$ = fix number. It can also be represented as y = fix number $\div x$

2 Fill in the table.

Speed x (km/h)	10	20	30	40	60	80	100	120
Time y (hours)	12	6	4	3	2	1.5	1.2	1

3 For speed of x and a corresponding time of y draw a point on the graph below that represents the values for x and y.



4 Fill in the table, if x is 15, 25, 75.

Speed x (km/h)	15	25	75
Time y (hours)	8	4.8	1.6

There is a job that will take 60 days if one person does the same amount of work per day. It will take y days if x people do this job. Let's consider the relationship between x and y.

1 How can you express the relationship between x and y in the math sentence if the number of people is x and the number of days?

Answer _____

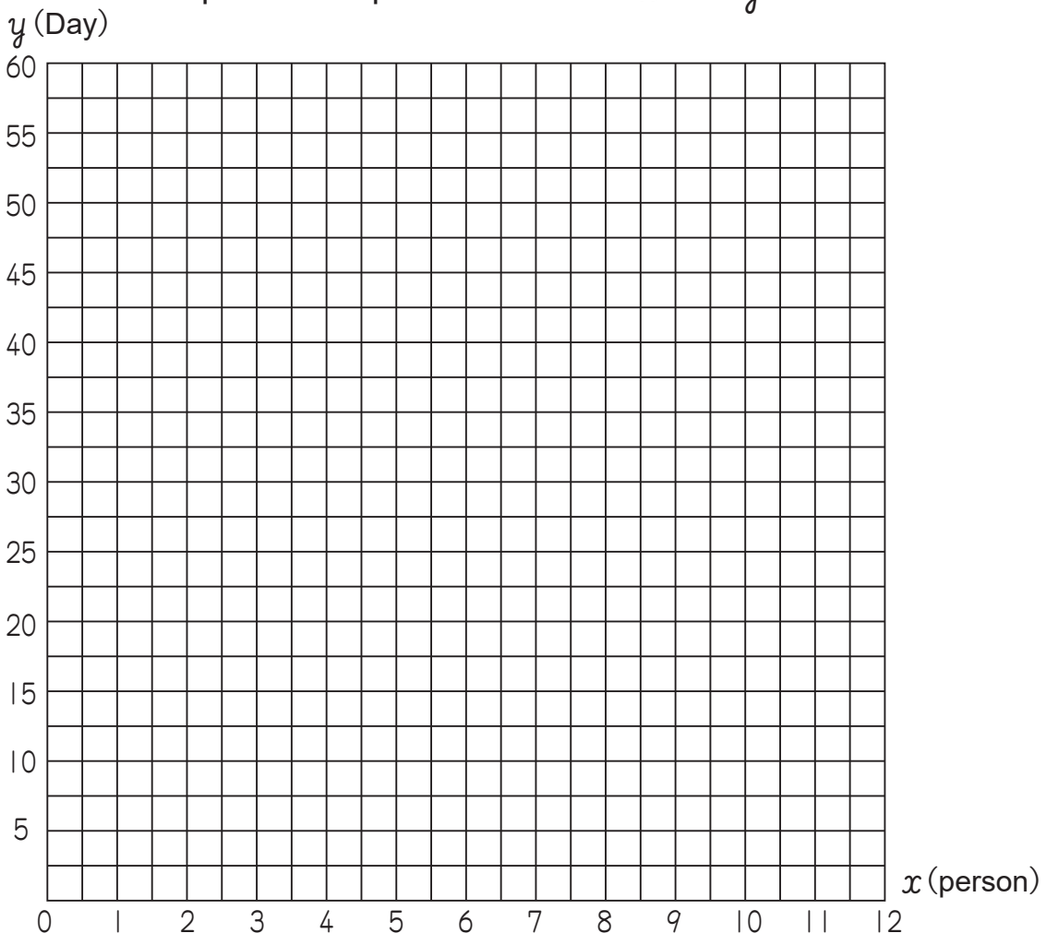
$x \times y =$ fix number. It can also be represented as $y =$ fix number $\div x$

2 Fill in the table.

People x (person)	1	2	3	4	5	6
Days y (Day)	60					

x (person)	8	10	12
y (Day)			

3 For x people and a corresponding y day, draw a point on the graph below that represents a pair of values for x and y.



4 Fill in the table, if x is 1.5, 2.5, 7.5.

People x (person)	1.5	2.5	7.5
Days y (Day)			

8 - 11 Proportion and Inverse Proportion

Review (2)

1 The table below shows the relationship between the base and the height of a parallelogram with an area of 30 cm^2.

1 Fill in the table.

Base x (cm)	1	2	3	4	5	6	…
Height y (cm)	30	15	10				…

2 What is the relationship between the value for base and the corresponding value for height?

Base x (cm)	1	2	3	4
	1 × — = □	2 × — = □	3 × — = □	4 × — = □
Height y (cm)	30	15	10	7.5

3 What is the relationship between the base of the parallelogram (x cm) and the height of the parallelogram (y cm)?

Answer _____

4 How can you express the relationship between x and y in the math sentence if the base of the parallelogram is x cm and the height of the parallelogram is y cm?

Answer _____

5 If the value of x is 10, what is the value of y?

Math sentence

Answer _____

6 If the height of the parallelogram is 8 cm, what is the base of the parallelogram?

Math sentence

Answer _____

2 The table below shows the relationship between the amount of water x L per minute and the time y minutes it takes to fill a water bottle with 18 L of water.

1 How can you express the relationship between x and y in the math sentence if the amount of water x L per minutes and the time is y minutes?

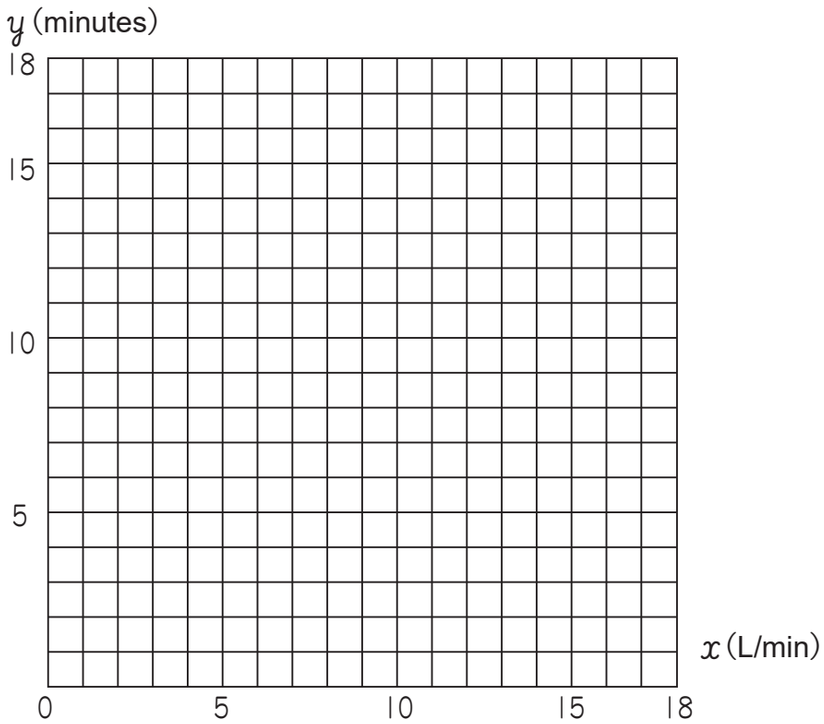
Answer _____

2 Fill in the table.

Water x (L/min)	1	2	3	4	5	6	9
Time y (minutes)	18						

x (L/min)	10	12	15	18
y (minutes)				

3 For x L/min and a corresponding y minutes, draw a point on the graph below that represents a pair of values of x and y.



4 Fill in the table, if x is 1.5, 2.5, 7.5.

Water x (L/min)	1.5	2.5	4.5	7.5
Time y (minutes)				

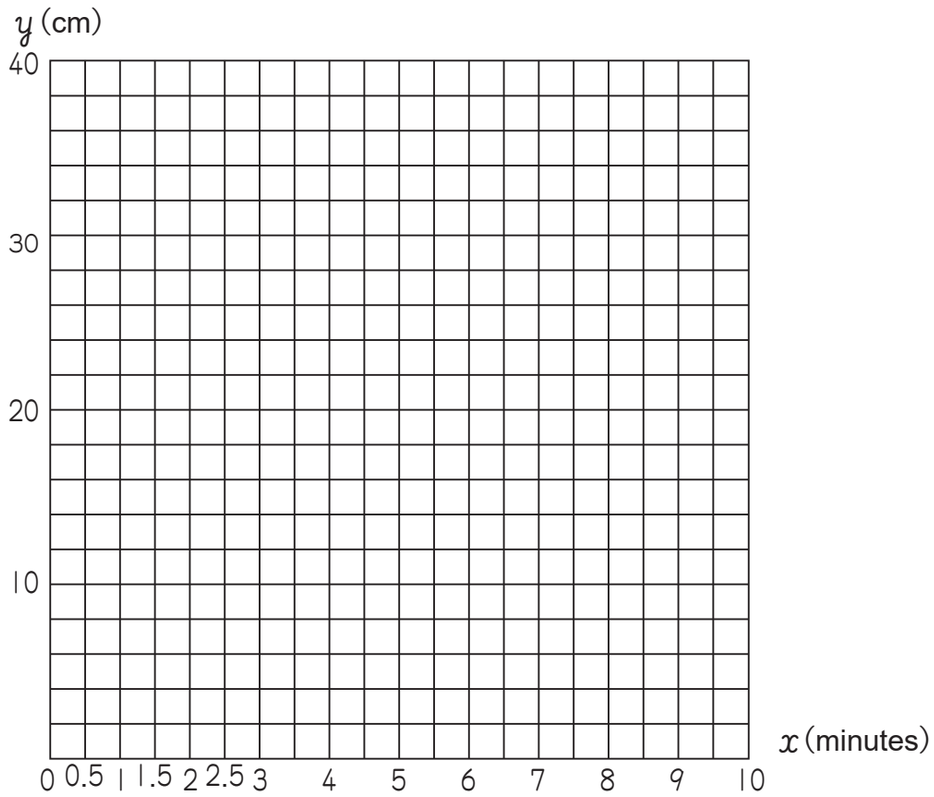
8 - 12 Proportion and Inverse Proportion

Review (3)

1 The table below shows the depth of water in a rectangular shaped tank at one minutes intervals.

Time (minutes)	1	2	3	4	5	6	7	…
Depth (cm)	4	8	12					…

1 Draw a graph to show the relationship between x and y.



2 Using the math sentence $y = 4 \times x$ to show the relationship between x and y, find the value of y when the value of x is 0, 0.5, 1.5 and 2.5. Draw a graph showing the points that represent the values of x and y.

3 Draw the graph representing the time (x minutes) to fill the water and the depth of the water (y cm).

4 How many cm is it if the time is 9.5 minutes?

Answer _____

5 How many minutes does it take if the depth is 22 cm?

Answer _____

2 This table summarizes the relationship between the length x cm and the width y cm of a rectangle with an area of 60 cm^2. Let's consider the relationship between x and y.

1 How can you express the relationship between x and y in the math sentence if the length is x cm and the width is y cm?

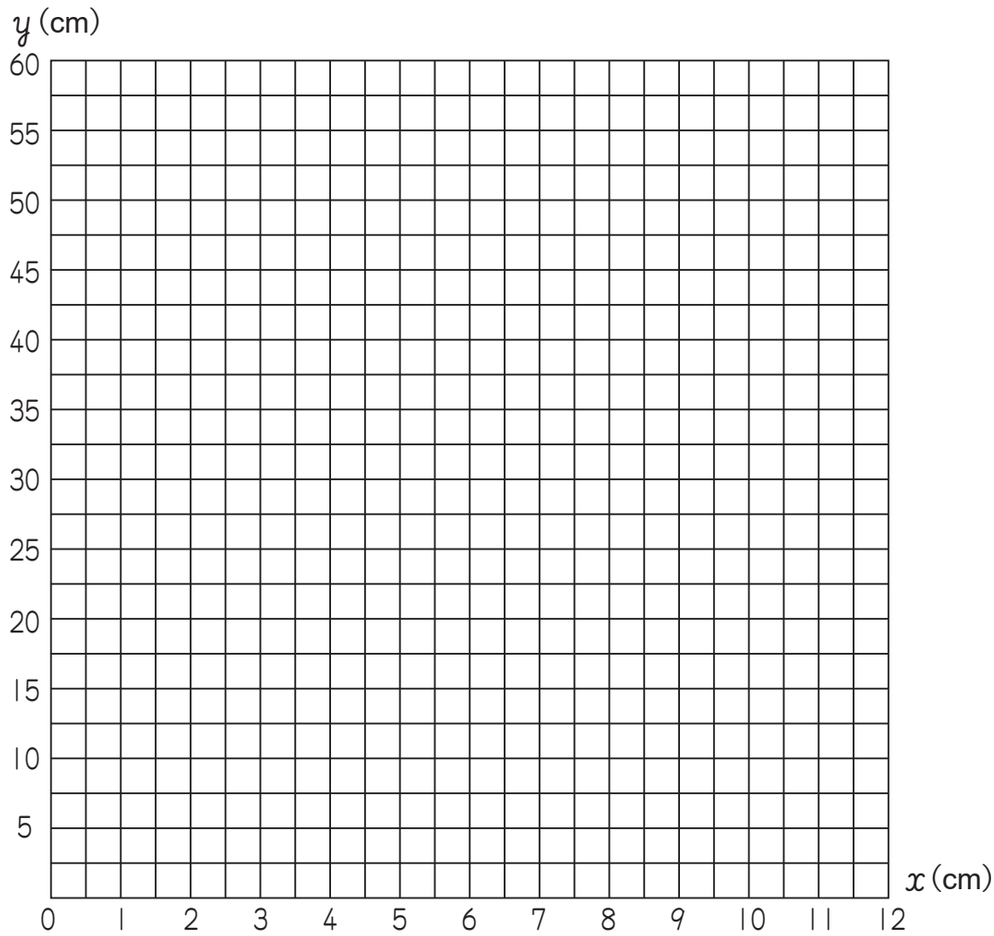
Answer _____

$x \times y =$ fix number.
So $y =$ fix number $\div x$

2 Fill in the table.

Length x (cm)	1	2	3	4	5	6	8	10	12
Width y (cm)	60								

3 For the length of x cm and a corresponding width of y cm, draw a point on the graph below that represents a pair of values of x and y.



4 Fill in the table, if x is 1.5, 2.5, 7.5.

Length x (cm)	1.5	2.5	7.5
Width y (cm)			