APPENDIX: Model for Developing Global Growth Scenarios

This study estimates GDP as a function of labor force, capital stock, and total factor productivity for 187 countries between 2013 and 2050 under three different growth scenarios, the "Convergence Scenario", "Business as Usual Scenario", and the "Downside Scenario". This section offers an abbreviated description of the model; a more detailed exposition, in Kohli, Szyf, and Arnold (2012), is available on request.[90]

As seen in equation (1), a Cobb-Douglas function with constant returns to scale is assumed, with α equal to two-thirds:

$$GDP = TFP \times L^{\alpha} \times K^{1-\alpha} \qquad (1)$$

where *TFP* is total factor productivity, *L* is labor, and *K* is capital stock.

GDP figures are generated for three different measures: real GDP (constant 2010 prices); GDP PPP (constant 2010 PPP prices); and GDP at expected market exchange rates, which incorporates expected exchange rate movements and serves as the best proxy for nominal GDP.

The model first estimates annual real GDP growth for each country between 2013 and 2050. These estimates are applied to the previous values of real GDP, GDP PPP, and a measure equal to nominal GDP deflated by US inflation (on which GDP at market exchange rates is based) to derive the full series. Finally, to derive GDP at market exchange rates, real exchange rate changes are estimated and multiplied by nominal GDP deflated by US inflation to obtain GDP at market exchange rates.

Labor force growth stems from population growth and from changes in labor force participation rates. Population growth is based on the medium variant of the 2010 Revision of the UN's World Population Prospects, while labor force participation rates are projected separately, by gender, for seven age cohorts (15–19, 20–24, 25–29, 30–49, 50–59, 60–64, and 65+) to better capture cohort-specific trends. Male rates are projected directly; female rates are derived by projecting the difference between male and female rates for each age group. Labor force participation rates from 1980 through 2012 are taken from the International Labor organization.

The cross-country, cohort-specific equations to forecast male rates are simple autoregressions of the following form:

$$\ln(M_{age,t}) = m_{age} \times \ln(M_{age,t-1}) \qquad (2)$$

where M is the percent of males in age group age who are active in the labor force and m age is a constant that varies for each age group.

[90] This appendix is based on Kohli, Szyf, & Arnold (2012)

The cross-country, cohort-specific equations to forecast the differentials between male and female participations rates are:

$$\ln(D_{age,t}) = d_{age} \times \ln(D_{age,t-1}) \tag{3}$$

where D_{age} equals the difference between the percentage of males in age group age in the labor force and the percentage of females in age group age in the labor force, and d_{age} is a constant that varies by age group. In both male and female models, for certain cohorts, rough upper or lower bounds are incorporated to address outliers. Observations that begin in 2012 beyond these bounds are not governed by the regressions but instead gradually converge over time towards the bounds.

Capital stock growth, based on an initial capital stock and yearly investment rates and depreciation, is defined as:

$$(1 + K\ Growth_t) = \frac{K_t}{K_{t-1}} = \left(\frac{I_{t-1}}{K_{t-1}}\right) - 0.06 \tag{4}$$

where *K* is the capital stock, 0.06 represents the yearly depreciation of 6%, and I_{t-1} is the capital investment from the previous year, which is defined as the previous year's GDP (measured in constant 2010 PPP dollars) multiplied by the investment rate as a share of GDP.

The initial capital stock is calculated using the Caselli method, with the following equation:

$$K_0 = \frac{I_0}{g + 0.06} \tag{5}$$

where *K* is the initial capital stock, *g* is the average GDP growth over the subsequent ten years, 0.06 is the depreciation rate, and I is the initial year's investment. For *I*, for each country, the earliest year for which there exists capital investment data (year y) is identified. The average of the investment rate values for year y and the two subsequent years is computed and treated as the initial investment rate. This smoothing out of fluctuations in the initial investment rate yields better estimates for certain countries with high volatility in the earliest investment rate values. This rate is then multiplied by the GDP in year y to determine I. The earliest year possible is chosen for this estimate because the longer the timeframe before the projections commence the more the yearly depreciations will reduce the effects on the model of any initial imprecisions in capital estimates.

The model is calibrated by calculating total factor productivity (TFP) for an initial year (2012)[91] based on labor force, capital stock, and historical GDP, with GDP and capital stock measured in purchasing-power-parity dollars at constant 2010 PPP prices. For subsequent years, TFP is projected.

For the TFP projections, we differentiate four categories: rich or developed; converging; non-converging; and fragile. All countries begin with a default TFP growth rate of 1 percent which, with a strong level of statistical significance, equals the average US rate over the past 40-, 30-, 25-, and 20-years, and which, also with a strong level of statistical significance, equals the average rate of all non-converging countries over the same four periods. In our model, this is the fixed rate of productivity growth for the category of non-converging countries. For this study, different scenarios have been created in which each country's convergence status falls under varying sets of assumptions. These scenarios are described in detail later in this section.

Research shows that some growth differences between developing countries can be successfully modeled by separating them into two groups: converging and non-converging countries (Gill and Kharas, 2007).

A country is deemed to be converging at the start of the projection period if its per-capita income has rapidly converged over a 20-year period to that of best practice economies or if its 2001–2011 TFP growth is closer to what the model would predict for a converger (see below) than to what it would predict for a non-converger; the lower a country's productivity relative to the global best practice, the faster the rate at which it converges. This convergence reflects technology transfers from richer innovating countries, technology leapfrogging, the diffusion of management and operational research from more developed countries, and other ways that a country can shortcut productivity-improvement processes by learning from economies that are already at the productivity frontier.

In the model, the lower a converging country's productivity relative to that of the US, the larger the boost and the quicker the pace of catch-up.[92] The productivity growth of 14 of the 36 rich countries is treated the same as that of converging countries. Non-converging countries and 22 of the 36 rich countries maintain the default 1 percent yearly productivity growth and hence experience no convergence boost. The rich countries are divided into these categories based on their past TFP performance. The general equation for TFP growth is:

$$TFP\ Growth = 1.0\% + CB - FP \qquad (6)$$

where *CB* is the convergence boost benefiting "converging" countries and *FP* is the productivity growth penalty suffered by fragile states.

[91] IMF WEO GDP growth projections are used for 2012 and 2013.

[92] TFP is used in the convergence term instead of the per-capita income used by others for three reasons: first, if the equation were to use GDP per capita, over time the TFP of a converging country would not converge to that of the US but instead to other values. Also, since the convergence equation represents convergence of TFP, we use TFP in order to make the equation consistent with its purpose. Third, using the convergence coefficient from past research in tandem with an income-based convergence term yields large discrepancies with the recent historical data for TFP growth for many countries; using TFP yields a better fit.

The convergence boost is defined as follows:

$$CB = c \times 2.69\% \times \ln\left(\frac{TFP_{USA,t-1}}{TFP_{i,t-1}}\right) \qquad (7)$$

where *i* is the country, 2.69 percent is the convergence coefficient (derived from historical data), *TFP* is total factor productivity, and *c* takes a value between 0 and 1 and identifies whether a country is treated as a converger (c =1) or as a non-converger or fragile state (c =0), or in an intermediate state of transition between being a converger and non-converger (0 < c < 1).

The productivity growth penalty for fragile states, FP, is defined as:

$$FP = f \times 1.5\% \qquad (8)$$

where *f* plays a role analogous to that of c in equation (7) above. For each fragile country, f is set equal to 1, corresponding to a penalty in productivity growth of 1.5 percent, so that its productivity is assumed to fall by 0.5 percent a year. The coefficient of negative 1.5 percent is derived by identifying state failures and debilitating wars prior to the global financial crisis that lasted at least 2 consecutive years in 44 countries, and analyzing their effects on growth. The list of fragile states in Africa is the harmonized list prepared by the African Development Bank and the World Bank.

The projections of GDP growth are completed by applying the labor growth, capital deepening, and productivity changes to each country over the period 2013–2050.

The measure of GDP at expected market exchange rates adjusts the GDP estimate by expected changes in the real exchange rate. First, an equation is derived to establish a theoretical relationship between a country's real exchange rate and its PPP income relative to that of the US. Then, the country's modeled exchange rate converges towards the value that corresponds to its income in this theoretical equation. These relationships are not linear, and the countries for which increases in GDP PPP per capita lead to the largest appreciation of their real exchange rates are the countries whose incomes are between a third and two-thirds that of the United States, and not the poorest or richest countries.

The model also projects the sizes of the low, middle, and high-income populations, again following Kharas, by measuring the number of people in each country with living standards—in PPP terms—within a certain absolute range. An income distribution for each country is derived from the World Bank's International Comparison Program.

The model calculates what share of the nation's income is available for consumption, and it distributes this consumption income over the population according to the income distribution. As the country's overall consumption income increases, the purchasing power of those at the bottom of the distribution increases, raising more to middle-income status.

For purposes of computing consumption income classes, the model projects changes in the share of the country's income available for consumption using the following equation:

$$\ln(C_{i,t}) = \alpha_1 \times \ln(C_{i,t-1}) + \alpha_2 \times \ln(GDPPCCap_{i,t}) + \alpha_0 \qquad (9)$$

where *t* is the year, *i* is the country, *C* is the ratio of consumption to GDP, *GDPPCCap* is the minimum of each country's GDP PPP PC and $50,000 PPP (in 2010 PPP international dollars), and α_0, α_1 , and α_2 are constants.

The study makes separate projections for three different growth scenarios: the "Convergence Scenario", the "Business as Usual Scenario", and the "Downside Scenario". The difference between the scenarios is how countries are classified at the outset, either as converging, non-converging, or fragile, and how countries gradually transition between classifications.

For 145 countries the initial classification is based on the Kharas classification and for an additional 42 countries on a similar analysis of recent data. Under this classification, four African countries (Botswana, Cape Verde, Mauritius, and Mozambique) are classified as "convergers".

For the Convergence Scenario, a group of 15 additional African countries join the convergers listed above, and are referred to in the study as "early convergers". This group of 15 countries begins to converge this decade (up to 2020). An additional group of 15 "late convergers" begins converging in the following decade (up to 2030). The remaining 20 countries currently considered "fragile" transition out of fragility over the next 30 years.

Under the "Business as Usual Scenario" the convergence picture remains the same as today. Four African countries that are currently converging are assumed to continue converging through 2050. All current non-convergers continue to not converge, and all fragile countries remain fragile.

In the "Downside Scenario" an additional five countries become fragile, non-convergers do not converge, and the four convergers stop converging. This scenario also includes cyclical fluctuations in Africa's terms of trade. Specifically starting in 2015 the terms of trade deteriorate by 15 percent over 5 years and then recover by 15 percent over the subsequent 10 years, after which this cycle repeats.

In all three scenarios, the transition of individual countries between converging and non-converging, or from fragile to non-converging, is gradual. That is, countries are made to adopt an intermediate state between fragile and not-fragile or between converging and non-converging, by varying the values of f and c in equations (7) and (8).