# HOW TO PREPARE FOR COLUMN METHOD IN FOUNDATION PHASE

- Prepare our learners for fundamental skills of calculation -

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### Introduction

• 79.5% in grade 5 and 60.3% in grade 7 learners rely on unit counting (Schollar. 2008)

#### We need to let learners move away from unit-counting in addition and subtraction.

We propose the following methods to understand base-ten number system conceptually:

- Subitising
- Make-a-ten method
- Column method

## Outline of today's activity

We do the following activities today.

Agenda	Contents • Subitising	Duration
Introduction	What is necessary for four basic operations in the Make-a-ten method?	0 minutes
Activity 1 (hands-on)	<ul> <li>In activity 1, we do:</li> <li>instant recognition of numbers up to 10 using a ten-frame; and</li> <li>calculation with bottle tops and ten-frames using the make-a-ten method.</li> </ul>	15 minutes
Activity 2 (hands-on)	<ul> <li>In activity 2, we calculate with:</li> <li>printed tens and hundreds</li> <li>printed tens and bottle tops; and</li> <li>column method.</li> </ul>	15 minutes
Discussion/Feedback	What advantages and disadvantages did the participants find?	10 minutes
Wrap up	Conclude the workshop.	5 minutes

## Basic Knowledge and skills for Activity 1-1

We use subitising with a ten-frame.

#### What is subitising?

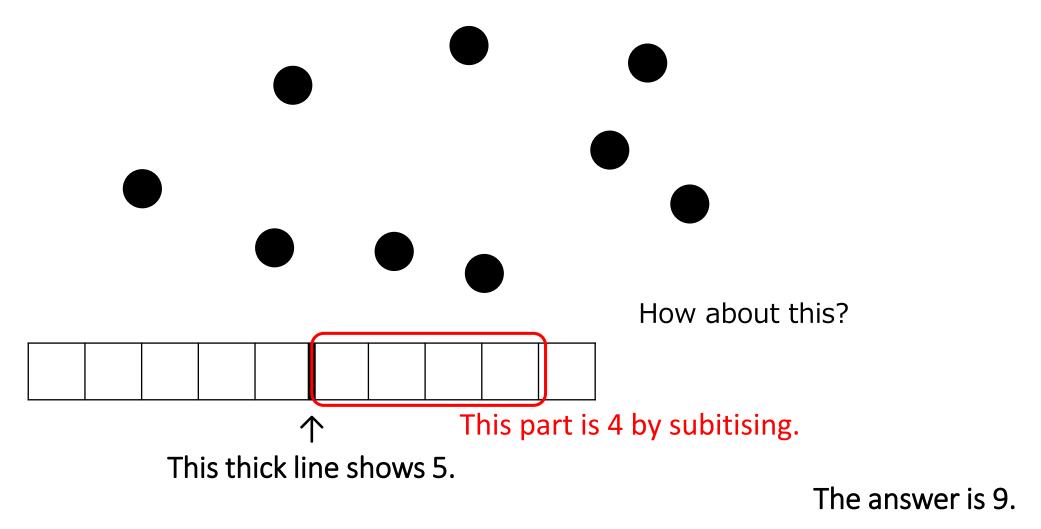
Subitising is the instant recognition of the number of objects in a collection without counting them.

What is a ten-frame?

A ten-frame is a frame showing 10 boxes. Each of the box holds a bottle top.

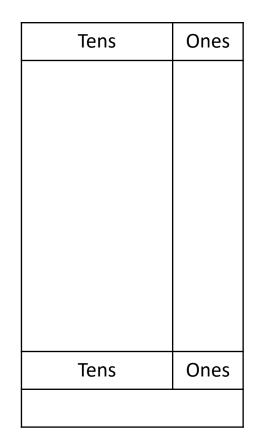
## Basic Knowledge and skills for Activity 1-1

Can we know the number of dots without counting?



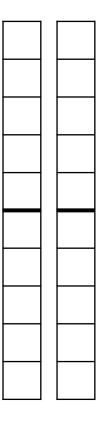
### Base-ten kit

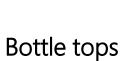
The following is the components of base-ten kit.



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Place-value table Hundreds

Tens

Ten-frames

•

<Tool>

10 bottle tops and a ten-frame.

<Steps>Work in pairs.

- 1. One of the participants take bottle tops (any number).
- 2. The other participants answer how many **without counting them**.
- 3. Check the answer by placing them in a ten-frame.
- 4. Repeat 1-3 by turn.

Do NOT count bottle tops in a ten-frame!!

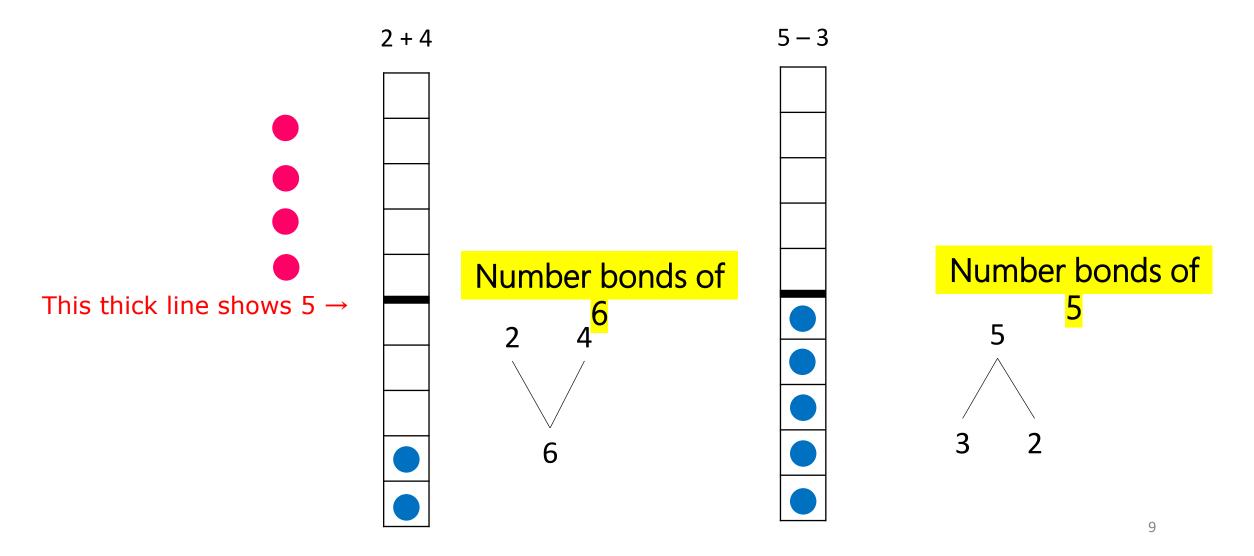
Addition and subtraction up to 20 are divided into the following types.

Туре	Addition	Subtraction
Without carrying or borrowing	(1-digit) + (1-digit) < 10 e.g. 2 + 4 = 6	(1-digit) – (1-digit) e.g. 5 – 3 = 2
	(numbers 10-18) + (1-digit) < 20 e.g. 12 + 4 = 16	(number 11-18) – (1-digit) ≧ 10 e.g. 15 – 3
With carrying and borrowing	(1-digit) + (1-digit) ≧ 10 e.g. 9 + 4	(number 11-18) – (1-digit) < 10 e.g. 15 – 9
		<u> </u>

They are inverse operations.

## Without carrying or borrowing (1-digit)

(1-digit) + (1-digit) < 10 and its inverse.



## Activity 1-2 (5 minutes)

<Tool>

Bottle tops with 2 ten-frames.

<Steps> Work in pairs.

- 1. Solve the following using a base-ten kit.
  - 12 + 4
  - 15 3
- 2. Solve the following using a base-ten kit.
  - 9 + 4
  - 15 9
- 3. Find the difference between 1 and 2.

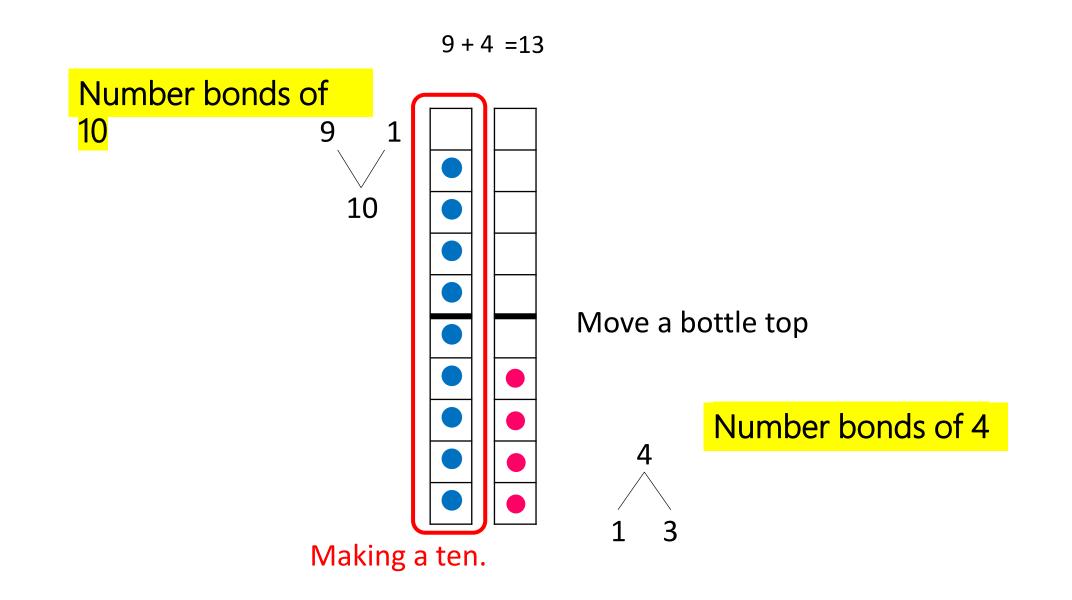
Do NOT count bottle tops in a ten-frame!!

## Without carrying or borrowing (1-digit)

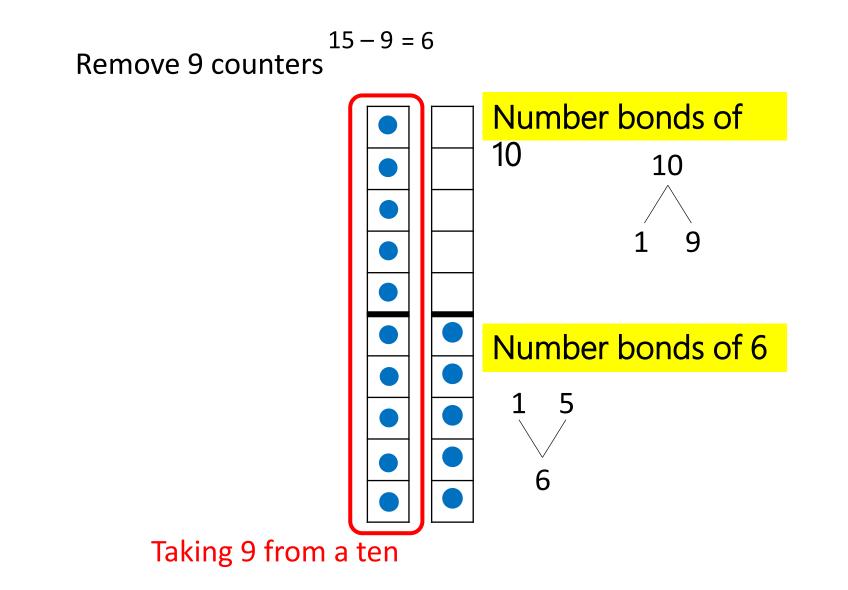
(numbers 10-18) + (1-digit) < 20 and its inverse.

15 – 3 =12 12 + 4 = 16No change in a ten.

## Addition with carrying (Make-a-ten Method)



## Subtraction with borrowing (Make-a-ten Method)



## Summary of Activity 1-2

- 1. No change in a ten.
  - 12 + 4 (addition without carrying)
  - 15 3 (subtraction without borrowing)
- 2. Making a ten.
  - 9 + 4 (addition with carrying)
  - 15 9 (subtraction with borrowing)

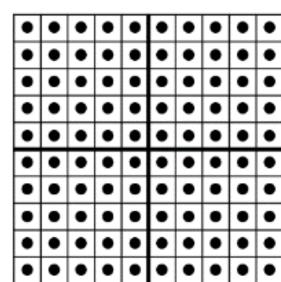
2. is called "make-a-ten" method.

## Basic Knowledge and skills for Activity 2

We will work on (2-digit number)  $\pm$  (2-digit number) with a base-ten kit as well as the column method.

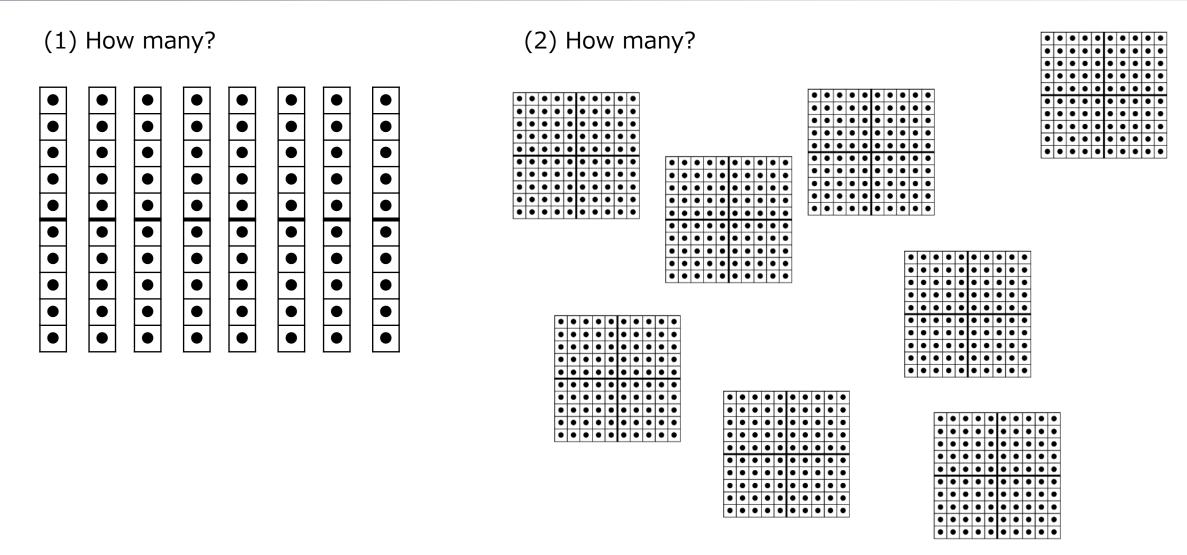
We use printed tens and hundreds.

a printed ten



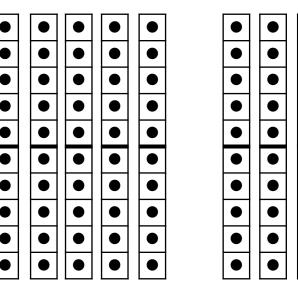
a printed hundred

### How to organise tens and hundreds better



### How to organise tens and hundreds better

#### (1) How many?



(2) How many?

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5 tens and 3 tens make 8 tens

#### 5 hundreds and 3 hundreds make 8 hundreds

Organise tens and hundreds in groups of 5. This will help you subitise numbers.

<Tool>

10 printed tens and 2 printed hundreds.

<Steps> Work in pairs.

Solve the following using a base-ten kit.

- 1. 40 + 30
- 2. 80 20
- 3. 110 + 40
- 4. 240 100

Activity 2-1

1.40 + 30 = 70• • • • • • • • • • • • •

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3.110 + 40 = 150•
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$$2.80 - 20 = 60$$

4. 240 - 100 = 140

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## Column method

Four basic operations are the foundation of numbers and operations.

We recommend the column method for the four basic operation because:

- Algorithm is simple;
- It represents base-ten number system;
- It is all-round; and
- It is universal method of calculation.

When the sum exceeds ten, the ten is carried to the next place.

$$\begin{array}{r}
1 & 1 & 1 \\
3 & 4 & 8 & 7 & 6 \\
+ 4 & 3 & 8 & 7 & 5 \\
\hline
7 & 8 & 7 & 5 & 1
\end{array}$$

We recommend introducing the column method for the four basic operation in early grade.

Why the column method in early grade?

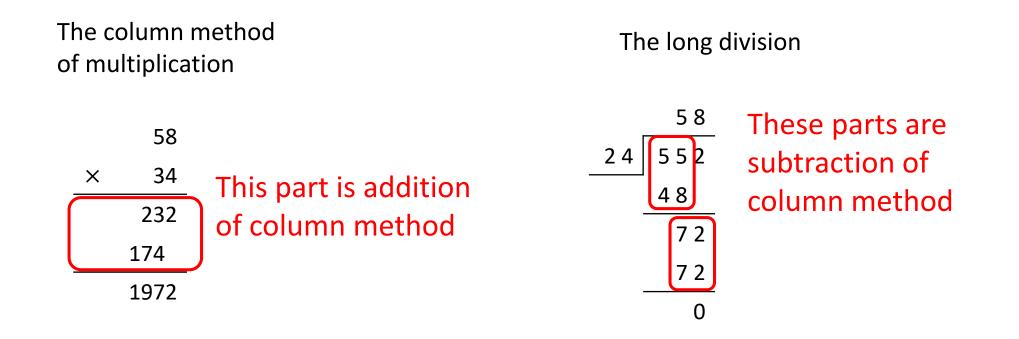
Because:

- it is easy to learn in small number; and
- learners have readiness for it.

## Column method

Addition and subtraction in columns are the keys to learning the four operations.

Why the column method of addition and subtraction is critical?



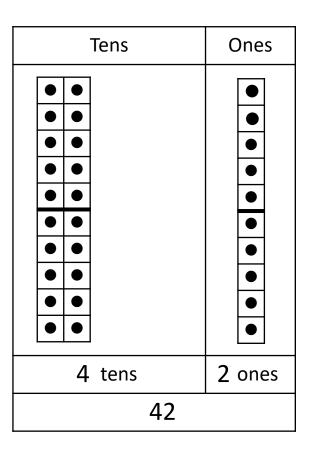
Because they are used in the column method of multiplication and long division.

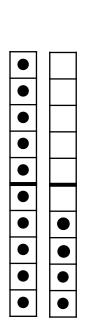
## Activity 2-2 (5 minutes)

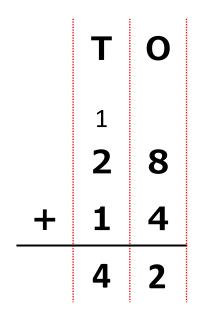
<Tool>

- 5 printed tens and 15 bottle tops.
- <Steps> Work in pairs.
- Solve the following using a base-ten kit.
  - 1. 28 + 14
  - 2. 53 26

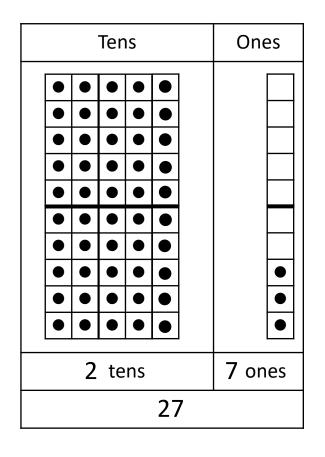
## Activity 2-2 (28+14)

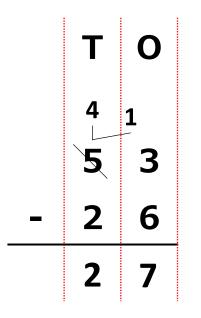






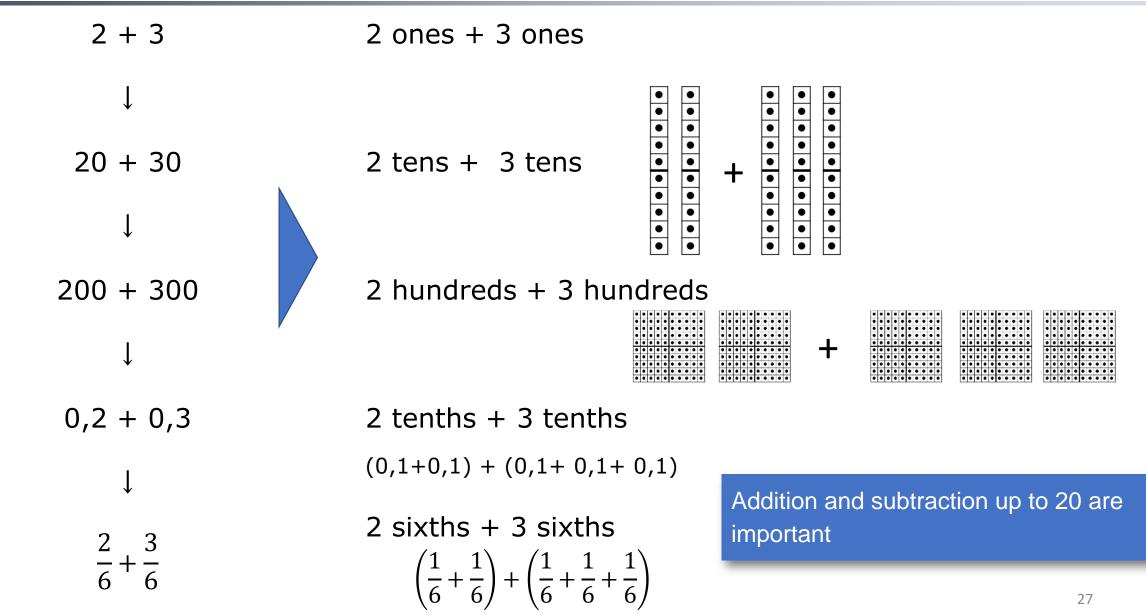
## Activity 2-2 (53-26)





- Using a base-ten kit helps learners move away from counting.
- Showing learners steps of the column method using a base-ten kit.
- Addition and subtraction up to 20 are crucial, which will be used in higher grades repeatedly. (See the next slide)

## Importance of addition and subtraction up to 20 [1]



## Importance of addition and subtraction up to 20 [2]

How many types of additions involves carrying?

	Without carrying (patterns)	With carrying (patterns)	With carrying (%)
(1-digit) + (1-digit)	55	45	45%
(2-digit) + (2-digit)*	3 025	6 975	69,8%
(3-digit) + (3-digit)*	166 375	833 625	83.4%

The more number increases, the more patterns of addition involve carrying.

\* (2-digit) + (2-digit) includes (2-digit) + (1-digit), (1-digit) + (2-digit) and (1-digit) + (1-digit)

## Bad Practices in TMU pilot

<Examples of Bad Practice>

Some teachers:

- counted bottle tops in a ten-frame. Bear in mind that the **thick line means 5**.
- let learners count bottle tops. Avoid counting as much as possible;
- did not organise tens and hundreds. Organise in groups of 5.

Always make groups of 5, 10, 100 etc.

This will help learners understand numbers as a group.

## Thank you so much!!

## ありがとうございました!

Arigato gozai masita.