12 - 1 Ways of Ordering (1)

Instruction How to put items in order without missing an item or choosing the same item twice. Three students, Emma, John, and Mia are relay athletes.

> Well, Emma, John, Mia
> Then, Mia, John, Emma…Oh it isn't very easy. Do you have any good ideas to decide?

Consider the case where Emma runs first. How will the running order of John, Mia, and Steven be decided?
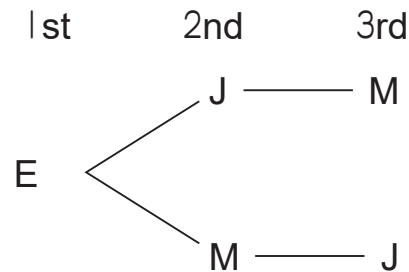
> To write down their names easily, you can use symbols such as
> For example, E for Emma, J for John, M for Mia, and S for Steven.

Idea 1. Using a table.	Idea 2. Using a diagram.

Idea 1. Using a table.

1st	2nd	3rd
E	J	M
E	M	J

Idea 2. Using a diagram.

1st   2nd   3rd

```
            J ——— M
E  <
            M ——— J
```

> Using a table or diagram is helpful when ordering items.

The following tips are effective for ordering items.
- Use symbols to write down their names easily.
- Choose the first item.
- Use tables and diagrams to avoid mistakes.

Example Draw tables and diagrams where the first runner is John or Mia respectively.

John is the first runner.

1st	2nd	3rd
J	E	M
J	M	E

1st   2nd   3rd

J — E — M

J — M — E

Mia is the first runner.

1st	2nd	3rd
M	J	E
M	E	J

1st   2nd   3rd

M — J — E

M — E — J

2 In total, how many ways of ordering are there?

In each case, we can find two ways.

6
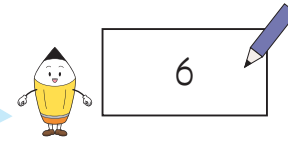
A farmer delivers her crops to the market, hotel, and a customer's house. As for how to decide the delivery order, let market, hotel, and customer's house be M, H, and C, respectively, and answer the following questions.

1 Fixing the first delivery place, draw tables and diagrams.

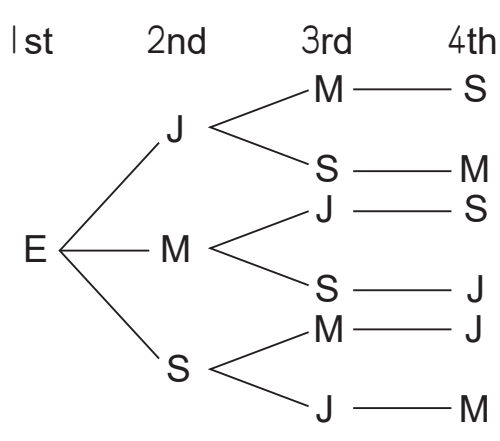| 1st	2nd	3rd

1st   2nd   3rd

M

H

C

2 In total, how many ways of ordering are there?
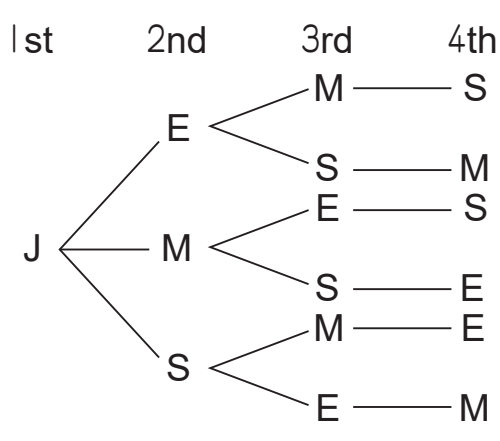
12 - 2 Ways of Ordering (2)

Example Four students, Emma, John, Mia, and Steven are relay athletes. Decide the running order. Answer the following questions.

1st	2nd	3rd	4th
E	J	M	S
E	J	S	M
E	M	J	S
E	M	S	J
E	S	M	J
E	S	J	M
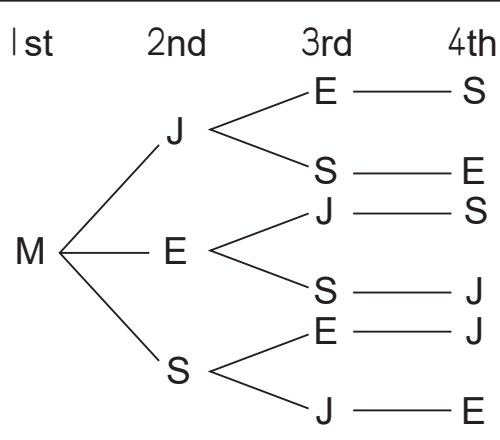
```
1st      2nd      3rd      4th
                  M ——— S
         J <
                  S ——— M
                  J ——— S
E <      M <
                  S ——— J
                  M ——— J
         S <
                  J ——— M
```

1st	2nd	3rd	4th
J	E	M	S
J	E	S	M
J	M	E	S
J	M	S	E
J	S	M	E
J	S	E	M

```
1st      2nd      3rd      4th
                  M ——— S
         E <
                  S ——— M
                  E ——— S
J <      M <
                  S ——— E
                  M ——— E
         S <
                  E ——— M
```

1st	2nd	3rd	4th
M	J	E	S
M	J	S	E
M	E	J	S
M	E	S	J
M	S	E	J
M	S	J	E

```
1st      2nd      3rd      4th
                  E ——— S
         J <
                  S ——— E
                  J ——— S
M <      E <
                  S ——— J
                  E ——— J
         S <
                  J ——— E
```

1st	2nd	3rd	4th
S	J	M	E
S	J	E	M
S	M	J	E
S	M	E	J
S	E	M	J
S	E	J	M

```
1st      2nd      3rd      4th
                   M ——— E
              J <
                   E ——— M
                   J ——— E
  S <     M <
                   E ——— J
                   M ——— J
              E <
                   J ——— M
```

2 In total, how many ways of ordering are there?   24

When we fix the first order, there are 6 ways to order.

A student visits a zoo to see four of her favorite animals: Rhino, Elephant, Giraffe, and Leopard. Rhino, Elephant, Giraffe, and Leopard are R, E, G, and L, respectively.

1 If you see the Rhino first, decide the visiting order by drawing tables and diagrams.

1st	2nd	3rd	4th

```
1st      2nd      3rd      4th
                          ____
                     <    ____
                          ____
      <              <
                          ____
                     <    ____
                          ____
```

2 Fill in the blanks with numbers.

In case we visit the Rhino first, there are ☐ ways.

There are ☐ cases.

And there are ☐ ways each. So in total, there are ☐ ways.

12 - 3 Ways of Ordering and Combination
Ways of Ordering (3)

Example 1 There are red, yellow, green and blue crayons. If you colour the flag below, how many ways of colouring are there?
Let red, yellow, green, and blue be R, Y, G, and B respectively.

Left Right

Colouring →

Left	Right
R	Y
R	G
R	B
Y	R
Y	G
Y	B

Left	Right
G	R
G	Y
G	B
B	R
B	Y
B	G

Left Right
R — Y
R — G
R — B
G — R
G — Y
G — B

Left Right
Y — R
Y — G
Y — B
B — R
B — Y
B — G

There are [12] ways in total.

1 There is a cookie, gum, potato chips, and a donut in a box.
Let cookie, gum, potato chips, and donut be C, G, P, and D respectively.
Complete the diagram and find the total ways of ordering.

C — G / P / D
G — C / P / □
P — C / □ / □
D — □ / □ / □

Left Right

There are [ ] ways in total.

Example 2 There is one card for each of the following numbers: 1, 3, 5, 7. From these 4 cards, use 3 cards to create 3-digit whole numbers. How many whole numbers can you make in total?
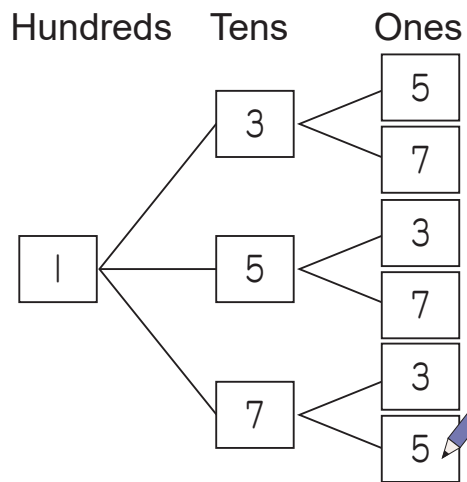
| 1 | 3 | 5 | 7 |

Hundreds   Tens   Ones

① Complete the table and diagram below when [1] is in the hundreds place.

Hundreds	Tens	Ones
1	3	5
1	3	7
1	5	3
1	5	7
1	7	3
1	7	5

Hundreds Tens Ones

1 → 3 → 5, 7
1 → 5 → 3, 7
1 → 7 → 3, 5

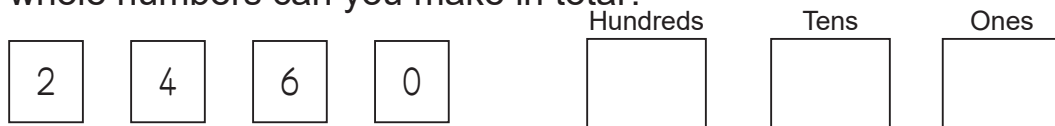**2** How many 3-digit whole numbers can you make in total?

When 1 is in the hundreds place, there are [6] ways.

There are [4] cases where we can put the number in the hundreds place.

And there are [6] ways each. So in total, there are [24] ways.

2 There is one card for each of the following numbers: 2, 4, 6, 0.
From these 4 cards, use 3 cards to create 3-digit whole numbers. How many whole numbers can you make in total?

[2] [4] [6] [0]

Hundreds   Tens   Ones
[   ]   [   ]   [   ]

1 Complete the table and diagram below when [2] is in the hundreds place.

Hundreds	Tens	Ones
2	4	6
2	4	
2	6	
2	6	
2		
2		

Hundreds Tens Ones

2 → 4 → 6, [ ]
2 → 6 → [ ], [ ]
2 → [ ] → [ ], [ ]
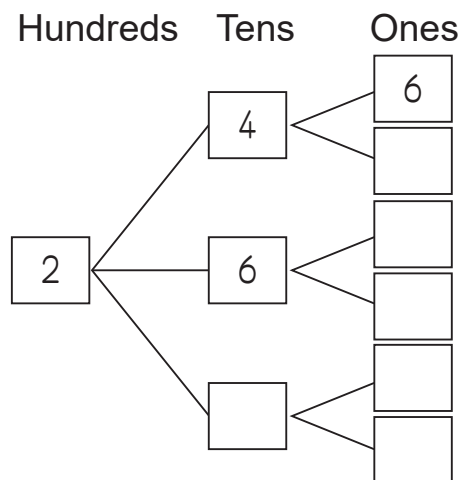
2 How many 3-digit whole numbers can you make in total?

[   ] numbers

If you place 0 in the hundreds place, you cannot make a 3-digit whole number.

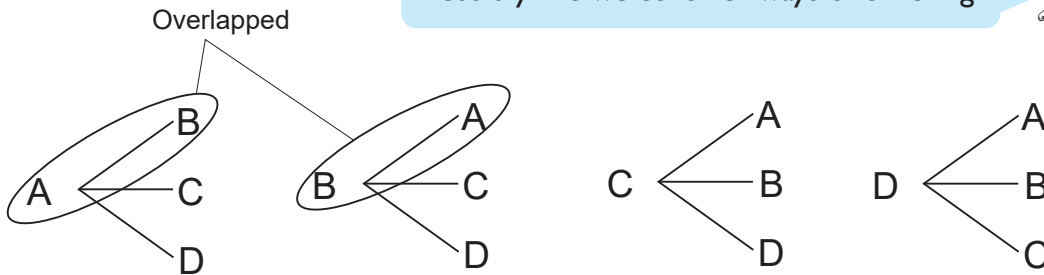**Instruction** Ways of combination.
Four teams (Team A, B, C, D) play football. If each team competes against the other team only one time, how many games will be played in total?

Let's try like we consider ways of ordering

Overlapped

B
A —— C
D

A
B —— C
D

A
C —— B
D

A
D —— B
C

Given that each team competes only once, A − B and B − A are overlapped.
Do we have any other ideas to make a table or diagram?

Idea 1. Using a table. Look at the horizontal array and find pairs of teams, like A − B, A − C, and A − D. Add a check mark.	Move on to the next array and continue to find pairs. For example, B − A is overlapped with A − B.

	A	B	C	D
A		✓	✓	✓
B				
C				
D				

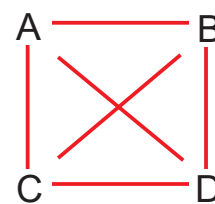|	A	B	C	D
A		✓	✓	✓
B			✓	✓
C				✓
D				

Idea 2. Using a diagram. Place A, B, C, D to form square	Draw lines to connect pairs and write them. A − B, A − C, A − D, B − C, C − D, and B − D

A   B

C   D

A ——— B

C ——— D

The same as when exploring ways of ordering, use a table or diagram to erase one of the repeated combinations and then count the total cases.

Example Three teams（Team A, B, C）play football. If each team competes with the other teams only one time, answer the following questions.

1 Complete the table and diagram below.

	A	B	C
A		✓	✓
B			✓
C			

When we consider pairs among three teams, we can draw a triangle.

2 In total, how many combinations are there?
According to the table or diagram above, the ways are A – B, A – C, and B – C.

3

There are 5 types of pastries, Apple pie, Banana cake, Chocolate cake, Donut, and Egg tart. Two types were bought. Answer the following questions. Let Apple pie, Banana cake, Chocolate cake, Donut, and Egg tart be A, B, C, D, and E, respectively.

1 Complete the table and diagram below.

	A	B	C	D	E
A		✓	✓		
B					
C					
D					
E					

2 Write down the combinations and find the total number.

Combinations

The number of cases

12 - 5　Combination (2)

Example 1　There is one card for each of the following numbers: 2, 4, 6, 8. From these 4 cards, add 2 cards to create whole numbers. Answer the following questions.

1　Complete the table and diagram below.

	2	4	6	8
2		✓	✓	✓
4			✓	✓
6				✓
8				

2　Write down the combinations and find the total number.

Combinations

$2 + 4 = 6, 2 + 6 = 8, 2 + 8 = 10$
$(4 + 6 = 10), 4 + 8 = 12, 6 + 8 = 14$

The number of cases　　5

1　There is one card for each of the following numbers: 1, 3, 5, 7. From these 4 cards, add 2 cards to create whole numbers. Answer the following questions.

1　Complete the table and diagram below.

	1	3	5	7
1				
3				
5				
7				

1	3
5	7

2　Write down the combinations and find the total number of cases.
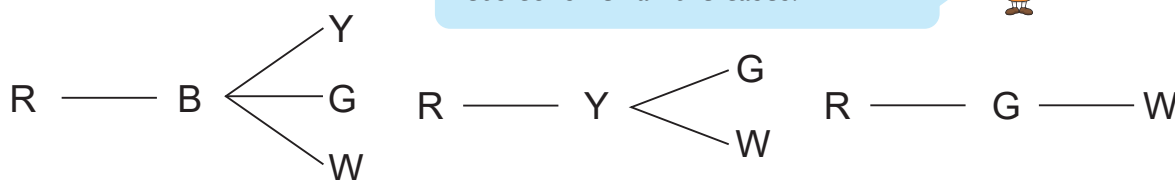
Combinations

The number of cases

Example 2 There are 5 coloured balls: red, blue, yellow, green, and white. From these 5 balls, pick up 3 balls and put them into a bag. Write down the combinations and find the total number of cases. Let red, blue, yellow, green, and white be R, B, Y, G, and W respectively.
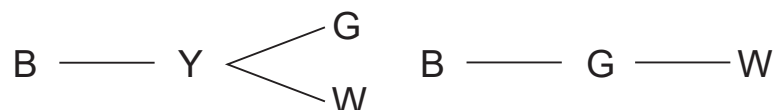
| R | B | Y | G | W |

Assuming red is picked,

> By fixing a case and using a diagram, let's consider all the cases.

$$R \longrightarrow B \begin{cases} Y \\ G \\ W \end{cases} \qquad R \longrightarrow Y \begin{cases} G \\ W \end{cases} \qquad R \longrightarrow G \longrightarrow W$$

Assuming blue is picked, and red is not picked,

$$B \longrightarrow Y \begin{cases} G \\ W \end{cases} \qquad B \longrightarrow G \longrightarrow W$$

Assuming yellow is picked, and red and blue are not picked,

$$Y \longrightarrow G \longrightarrow W$$

> Since we pick 3 balls, there is no case where green or white is picked, while red, blue, and yellow are not picked.

The number of cases | 10 |

2 There is one coin for each amount: 1 zed, 10 zeds, 50 zeds, and 100 zeds. From these 4 coins, choose 3 coins and find the total amount of money. ("zed(s)" is the fictional currency unit.)
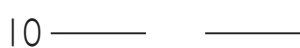
| 1 | 10 | 50 | 100 |

1 Start with 1 zed and write down all the different combinations of money.

$$1 \longrightarrow \quad < \qquad\qquad 1 \longrightarrow \quad \longrightarrow$$

The combinations of money

2 When you use 10 zed and do not use 1 zed, write down all the different combinations of money.

$$10 \longrightarrow \quad \longrightarrow$$

The combinations of money

3 How many the different combinations of money are there in total?

Combination (3)

Example 1　Four students, Emma, John, Mia, and Steven are nominees for student council. From these 4 students, 3 students will be elected. Write down the combinations of selected students. Let Emma, John, Mia, and Steven be E, J, M, and S respectively.

"Select 3 students out of 4 students" also means the same as "do not select 1 student out of 4 students".

E	J	M	S

Selected　　　Not selected
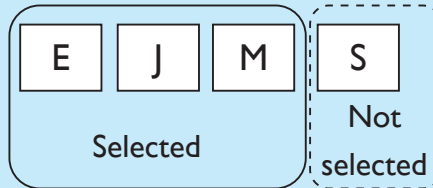
Let's consider cases where "not selected 1 student out of 4 students" is.

Combinations

E – J – M, E – J – S, E – M – S, J – M – S

The number of cases　　4

1　There are 5 different coloured flours: red, blue, yellow, purple, and white. From these 5 flowers, choose 4 flowers and make a bouquet. Write down the combinations and find the total number of cases. Let red, blue, yellow, purple, and white be R, B, Y, P, and W respectively.

R	B	Y	P	W

Combinations

The number of cases

Example 2 You go to a school canteen for lunch. You can choose one from each category A, B, C. How many different cases are there? (Let the name of foods represent their first letter.)

```
        F
      B
        S
        F
C     R
        S
      P F
        S
        F
      B
        S
        F
F     R
        S
      P F
        S
```

A	Chicken	Fish
B	Bread	Rice	Potato
C	Fruits	Salad

The number of cases |2

2 You go to a café for afternoon tea. You can choose one from each category A, B, C.

1 Complete the diagram below. Let the name of foods represent their first letter. If there are the same first letter, use second letter to distinguish.

```
C     Ch
      Cu

T
```

A	Coffee	Tea
B	Pancake	Chocolate cake	Cupcake
C	Fruits	Salad

2 Find the number of cases you can choose.

The number of cases

Review

1 There is one card for each of the following numbers: 1, 2, 3, 4.
From these 4 cards, use 3 cards to create 3-digit whole numbers. How many whole numbers can you make in total?

| 1 | 2 | 3 | 4 |

| Hundreds | Tens | Ones |

1 Complete the table and diagram below when | 1 | is in the hundreds place.

Hundreds	Tens	Ones
1	2	
1	2	
1	3	
1	3	
1		
1		

Hundreds　Tens　Ones

2 How many 3-digit whole numbers can you make in total?

2 There is one card for each of the following numbers: 1, 2, 3, 0.
From these 4 cards, use 3 cards to create 3-digit whole numbers. How many whole numbers can you make in total?

| 1 | 2 | 3 | 0 |

| Thousands | Hundreds | Tens | Ones |

1 Complete the table and diagram below when | 2 | is in the thousands place.

Thousands	Hundreds	Tens	Ones
2	1		
2	1		
2	3		
2	3		
2			
2			

2 How many 3-digit whole numbers can you make in total?

_____ numbers

3 There are 5 students, A, B, C, D, E, and F. 2 students were elected as school council members.

1 Complete the table below.

	A	B	C	D	E	F
A						
B						
C						
D						
E						
F						

2 Write down the combinations and find the total number.

Combinations

The number of cases

4 There is one coin for each: 1 zed, 50 zeds, 100 zeds, and 500 zeds. From these 4 coins, choose 3 coins and find the total amount of money.
("zed(s)" is the fictional currency unit.)

| 1 | | 50 | | 100 | | 500 |

1 When you use 1 zed, write down all the different combinations of money.

1 ——— < ———   1 ———   ———

The amount of moneys

2 When you use 50 zed and do not use 1 zed, write down all the different combinations of money.

50 ———   ———   The amount of moneys

3 How many the different combinations of money in total?