Financial Internal Rate of Return (FIRR)[1]

—Revisited—

1. Introduction

The FIRR is an indicator to measure the financial return on investment of an income generation project and is used to make the investment decision. The general approach to calculating the FIRR has long been discussed and seems well-established in such a way that the cash flow analysis induces uniformly the FIRR. While this may hold true, a closer look at the FIRR from a different investor's point of view can result in a different implication for the FIRR. This is the very issue which we deal with in this paper.

2. Definition of FIRR

The FIRR is obtained by equating the present value of investment costs (as cash out-flows) and the present value of net incomes (as cash in-flows). This can be shown by the following equality.

$$I_0 + \frac{I_1}{(1+r)^1} + \frac{I_2}{(1+r)^2} + \cdots + \frac{I_m}{(1+r)^m} = \frac{B_1}{(1+r)^1} + \frac{B_2}{(1+r)^2} + \cdots + \frac{B_m}{(1+r)^m}$$

namely,

$$\sum_{n=0}^{m} \frac{In}{(1+r)^n} = \sum_{n=1}^{m} \frac{Bn}{(1+r)^n}$$

where; I_0 is the initial investment costs in the year 0 (the first year during which the project is constructed) and $I_1 \sim I_m$ are the additional investment costs for maintenance and rehabilitation for the entire project life period from year 1 (the second year) to year m.

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1. This paper is a revised version of the original paper prepared by Mr. Koji Fujimoto (Chief Representative of the OECF Jakarta Office) in collaboration with Mr. Kazuhiro Suzuki (C.P.A.,Chuo Audit Corporation, Japan) as an internal office document in October, 1994 for the former Japanese ODA institution, the Overseas Economic Cooperation Fund of Japan (OECF).

$B_1 \sim B_m$ are the annual net incomes for the entire operation period (the entire project life period) from year 1 (the second year) to year m.

By solving the above equality, we can obtain the value of r and this r is the Financial Internal Rate of Return(FIRR).

3. Income Statement as Basis of Cash Flow Analysis

The FIRR represents the level of financial return on the investment and, therefore, the investor's main concern centers around expected cash in-flows. In identifying and projecting cash flows from an income generating project, say, Project X, an Income Statement (Profit and Loss Statement) with some qualification is commonly employed.

In an attempt to prepare a sample income statement, let us assume that Project X is featured as follows.

(i) Construction of the project is completed during the first year (the year 0).
(ii) Operation of the project starts from the second year (the year 1) and lasts for 5 years (the project life / the operation period is 5 years).
(iii) Initial investment costs amount to $ 200 million (No additional investments including working capital are required throughout the operation period).
(iv) The Straight Line Method is applied to calculate depreciation and the scrap value of the project is zero.
(v) The income tax rate is 50 %.
(vi) Initial investment costs are financed by shareholders ($ 150 million) in the form of equity capital investment and by banks ($ 50 million) in the form of loans.
(vii) The shareholders receive dividends annually at the end of year throughout the operation period.
(viii) The interest rate of the loan(s) is 10 % per annum and its principal is repaid by equal annual installments at the year end with interest throughout the operation period.
(ix) Annual operating income is $ 60 million throughout the operation period.

Based on these assumptions, we can complete the following income statement of Project X (Table 1) for the sake of the cash flow analysis.

Table 1 Income Statement

(Unit : $ million)

Year	0	1	2	3	4	5
Operating Income	-	60	60	60	60	60
Depreciation	-	40	40	40	40	40
Interest	-	5	4	3	2	1
Income before Tax	-	15	16	17	18	19
Tax	-	7.5	8.0	8.5	9	9.5
Net Income	-	7.5	8.0	8.5	9	9.5

This income statement, however, differs from regular accounting practices, in that the Operating Income is defined as one from which depreciation is not deducted. In other words, the annual operating income of $ 60 million is the income " before depreciation." This is because (a) depreciation is not a cash flow, but a mere accounting cost, while the operating income has to be a real cash flow for the cash flow analysis, and (b) in addition, in order to calculate tax (which is a cash flow), depreciation as an eligible cost for its calculation has to be deducted from taxable income which is shown as Income before Tax in Table 1.

4. FIRRs

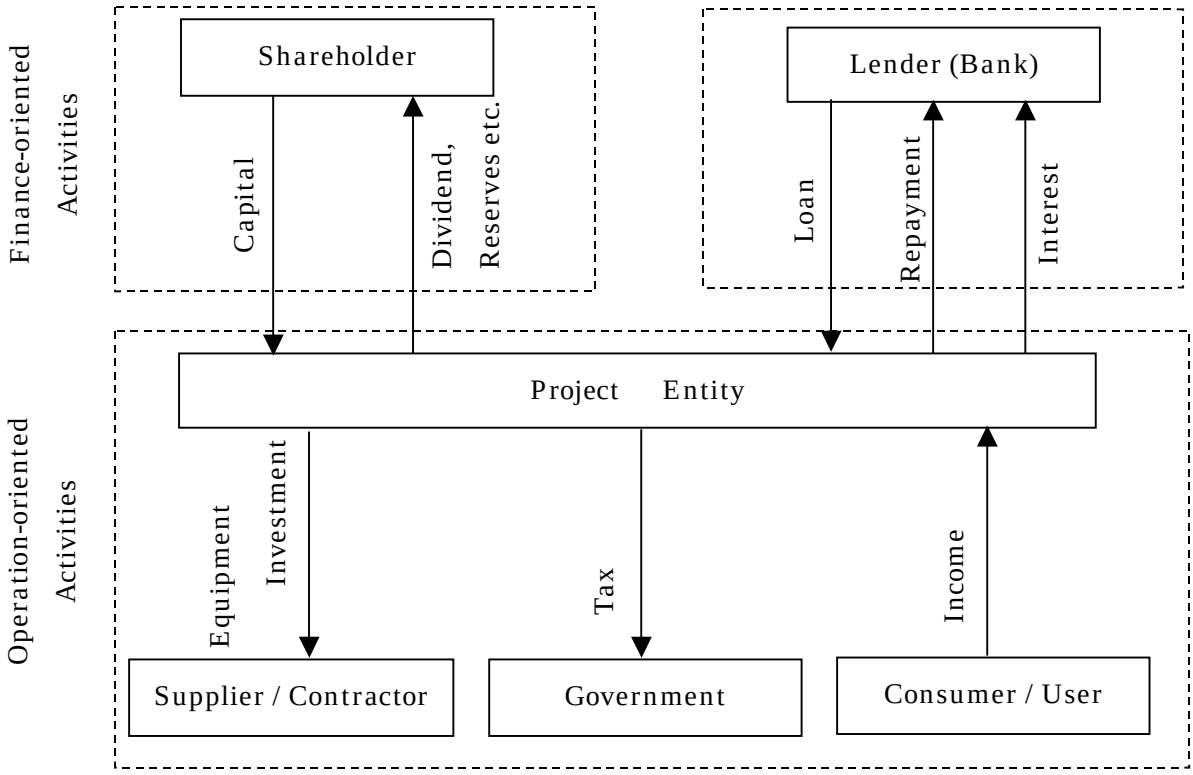
The simple case of the investment project assumed above and its income statement projection summarized in Table 1 lay the foundations for the cash flow analysis whose eventual purpose is to estimate FIRRs.

In addition to the cash flows identified in the discussion above, Project X produces several other cash flows. To understand these cash flows more easily and clearly, Figure 1 would be of great help. Project X involves at least six stakeholders; namely, the project entity (such as a joint venture company), the shareholder, the lender, the supplier/contractor, the government and the consumer/user, and cash flows are recognized individually by each one of these stakeholders. The project

entity deals with all the cash transactions, while the shareholder invests equity capital and receives dividends, the lender extends a loan and receives principal repayment and interest payment, the supplier/contractor builds production facilities and receives payment, the government receives tax, and consumer/user buys products.

In understanding the project a step further, we should note that there exist two kinds of distinct activities perceived by the project entity. One is finance-oriented activities which relate directly to the movement of investment funds between the project entity and the investors such as the shareholder and the lender. The other is operation-oriented activities which relate to business transactions between the project entity and the related parties such as the supplier/contractor, the government and the consumer/user. The former activities are shown in the upper half of Figure 1, and the latter in the lower half of the same figure.

Figure 1 Cash Flows of Project X



（１）Overall Cash Flow Analysis (Involving Operation-oriented as well as Finance-oriented activities) by Project Entity

Figure 1 identifies all the cash flows of Project X. Let us, then, seek the FIRR by taking all those cash flows into account. Based on the assumptions given in section 3 above together with an additional assumption that the dividend is $ 2 million per annum throughout the operation period, we can obtain in Table 2 in which out-flows of cash from the project entity are as negative and in-flows to it as positive.

During the year 0, the investment funds collected in the form of equity capital and the loan are spent on procurement and construction of production facilities. Consequently, the net cash flow is zero. From year 1 to year 5, the project entity receives constantly the net cash inflows, which means that it enjoys a good cash position throughout the project life.

Thus, in this case it becomes apparent that the equality discussed in section 2 above cannot be solved, where the net cash flows on investment is zero and the annual net cash flow during the operation period is positive. That is,

$$\sum_{n=0}^{m} \frac{\mathrm{In}}{(1+\mathrm{r})^{n}} = 0$$

and

$$\sum_{n=1}^{m} \frac{\mathrm{Bn}}{(1+\mathrm{r})^{n}} = \frac{35.5}{(1+r)^{1}} + \frac{36.0}{(1+r)^{2}} + \frac{36.5}{(1+r)^{3}} + \frac{37.0}{(1+r)^{4}} + \frac{37.5}{(1+r)^{5}}$$

In other words, once we take into account all the occurrences of cash flow of any project, we cannot obtain the FIRR. This, in turn, implies implicitly that the cash flow should be analyzed from the investor's point of view (the finance-oriented activities) and/or the operator's point of view (the operation-oriented activities).

Table 2　Overall Cash Flows Recognized by Project Entity

(Unit : $ million)

Year	0	1	2	3	4	5	Total
Operating Income-	-	60	60	60	60	60	300
Principal Repaym't	-	△10	△10	△10	△10	△10	△50
Interest	-	△5	△4	△3	△2	△1	△15
New Loan	50	-	-	-	-	-	50
New Capital	150	-	-	-	-	-	150
Investment	△200	-	-	-	-	-	△200
(Income) Tax	-	△7.5	△8.0	△8.5	△9.0	△9.5	△42.5
Dividend	-	△2	△2	△2	△2	△2	△10
Net Cash Flow	-	35.5	36.0	36.5	37.0	37.5	182.5

(2)　Cash Flow Analysis by Shareholder

The equity capital investor (shareholder) makes an investment decision based on a reasonable and expected rate of return on capital investment.　The main concern of the shareholder is how much the project yields in disposable income, which often takes a form of dividend, for the shareholder.

This case is illustrated in the dotted rectangle at the upper left-hand corner of Figure 1 and the investor's cash flows are projected in Table 3.　The net cash flows during the operation period show the available funds that the project entity can dispose of for the shareholder.　These funds, therefore, may either be totally expended as dividend or else divided between dividend and reserves which are disposed of at later opportunity.

Table 3　Cash Flows Recognized by Shareholder

(Unit : $ million)

Year	0	1	2	3	4	5	Total
Operating Income	-	60	60	60	60	60	300
(New) Capital	△150	-	-	-	-	-	△150
Repayment	-	△10	△10	△10	△10	△10	△50
Interest	-	△5	△4	△3	△2	△1	△15
(Income) Tax	-	△7.5	△8.0	△8.5	△9.0	△9.5	△42.5
Net Cash Flow	△150	37.5	38.0	38.5	39.0	39.5	42.5

The net cash in-flows shown in Table 3 can, thus, be regarded as the maximum disposable income (dividend) for the shareholder.

By equating the present value of equity capital investment and the present value of net cash inflows during the operation period (namely, the equality discussed in section 2 above), we can obtain a FIRR of 8.9 %.

(3) Cash Flow Analysis by Lender

From the view point of the lender (or bank), what is of most concerns is securing the repayment of the loan (principal) and payment of interest. It is usually the case that conditions of a loan for any project are usually agreed upon beforehand between the project entity and the lender. In our present case of Project X, we assumed that the interest is 10 % per annum and the repayment period is 5 years with equal annual installments.

This case is illustrated in the dotted rectangle at the upper right-hand corner of Figure 1 and the cash flows are sorted out in Table 4. By inserting the values of the net cash flow in the present value equality in section 2 above, we obtain the FIRR of 10 % which is equal to the assumed interest rate. It always holds true that the FIRR and the interest rate are the same as long as the interest rate is pre-determined and repayment of the loan principal and payment of interest are secured. Needless to say, the lender could be satisfied to invest if Project X is well-prepared and projected to guarantee the capital repayment and the interest payment.

Table 4 Cash Flows Recognized by Lender

(Unit : $ million)

Year	0	1	2	3	4	5	Total
(New) Loan	△50	-	-		-	-	△50
Repayment	-	10	10	10	10	10	50
Interest	-	5	4	3	2	1	15
Net Cash Flow	△50	15	14	13	12	11	15

(4) Operation-oriented Cash Flow Analysis by Project Entity

From the viewpoint of the project entity, the actual concerns is whether the project entity can make Project X profitable and viable through its production and sales operations. And this case is illustrated in the dotted large rectangle at the lower half of Figure 1. The project entity recognizes projected cash flows as shown in Table 5. The total investment is $ 200 million and, during the operation period, substantial net cash flows are expected. We should, however, carefully observe that these positive net cash flows are expected to suffice simultaneously for principal payment and interest payment to the shareholder and dividend payment to the lender. In other words, Operating Income after Tax (which is Net Cash Flow) during the operation period is the sum of the principal repayment, interest payment and dividend. Based on the net cash flows in Table 5, we can analyze a level of return on the entire investment by the project entity and obtain the FIRR of 9.1 %.

Table 5 Operation-oriented Cash Flows Recognized by Project Entity

(Unit : $ million)

Year	0	1	2	3	4	5	Total
Investment	△200	-	-	-	-	-	△200
Operating Income	-	60	60	60	60	60	300
(Income) Tax	-	△7.5	△8.0	△8.5	△9.0	△9.5	△42.5
Net Cash Flow*	△200	52.5	52.0	51.5	51.0	50.5	57.5

*Operating Income after Tax = Repayment + Interest + Dividend (and Reserves)

(5) Three Types of FIRRs

As discussed above, there exist three types of FIRRs; that is, the FIRR on equity capital (the FIRR of the shareholder), the FIRR on loan (the FIRR of the lender/bank) and the FIRR on total investment (the FIRR of the project entity). In principle, therefore, it is possible to calculate these three FIRRs in any income generating project. In practice, however, it is not necessarily required to obtain all of the three FIRRs. Rather, it would be important to identify what kinds of

investments (investors) are involved and calculate the FIRR selectively for the specific decision making purposes.

Generally speaking, however, if the FIRR of the project entity, namely, the FIRR on total investment, is at a certain satisfactory level, 10 % and over, for instance, the other two FIRRs will be guaranteed and be within the similar vicinity. We may, therefore, say that this must have been the reason the FIRR on total investment is treated as if it is the only FIRR, without any particular mention and specification.

5. Further Considerations

The FIRR which is scrutinized in the pre-investment study plays a crucial role in judging the financial feasibility and viability of the project, or in other words, in making an investment decision. In understanding the practical implication of the FIRR, however, a variety of further considerations are required.

(1) Sensitivity Analysis vis-à-vis Assumed Variables

We have already learned that the FIRRs are obtainable on a variety of assumptions. These assumptions usually contain variables. When some particular variables alter, values of the FIRR may change substantially. In such cases, investment decision can be influenced in one way or another. In the cash flow analysis, therefore, it is of utmost importance to scrutinize these variables, justify their appropriateness and understand their limitation. We should not forget that the variables are identified not only from the similar assumptions set forth in section 3 above, but also from a detailed projection of Income Statement（Profit and Loss Statement ） which includes such items as “Sales,” “Cost of Sales,” “Selling and Administrative Expenses,” “Non-Operating Income” and “Non-Operating Expenses,” Statement of the Cost of Goods Manufactured (or Services Produced) and Surplus Statement. In this connection, change of the inflation rate and that of the exchange rate are often taken into consideration as a factor to alter some of the variables to an unavoidable degree.

We may obtain different values of the FIRR, when, for instance, the following variables are altered;

(i) disposal of the shares

(ii) disposal of the reserves

(iii) tariff rates (or prices of manufactured goods).

Thus, it is a general practice to recalculate the FIRR time and time again by substituting a single or a set of responsive and influential variables to lessen risks involved. This is, in fact, what we call the “sensitivity analysis.”

(2) Other Considerations

In addition to the sensitivity analysis, there are at least the following two aspects which should be carefully analyzed.

First, although the relationship between the three FIRRs is briefly stated in (5) of section 4 above in the “friendly to each other” manner, we should not rule out a possibility of a more complicated and conflicting relationship between them as projects have their own characteristics. It is recommended, therefore, to examine their relationship and the impact of one FIRR on another or the others.

Second, when a new project is planned by a newly established venture company (the project entity), in other words, when the new project is the only business venture for the company, the analytical framework discussed in this paper is fully applicable. However, if the new project is an additionality to the other business operations the company is undertaking, then, it becomes imperative to analyze overall impacts on the entire (operational and) financial status of the company.

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